Portfolio Optimization by Using Birds Flight Algorithm

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Abstract
Markowitz optimization problem so determination of investment efficient set, while the number of investable assets and constraints in the market is low can be solved by mathematical models. But when real-world conditions and limitations to be considered, portfolio optimization problem cannot solve easily using a methods of mathematics. For this reason, portfolio optimization using evolutionary algorithms has been an important topic of discussion today. The main objective of the present study is the solving of portfolio optimization problem (mean – variance model) by using Particle Swarm Optimization (PSO). for this purpose by use the price of 40 accepted shares in stock during the 1385 till 1390 years, is plotted the investment efficient set. Results from this study show that particle Swarm optimization method in portfolio optimization, despite the current limitations is successful.

Key words
Portfolio optimization, Particle Swarm Optimization movement technique, mean-variance model, efficient set

1. Introduction
One of the important issues has raised in the capital markets that should be considered by both investors, natural and legal persons is choosing optimize portfolio, in this regard reviews and study have been done by. Investors to select the best portfolio by attention to its risk and returns. Usually it is assumed that investors do not like risk and are risk-averse and always attempt to invest in assets that have max. returns and min. risk. In other words, investing returns are viewed as a favorable factor by investor and the variance of returns (risk) are considered as an undesirable element. When the number of assets and restrictions on investment in the market is low, the optimization problem is solved by mathematical models but when real-world conditions and limitations to be considered, portfolio optimization problem cannot easily be solved using mathematical methods. For this reason, initiative methods such as neural networks and evolutionary algorithms in portfolios optimizing, has been one major issue in recent times. The main purpose of survey is solution of portfolio optimization problem using the Particle Swarm Optimization method or birds flight algorithm.

2. Literature review
Harry, M. Markowitz In 1952 provides his model to selecting portfolio. Markowitz’s mean - variance Model is the most famous and prevalent approach in investment selecting matter.

Today, ultra-initiated methods are the matter of public interest for Complete Solution of optimization problems. in 2006, Makoto Koshino and his colleagues were use of birds flight algorithm to select optimize portfolio They do this, using of birds flight algorithm against Sublimation and genetic algorithms to choose optimal portfolio. The result of this research was led to the rejection of Chang and his colleagues in 2000
Chim and colleagues in 2009, were used a mimicking model of birds flight algorithms for finance computational applications. The purpose of this study was to predict the time series of stock prices and formation of optimal portfolio. Hanhng Zhou and Yi Wang and his colleagues in 2011, were applied the birds flight algorithm for bound optimal portfolio selection and showed that birds fly algorithm is more efficient than genetic algorithm.

In Iran Gholamreza Eslami Bidgoliand and Ahmad Talangy (1378) in their paper were considered the optimal portfolio choice programming models, and they generally came to the conclusion that the most appropriate framework for portfolio selection is portfolio modeling analysis based on multiple criteria techniques. Raii (1381) in an article entitled “portfolio formation for risk-love investor: comparison of neural network and Markowitz” is discussed about portfolio optimization problem.

In recent years several studies is carried out to develop methods of risk management based on the theory of value at risk. Khalouzadeh and Amini (1384) conducted research in regards that the optimal portfolio is obtained by using a genetic algorithm that has maximum of, while have bound on the portfolio risk. In 1381, Ramin Vatankhah was considered, control and optimize the cluster movement by a method inspired by nature, including flying birds, genetics, sublimation, neural networks and fuzzy neural networks in the thesis of his dissertation. This study revealed that the improved method of birds fly algorithm versus other methods is the best way to move a bunch of robots and this algorithm was used to making guard and control robots.

Morteza Ashhr (1390), predict the Tehran Stock Exchange Index using the birds fly algorithm and compare it with the traditional patterns in his master's thesis. And show that the amount of error variance for intelligent model is very small compared to best traditional models and error in the best tradition model, ARIMA, is more than of intelligent model.

### 3. Portfolio optimization problem

Markowitz's Mean –variance Model is the best-known and most popular approach in investment choice matter. The most efficient tool for the optimal portfolio choice is the mathematical programming model that is proposed by Markowitz. The most significant concerns in the Markowitz model is regarding to the risk of investing not only based on the standard deviation of the one share but based on risk of investing collection.

Markowitz model is based on the following assumptions:

1. Investors are risk averter and have an increasing expected utility and marginal utility of wealth curve is decreasing.
2. Investors, choose their portfolios based on the mea and variance of expected returns. So, Indifference curves are a function of the rate of return and the expected variance.
3. Each investment option is infinitely divisible.
4. Investors have time horizon "a period" and it is the same for all investors.
5. Investors at a certain level of risk prefer higher returns and vice versa, for a given level of returns, are demanding less risk.

In This model assets portfolio returns is equal weighted average return of each assets. The weight used for each returns would be ratio of done investment in the mentioned asset is as below:

\[
 r_p = w_1 r_1 + w_2 r_2 + \ldots + w_n r_n = \sum_{i=1}^{n} w_i r_i 
\]  

(1)

Returns of each asset in (1) relation is calculated by return arithmetic mean last periods.

\[
 r_i = \frac{\sum_{1}^{n} w_i}{N} 
\]  

(2)

Used weight for each returns would be ratio of done investing in mentioned asset and is as below:

\[
 w_i = (\text{amount of investing in I asset})/(\text{The total investment in portfolio}) 
\]  

(3)

Portfolio risk is function of single shares and covariance between the returns of the single shares. Portfolio risk is given in terms of variance for two share as following relationship:

\[
 \delta_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \times \text{cov}(r_1,r_2) 
\]  

(4)
4. Mean-variance Non-linear programming model

Mean-variance Model of this survey is according to the following equation:

\[
\begin{align*}
\text{Min} & \quad \sum_{i=1}^{n} w_i \times \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j \\
R & = \sum_{i=1}^{n} w_i r_i \\
\sum_{i=1}^{n} w_i & = 1 \\
0 \leq w_i & \leq 1
\end{align*}
\]  

The objective of relation (5) is to minimize portfolio risk in a certain level of returns. For the purpose of objective function formation use of different shares returns mean that studied in survey. To form of objective function of relation (5) is needed to covariance matrix.

\[
\sum_{i=1}^{40} w_i r_i = 15.27 w_{14} \cdot 1.95 w_{24} \cdot 1.47 w_{36} \cdot 15.85 w_{40}
\]  

5. Birds Flight Algorithm optimization method

Particle mass optimization algorithm is one of the most important algorithms that take place in the field of collective intelligence. Kennedy and Eberhart (1995) demonstrated that the PSO of each particle representing a possible solution that moves randomly in problem space Displacement of each particle in the search space is influenced by self knowledge and their neighbors. Therefore, the position of the other congestion particles influence on how searching of a particle. The results of this social behavior modeling are, searching process in which particles move into appropriate area. Particle Learn from each other in group and based on the gained knowledge move to side the best of their neighbors. The base of pso is on this: In every moment, every particle set its location in the search space due to it being the best place ever and the best place that exist in whole neighbors. The following equation is used to update rapidly:

\[
V[t+1]=v[t]+c1 \times \text{rand}(t) \times (\text{pbest}[t]-\text{position}[t])+c2 \times \text{rand}(t) \times (\text{gbest}[t]-\text{position}[t])
\]  

Position, v: Both are arrays to the length of problem dimensions number. Parameters are applied to following purpose:

V: Particle velocity
Position; Current location of the particle
Rand: A random number in the interval (0, 1)
C1, c2: Learning factors
T: Step down

The new position of the particle in each dimension calculated corresponding to the position update equation that is presented below:

\[
\text{Position} [t+1] = \text{Position}[t]+V[t]
\]  

They are same the parameters of updating velocity equation v: particle velocity, position: current position of particle that both are arrays to the length of problem dimensions number.

6. The process of investigation

Several steps are necessary for optimal portfolio selection, using the mean-variance model and birds flight algorithm optimization method. For the simplicity and understandability, the necessary steps are shown in the diagram (1). Descriptive statistics have been calculated of the studied companies by using of corporate quarterly returns. Table (1) is shown Descriptive statistics of selected companies, including the name of the stock, mean and standard deviation of each stock.
The optimal portfolio selection by birds flight algorithm

Basic concepts of risk and return, portfolio and more

Introduction to birds flight Algorithms optimization method

The selection of an appropriate statically society and sample

Gathering data survey from databases

Consists of the mean – variance model to portfolio optimization

Solving mean – variance model with bird flight algorithm

Determine the weight of each constituent stock portfolio

Determine the efficient set in mean - variance model

**Figure 1.** The process of survey

<table>
<thead>
<tr>
<th>S8</th>
<th>S7</th>
<th>S6</th>
<th>S5</th>
<th>S4</th>
<th>S3</th>
<th>S2</th>
<th>S1</th>
<th>stock</th>
<th>return</th>
<th>risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.53</td>
<td>9.74</td>
<td>13.59</td>
<td>5.10</td>
<td>2.30</td>
<td>1.47</td>
<td>1.95</td>
<td>15.27</td>
<td>return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.47</td>
<td>19.65</td>
<td>18.93</td>
<td>32.29</td>
<td>14.3</td>
<td>26.79</td>
<td>19.69</td>
<td>13.86</td>
<td>risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S16</td>
<td>S15</td>
<td>S14</td>
<td>S13</td>
<td>S12</td>
<td>S11</td>
<td>S10</td>
<td>S9</td>
<td>stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.68</td>
<td>11.44</td>
<td>8.52</td>
<td>6.96</td>
<td>5.76</td>
<td>4.31</td>
<td>14.29</td>
<td>7.20</td>
<td>return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.87</td>
<td>15.43</td>
<td>14.04</td>
<td>16.62</td>
<td>17.9</td>
<td>10.22</td>
<td>20.78</td>
<td>14.74</td>
<td>risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S24</td>
<td>S23</td>
<td>S22</td>
<td>S21</td>
<td>S20</td>
<td>S19</td>
<td>S18</td>
<td>S17</td>
<td>stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.41</td>
<td>10.31</td>
<td>2.55</td>
<td>6.35</td>
<td>6.67</td>
<td>6.77</td>
<td>5.77</td>
<td>7.72</td>
<td>return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18.67</td>
<td>16.68</td>
<td>9.23</td>
<td>30.77</td>
<td>10.4</td>
<td>15.32</td>
<td>9.12</td>
<td>15.49</td>
<td>risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S32</td>
<td>S31</td>
<td>S30</td>
<td>S29</td>
<td>S28</td>
<td>S27</td>
<td>S26</td>
<td>S25</td>
<td>stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.64</td>
<td>2.96</td>
<td>15.11</td>
<td>7.44</td>
<td>6.53</td>
<td>7.83</td>
<td>10.65</td>
<td>5.36</td>
<td>return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37.94</td>
<td>13.71</td>
<td>46.81</td>
<td>19.58</td>
<td>15.71</td>
<td>12.87</td>
<td>20.11</td>
<td>11.17</td>
<td>risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S40</td>
<td>S39</td>
<td>S38</td>
<td>S37</td>
<td>S36</td>
<td>S35</td>
<td>S34</td>
<td>S33</td>
<td>stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.85</td>
<td>19.80</td>
<td>16.66</td>
<td>7.17</td>
<td>9.36</td>
<td>1.86</td>
<td>13.71</td>
<td>11.98</td>
<td>return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.35</td>
<td>28.58</td>
<td>33.95</td>
<td>31.74</td>
<td>32.01</td>
<td>29.77</td>
<td>44.93</td>
<td>14.4</td>
<td>risk</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**7. Determine the algorithm parameters**

To determine the relative performance of the current parameters in birds’ flight algorithm main problem was carried out in 10% return and the results can be considered according to below graphs. This parameter is as follows:

<table>
<thead>
<tr>
<th>W</th>
<th>C1</th>
<th>C2</th>
<th>MaxIt</th>
<th>npop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4692</td>
<td>1.4692</td>
<td>400</td>
<td>200</td>
</tr>
</tbody>
</table>
Figure 2. The relationship between risk and returns in the implementation of the algorithm by table 2 parameters

Table 3. The algorithm coefficient in the second implementation

<table>
<thead>
<tr>
<th>W</th>
<th>C1</th>
<th>C2</th>
<th>MaxIt</th>
<th>npop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4692</td>
<td>1.4692</td>
<td>300</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 3. The relationship between risk and returns in the implementation of the algorithm by table 3 parameters

Table 4. The algorithm coefficient in the third implementation

<table>
<thead>
<tr>
<th>W</th>
<th>C1</th>
<th>C2</th>
<th>MaxIt</th>
<th>npop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4692</td>
<td>1.4692</td>
<td>300</td>
<td>400</td>
</tr>
</tbody>
</table>
Table 5. The algorithm coefficient in the forth implementation

<table>
<thead>
<tr>
<th>W</th>
<th>C1</th>
<th>C2</th>
<th>MaxIt</th>
<th>npop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4692</td>
<td>1.4692</td>
<td>400</td>
<td>400</td>
</tr>
</tbody>
</table>

Figure 5. The relationship between risk and return in the implementation of the algorithm by table 5 parameters

Table 6. The algorithm coefficient in the fifth implementation

<table>
<thead>
<tr>
<th>W</th>
<th>C1</th>
<th>C2</th>
<th>MaxIt</th>
<th>npop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4692</td>
<td>1.4692</td>
<td>500</td>
<td>400</td>
</tr>
</tbody>
</table>

Figure 6. The relationship between risk and return in the implementation of the algorithm by table 6 parameters

Table 7. The algorithm coefficient in the final implementation

<table>
<thead>
<tr>
<th>W</th>
<th>C1</th>
<th>C2</th>
<th>MaxIt</th>
<th>npop</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4692</td>
<td>1.4692</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>
Then to get the logical relationship between risk and return for this share, survey’s problem was solved at different levels finally was computed risk minimum that identified by bird flight algorithm and the result is as table 2. For example with placing the investor’s expected return equal to 6% the minimum risk of portfolio invested in 40 shares 85.6% was achieved using the technique of birds flight. In each stage of the implementation with increasing of the expected returns of investor minimum risk was calculated:

<table>
<thead>
<tr>
<th>Acceptable returns (%)</th>
<th>Calculated Risk (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.77</td>
<td>5.12</td>
</tr>
<tr>
<td>35.8</td>
<td>12</td>
</tr>
<tr>
<td>18.7</td>
<td>5.11</td>
</tr>
<tr>
<td>99.6</td>
<td>11</td>
</tr>
<tr>
<td>85.6</td>
<td>5.10</td>
</tr>
<tr>
<td>93.15</td>
<td>6.11</td>
</tr>
</tbody>
</table>

Finally, according to the data in table 8 the efficient set graph was obtained based on the mean – variance model. Marked Points in the diagram (8) are that have been identified following the implementation of the algorithm. As has been indicated in the diagram by increasing the investor’s expected returns, risk also increases. And it is investor risk-averse principle which has been referred in Markowitz's theory.

8. Conclusions

The main goal of this research is how to select an optimal portfolio, using the mean – variance model and optimization methods of bird flight algorithms. In this research Markowitz’s Mean - Variance model has solved by applying some kind of constraint by birds flight ultra- initiative method. In this regard to review of performance of mean-variance model diagram (2) of the efficient set is obtained.
The main results obtained from the use of birds flight algorithm is as follows:
- Due to the risk and uncertainty prevailing stock and portfolio management can use of birds fly algorithm to select and manage the appropriate collection of securities.
- Using birds’ flight algorithm method to determine the weight of each share constituting the portfolio led to choose the most suitable optimal portfolios. Mean - variance model is one of the hard problems and using of traditional methods to optimize them aren’t obtained suitable results. Because these methods fall into the trap of local optimum Points while birds’ flight algorithm by techniques and regulating of parameters like inertia parameter can leave of local optimum points and will converge toward the optimal solution or one of the most suitable solutions.
- Observing of diversification principle in portfolio reduces unsystematic risk. The variation principle was considered via the mean – variance model and model optimization with birds’ flight algorithms and causing a significant reduction in the risk of investing in the Stock Exchange.
- According to survey findings, the use of initiative algorithms for the selection and portfolio optimization are confirmed and recommended. Successful performance of this algorithm in consistently superior to the market portfolio is evidence of the consistent claim with problem that is undeniable and indisputable.

By use of Birds flying algorithm than other intelligent optimization algorithms the following results is perceived:
- High-speed than other intelligent models;
- Implement it easier than other smart models;
- Less parameter for modeling than other intelligent models;
- Easier set and better solutions than other smart models.

As the birds’ flight algorithm, is a very powerful technique with high precision, therefore recommended to activators in capital markets and great decision makers use it to select an optimum portfolio of company stock. Birds fly algorithm is accurate method for predictions in the areas of economics and management so it is proposed to use in other area like: Rate of exchange, gold, oil etc.

In addition, this survey was examined a preliminary and general discussion about using of the mean - variance model to portfolio formation. Therefore, better results and more function can be obtained by extension and following research:
- Application of Mean – variance model and birds flight algorithms to selection portfolio with the equations operational constraints.
- The optimal portfolio selection using three variable returns, risk and the number of securities.

References


