An Economic Analysis of Compulsory and Voluntary Insurance

Kazuhiko SAKAI
Faculty of Economics
Kurume University
E-mail: sakai_kazuhiko@kurume-u.ac.jp

Mahito OKURA
Faculty of Economics
Nagasaki University
E-mail: okura@nagasaki-u.ac.jp

(Corresponding author)

ABSTRACT
This research analyzes an insurance market in which compulsory and voluntary insurance coexist. In particular, we investigate whether compulsory insurance provides an incentive to purchase voluntary insurance. The main conclusions of this article are as follows. When only voluntary insurance exists, we find that (1) an individual has a stronger incentive to purchase insurance when his or her future utility is high, (2) whether an individual’s incentive to purchase insurance becomes stronger when his or her initial wealth increases is ambiguous, and (3) an individual’s incentive to purchase insurance tends to become stronger when his or her initial wealth increases if both effort levels to lower the accident probabilities of individual in the case of higher and lower insurance coverage rates are relatively high. When both insurance coexist, we find that (1) when the compulsory insurance coverage rate is relatively low such that an individual may become personally bankrupt, introducing compulsory insurance increases the incentive to purchase voluntary insurance, (2) an increase in the coverage rate of compulsory insurance increases the incentive to purchase voluntary insurance when the compulsory insurance coverage rate is relatively low, and (3) when the compulsory insurance coverage rate is relatively high such that an individual never becomes personally bankrupt, introducing compulsory insurance does not provide an incentive to purchase voluntary insurance.

KEY WORDS
Compulsory insurance, voluntary insurance, economic analysis, incentive

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1. Introduction
All insurance can be classified into two categories. The first category is referred to as compulsory insurance, which all people must carry. For example, in many countries, some social securities, such as national medical insurance, national healthcare insurance, and national pension insurance, are compulsory insurance. The second category is referred to as voluntary insurance, which people can carry if they wish. Many kinds of insurance, such as fire insurance and life insurance, are included in this category.

There are some situations in which compulsory and voluntary insurance coexist. For example, in many countries, governments or other public sectors provide national medical insurance services. However, at the same time, some people purchase additional insurance services from
private insurance firms. That is, they can receive benefits from both public and private entities if payment conditions are met.

Given this situation, we first explain why compulsory insurance exists when voluntary insurance also exists. In economic analysis, the main explanation relates to asymmetric information in an insurance market. Suppose that an insurer cannot know each individual's risk type. In this case, the insurer proposes an average insurance rate, regardless of each individual's risk type. However, low-risk individuals evaluate this insurance rate as high and do not want to purchase insurance, whereas high-risk individuals are willing to purchase the insurance, regarding it as cheap. This well-known phenomenon is referred to as the adverse selection problem. In contrast, in the case of compulsory insurance, the adverse selection problem disappears because individuals do not have the right to refuse to purchase insurance. There are several studies that consider the efficiency of compulsory insurance under adverse selection, such as Pauly (1974), Johnson (1977), Dahlby (1981), Pitchford (1985), Balkenborg (2001), and Chen and Zhou (2010). In addition, Sandroni and Squintani (2007), Petretto (1999), and Hindriks (2001) discussed efficiency in the situation where compulsory and voluntary insurance coexist in a market with adverse selection.

These studies have provided reasonable explanations, but they investigated compulsory and voluntary insurance in an insurance market with adverse selection. In contrast, our research maintains that compulsory and voluntary insurance coexist even if there is no adverse selection in an insurance market. In particular, we will confirm whether compulsory insurance provides an incentive to purchase voluntary insurance.

2. The model

Suppose that an individual is faced with risks that will result in damage if they occur and that compulsory and voluntary insurance to protect the individual from the risk coexist. In this situation, the following three-stage game is considered.

First stage:

Government determines a compulsory insurance premium \( f_G \) and a compulsory insurance coverage rate \( q_G \).

Second stage:

The individual decides whether to purchase voluntary insurance. If he or she wants to purchase it, then the individual chooses the insurance coverage rate. The voluntary insurance premium and the insurance coverage rate are denoted by \( f_p \) and \( q_p \), respectively. Because over

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1 Of course, there are noneconomic reasons for both compulsory and voluntary insurance existing. For example, we can provide an explanation in terms of social policy. One purpose of social policy is to maintain the welfare of all national citizens. Income redistribution through social securities is a well-known method to prevent extreme income gaps between rich and poor. For example, in many countries, the insurance premium for national (public) medical insurance depends not on individuals’ risk levels, but on their income levels. In this situation, part of the insurance premiums paid by high-income individuals subsidizes the insurance premiums of low-income individuals. Thus, this situation is not the result of market discipline, but arises from a civil minimum legislated by the constitution.

2 The following explanation is mainly studied in information economics. For example, Rothschild and Stiglitz (1976) and Shavell (1979, 1982) are well-known pioneering works in the field.

3 In addition, there are some studies that use other approaches. For example, Briys et al. (1988) demonstrated how compulsory insurance affects the demand for voluntary insurance using portfolio theory. Hoel and Iverson (2002) investigated a genetic information problem in an insurance market in which compulsory and voluntary insurance coexist.
insurance is prohibited under insurance business law, the individual can only choose \( q_p \leq 1 - q_g \).
For simplicity of expression, we denote that \( f = f_g + f_p \) and \( q = q_g + q_p \).

Third stage:
The individual chooses his or her effort level \( x \) to lower his or her accident probability. Assume that \( x \) also represents the disutility from making an effort and that it is the private information of the individual. Then, \( p(x) \) represents the accident probability of the individual, and we assume that \( dp(x)/dx < 0 \), \( d^2p(x)/dx^2 > 0 \), \( dp(0)/dx = -\infty \), and \( dp(\infty)/dx = 0 \).

Next, we describe the situation when an accident occurs. If an accident occurs, the individual incurs a constant damage, denoted by \( h \). In this case, this individual can receive insurance money as a result of both compulsory and voluntary insurance. Thus, the individual finally has to bear \((1 - q)h\) by himself or herself. Assume that this individual has an initial wealth \( w \) that is larger than the insurance premium and smaller than the damage, that is, \( h > w > f \). Thus, there are some cases in which the individual becomes personally bankrupt if the accident occurs. The individual’s bankruptcy condition is written as \( w - f < (1 - q)h \). Let \( \hat{q} \) be the coverage rate that satisfies \( w - f = (1 - q)h \), that is, \( \hat{q} = (h - w + f)/h \). Thus, the individual becomes personally bankrupt if the accident occurs when the insurance coverage rate is \( q < \hat{q} \).

Furthermore, we assume that the individual loses future utility, which is denoted by \( \pi \), if he or she experiences personal bankruptcy. For example, suppose that the individual has some valuable assets such as a car or a house. The individual has to sell those assets to pay for the damage if he or she becomes personally bankrupt, and then the individual will not be able to obtain future utility from using his or her assets. In other words, the individual can obtain \( \pi \) only if he or she does not become personally bankrupt.

3. Analysis of a market without compulsory insurance

We first consider the situation in which only voluntary insurance exists. In this section, \( f = f_p \) and \( q = q_p \). In the third stage, there are two possible cases, \( q < \hat{q} \) and \( q \geq \hat{q} \).

In the case where \( q < \hat{q} \), if the accident does not occur, individual income is \((w + \pi - f)\). In contrast, if the accident occurs, individual income is zero because the individual becomes personally bankrupt. Thus, his or her expected income, which is denoted by \( u_1 \), can be written as follows:

\[
\begin{align*}
u_1 &= (1 - p(x))(w + \pi - f) + p(x) \cdot 0 - x \\
&= (1 - p(x))(w + \pi - f) - x.
\end{align*}
\]

Then, the individual’s optimal effort level, which is denoted by \( x_i \), is determined to satisfy the following first-order condition:

\[
-p'(x)(w+\pi-f) - 1 = 0.
\]  
(2)

In the case where \( q \geq \hat{q} \), if the accident does not occur, individual income is \((w + \pi - f)\). In contrast, if the accident occurs, individual income is \((w + \pi - f - (1 - q)h)\) because the individual does not become personally bankrupt even if the accident occurs. Given this, the individual’s expected income, which is denoted by \( u_2 \), can be written as follows:

\[
\frac{d}{dx}(x)(w)(\pi)(w+\pi-f) - 1 = 0.
\]  
(2)

Because \( h - w + f > 0 \) and \( w - f > 0 \), then it follows that \( \hat{q} \in (0,1) \) is satisfied.
$u_2 = (1-p(x))(w+\pi-f)+p(x)(w+\pi-f(1-q)h-x$}

$$= w+\pi-p(x)(1-q)h-x-f.$$ \hspace{1cm} (3)

Then, the individual’s optimal effort level, which is denoted by $x_2$, is determined to satisfy the following first-order condition:

$$-p'(x)(1-q)h-1=0.$$ \hspace{1cm} (4)

In the second stage, we consider the two possible cases, $q<\hat{q}$ and $q\geq \hat{q}$, separately.

In the case where $q<\hat{q}$, it is easy to verify that insurance premium $f$ depends on $q$. Let $f_1(q)$ be the insurance premium in the case where $q<\hat{q}$. The individual’s expected income, which is denoted by $U_1(q)$, can be rewritten as follows:

$$U_1(q) = (1-p(x_1))(w+\pi-f_1(q))-x_1.$$ \hspace{1cm} (5)

Whether equation (5) is an increasing or decreasing function of $q$ is ambiguous. However, we know that equation (5) is a decreasing function of $q$ if we introduce the natural assumption that $df_1(q)/dq > 0$. Because equation (5) is a decreasing function of $q$, the optimal insurance coverage rate is zero. In other words, the individual does not want to purchase any insurance. Then, substituting $q=0$ and $f_1(q)=0$ into equation (5), we show that:

$$u_1^* = U_1(q=0) = (1-p(x_1))(w+\pi)-x_1.$$ \hspace{1cm} (6)

In the case where $q\geq \hat{q}$, let $f_2(q)$ be the insurance premium. The individual’s expected income, which is denoted by $U_2(q)$, can be rewritten as follows:

$$U_2(q) = w+\pi-p(x_2)(1-q)h-x_2-f_2(q).$$ \hspace{1cm} (7)

Then:

$$\frac{dU_2(q)}{dq} = p(x_2)h - \frac{df_2(q)}{dq}.$$ \hspace{1cm} (8)

The first and second terms on the right-hand side of equation (8) represent marginal changes in the insurance money and the insurance premium, respectively. In order for the insurer to achieve a nonnegative expected profit, the insurance premium must outweigh the insurance money and, therefore, $p(x_2)h \leq df_2(q)/dq$. Thus, we find that $dU_2(q)/dq \leq 0$. Because equation (7) is a non-increasing function of $q$, the optimal insurance coverage rate is $\hat{q}$. Then, substituting $q=\hat{q}$ into equation (7), we show that:

$$u_2^* = U_2(q=\hat{q}) = w+\pi-p(x_2)(1-\hat{q})h-x_2-f_2(\hat{q}).$$ \hspace{1cm} (9)

Using $\hat{q}=(h-w+f)/h$, the condition under which the individual purchases insurance is:

$$\Delta U = U_2^*-U_1^* = [p(x_1)(w+\pi)+x_1]-[p(x_2)w+(1-p(x_2))f(\hat{q})x_2] \geq 0.$$ \hspace{1cm} (10)

From equation (10), there are two main results. First, $\Delta U$ increases when $\pi$ increases. This result can be easily understood because the individual cannot obtain any future utility if he or she becomes personally bankrupt, and we find that the individual has a stronger incentive to purchase insurance so as not to become bankrupt when his or her future utility is high.
The second result relates to the relationship between $w$ and $\Delta U$. In other words, if the individual’s initial wealth changes, how does his or her incentive to purchase insurance change? To investigate this, we partially differentiate equation (10) with respect to $w$. Then, we have:

$$\frac{\partial \Delta U}{\partial w} = p(x_1) - p(x_2) - \left(1 - p(x_2)\right) \frac{\partial \hat{f}(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w}. \quad (11)$$

Because $x_1$ and $x_2$ must satisfy the first-order conditions denoted in equations (2) and (4), we can show that:

$$-p(x_1) < -p(x_2) \Rightarrow p(x_1) > p(x_2) \Rightarrow p(x_1) < p(x_2). \quad (12)$$

However, because $\frac{\partial f(q)}{\partial q} > 0$ and $\frac{\partial q}{\partial w} = -1/h < 0$, then $\frac{\partial f(q)}{\partial q} \frac{\partial q}{\partial w} < 0$. Thus, the sign of equation (11) is not uniquely decided. From equation (11), we find that $\frac{\partial \Delta U}{\partial w} > 0$ ($\frac{\partial \Delta U}{\partial w} < 0$) is highly likely (unlikely) to be realized when both $x_1$ and $x_2$ are relatively large (small).\(^5\)

From these discussions, the following proposition can be derived.

**Proposition 1:**

Suppose there is a situation in which only voluntary insurance exists. From the analysis, the following results are derived.

1. The individual has a stronger incentive to purchase insurance when his or her future utility is high.
2. Whether an individual’s incentive to purchase insurance becomes stronger when his or her initial wealth increases is ambiguous.
3. The individual’s incentive to purchase insurance tends to become higher when his or her initial wealth increases if both effort levels to the accident probabilities of individual in the case of higher and lower insurance coverage rates are relatively high.

4. Analysis of a market with compulsory insurance

Next, we consider the situation in which compulsory and voluntary insurance coexist. In this situation, government compels individuals to carry compulsory insurance, the coverage rate of which is $q_G$. As well as carrying compulsory insurance, the individual can purchase additional voluntary insurance if he or she wants to do so. Thus, constraint $q \geq q_G$ must be satisfied.

First, consider the case where $q_G \geq \hat{q}$. In this case, the optimal effort level is $x_2$ because the individual never becomes personally bankrupt. Thus, the individual’s expected income in the second stage is depicted in equation (7) and the optimal coverage rate is $q_G$ because equation (7) is a decreasing function of $q$ and the constraint $q \geq q_G$ must be satisfied. This result means that the individual does not have an incentive to purchase voluntary insurance.

Next, consider the case where $q_G < \hat{q}$. In this case, there are two subcases regarding whether the individual may become personally bankrupt. First, we investigate the subcase in which the individual may become personally bankrupt. In this subcase, the optimal effort level is $x_1$, the

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\(^5\) This result can be derived from the assumption regarding the form of the accident probability function $p(\bullet)$. 

individual’s expected income in the second stage is as depicted in equation (5), and the optimal coverage rate is \( q^* \) because equation (5) is a decreasing function of \( q \) and constraint \( q \geq q^* \) must be satisfied. This result means that the individual does not have an incentive to purchase voluntary insurance. Moreover, we can easily check the following equation:

\[
U_1^* \equiv U_1(q=0) > U_1(q=q^*) \equiv U_1^G.
\]  

(13)

Next, we investigate another subcase in which the individual never becomes personally bankrupt. In this subcase, the optimal effort level is \( x^*_2 \), the individual’s expected income in the second stage is as depicted in equation (7), and we find that the optimal insurance coverage rate is \( \hat{q} \). Thus, the individual purchases \( q = \hat{q} - q^*_G \) voluntary insurance.

The condition under which the individual chooses the second subcase in which he or she purchases voluntary insurance can be written as follows:

\[
\Delta U^G \equiv U_2^* - U_1^G \geq 0.
\]  

(14)

From equation (13), we show that:

\[
\Delta U^G > \Delta U.
\]  

(15)

Equation (15) indicates that introducing compulsory insurance leads to a stronger incentive to purchase voluntary insurance. In addition, an increase in the coverage rate of compulsory insurance increases the incentive to purchase voluntary insurance because \( \Delta U^G \) is an increasing function of \( q^*_G \). However, if the coverage rate of compulsory insurance becomes too high, that is \( q^*_G \geq \hat{q} \), the individual does not have an incentive to purchase voluntary insurance.

From these discussions, the following proposition can be derived.

**Proposition 2:**

Suppose there is a situation in which compulsory and voluntary insurance coexist. From the analysis, the following results are derived.

1. When the compulsory insurance coverage rate is relatively low such that an individual may become personally bankrupt, introducing compulsory insurance increases the incentive to purchase voluntary insurance.
2. An increase in the coverage rate of compulsory insurance increases the incentive to purchase voluntary insurance when the compulsory insurance coverage rate is relatively low.
3. When the compulsory insurance coverage rate is relatively high such that an individual can never become personally bankrupt, introducing compulsory insurance does not result in an incentive to purchase voluntary insurance.

5. Concluding remarks

This research analyzed an insurance market in which compulsory and voluntary insurance coexist. In particular, this research investigated whether compulsory insurance provides an incentive to purchase voluntary insurance. The main conclusions of our article are as follows.

When only voluntary insurance exists, we found that (1) an individual has a stronger incentive to purchase insurance when his or her future utility is high, (2) whether an individual’s incentive to purchase insurance becomes stronger when his or her initial wealth increases is

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6 This is because \( U_2^* \) is constant and \( U_1^G \) is a decreasing function of \( q^*_G \).
ambiguous, and (3) an individual’s incentive to purchase insurance tends to become stronger when his or her initial wealth increases if both effort levels to lower the accident probabilities of individual in the case of higher and lower insurance coverage rates are relatively high.

When compulsory and voluntary insurance coexist, we found that (1) when the compulsory insurance coverage rate is relatively low such that an individual may become personally bankrupt, introducing compulsory insurance increases the incentive to purchase voluntary insurance, (2) an increase in the coverage rate of compulsory insurance increases the incentive to purchase voluntary insurance when the compulsory insurance coverage rate is relatively low, and (3) when the compulsory insurance coverage rate is relatively high such that an individual does not become personally bankrupt, introducing compulsory insurance is not associated with an incentive to purchase voluntary insurance.

Our analysis of the research results is that they suggest a reexamination of the existence of compulsory insurance. Further, this research has shed light on the relationship between compulsory and voluntary insurance in terms of the individual’s incentive to purchase voluntary insurance. However, there are several possible future extensions of this model. For example, the individual is implicitly assumed to be risk neutral in this article. If the individual is risk averse, we are interested in how the effect of compulsory insurance on the incentive to purchase voluntary insurance changes. Another example is that insurance premiums are an exogenous variable in our model. If insurance premiums are endogenous, particularly if compulsory and voluntary insurance premiums are determined through different mechanisms, how do our results change? The insurance premium in the case of voluntary insurance is determined to maximize a private insurance firm’s profits and market share. In contrast, the insurance premium for compulsory insurance is determined by several motives such as maximizing social welfare, maintaining a civil minimum, minimizing social inequalities, and so forth.

These points are still open questions. Much additional work is required to investigate the above aspects, and they are left to possible future research. However, several results in this article have important implications for the insurance market.

References


