Education and Tourism in a Small Open Growth Economy

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Abstract
The purpose of this study is to build an economic growth of a small-open economy with endogenous education and tourism. The growth machines of the economy are endogenous human capital, wealth and tourism. The national economy is composed of one industrial sector, one service sector, and one education sector. The production side is based on Solow’s one-sector growth model and Uzawa’s two-sector growth model. The education sector is based on the Uzawa-Lucas two-sector growth model. International interactions are based on the literature of growth models for small open economies and the literature of growth and tourism. We apply Zhang’s utility function to describe household behavior. The household chooses consumption of goods and services, education time, and saving at each point of time. We simulate the nonlinear dynamic system. With the chosen parameter values the dynamic system is characterized of a unique stable equilibrium point. We carry out comparative dynamic analysis to demonstrate transitory as well as long-term effects of changes in different parameters.

Keywords: Human Capital, Tourism, Education, Tourism, Wealth Accumulation

1. Introduction
Education is an important determinant of sustainable economic growth and development. For some small economies tourist industry plays an important role in economic development. As education and tourist industry make contributions to national growth in different ways and use national resources differently, it is necessary to introduce these two sectors in a general equilibrium framework. Nevertheless, theoretical economics still lacks such an analytical framework to properly examine interdependence between tourism, education, and wealth accumulation. The purpose of this study is to construct a dynamic general equilibrium model for a small-open economy with endogenous education and tourism.

We are concerned with a national economy which composed of one industrial sector, one service sector, and one education sector. The production side is based on Solow’s one-sector growth model and Uzawa’s two-sector growth model (Solow, 1956; Uzawa, 1961). The education sector is based on the Uzawa-Lucas two-sector growth model. International interactions are based on the literature of growth models for small open economies and the literature of growth and tourism. A special feature of this model is to include tourism industry which has different characters from tradable goods in traditional trade theory. Tourism goods such as monuments of national heritage, historical sites, beaches, and hot springs, are not-tradable in traditional international trade theory.
(Copeland, 2012). As tourism has become increasingly more important role in national economies, it is not relevant to aggregate tourism with other economic activities (Corden and Neary, 1982; Copeland, 1991, and Chao et al., 2009). This study makes a contribution to the literature by designing a dynamic model of economic growth with tourism as a sector. There are many empirical studies about relationships between tourism and economic development (see, e.g., Hazari and Sgro, 1995; Dritsakis, 2004; Kim et al. 2006). According to Chao et al. (2006), an expansion of tourism can lead to result in capital decumulation in a two-sector dynamic model with a capital-generating externality. Dwyer et al. (2004) argue for the necessity to examine tourism and its interaction with the rest economy within a dynamic general equilibrium modeling (see also Blake et al. 2006). This study studies tourism and economic structural change in context of a small-open economy. There are only a few theoretical models on interactions between growth and education with endogenous wealth accumulation. It is well known that an early formal modelling of education and growth Uzawa’s two sector growth model which is now often referred as the Uzawa-Lucas two-sector growth model (Uzawa, 1965; Lucas, 1988). In this approach capital is distributed between education and production in perfectly competitive markets (Jones et al. 1993; Stokey and Rebelo, 1995; De Hek, 2005; Chakraborty and Gupta, 2009; and Sano and Tomoda, 2010). Nevertheless, in this traditional approach one problem still reminds. Households’ preferences for education is not explicitly taken into account. This study overcomes this problem by applying Zhang’s approach to household.

Most of the growth models in the literature of tourism economics are developed for a small open economy (Zeng and Zhu, 2011). There are many studies on growth of small-open economies (e.g., Obstfeld and Rogoff, 1996; Benigno and Benigno, 2003; Gáli and Monacelli, 2005; and Ilzetzki, et al. 2013). We follow this tradition. It should be noted that the model is an extension of a growth model with tourism by Zhang (2012) and Zhang’s growth model with learning by dong (2016). Rather than learning by doing we consider education as the main determinant of human capital growth. This model is different from Zhang’s 2012 model is that this study introduces endogenous human capital and endogenous time distribution. We apply Zhang’s utility function to model decision making of household (Zhang, 1993, 2005). The household decides education time, consumption of goods and services, and saving at each point in time. The study is organized as follows. Section 2 proposes a small-open economic growth model with economic structure, education and tourism. Section 3 provides a computational procedure to plot motion of the economic system and simulates the model. Section 4 deals with the impact of changes in some parameters. Section 5 concludes the study.

2. The growth model with education and tourism
We now build a small open growth model with education and tourism. The main determinants of economic growth are through wealth accumulation (like in neoclassical growth theory), human capital accumulation (like in the Uzawa-Lucas model), and international trade and tourism. Open economy has free trade with the rest of the world. There are no barriers or transactions on importing or exporting goods and borrowing or lending resources. The model is built on the main features of some well-known economic models in the literature of economic growth. The capital
accumulation and economic structure are based on neoclassical growth theory, especially the Solow one-sector growth model and Uzawa two-sector growth model. Modelling education and human capital are influenced by the Uzawa-Lucas two-sector growth model. Some features of tourism are based on some models of tourism.

There is a single industrial good which is freely internationally traded in world economy. The price of the industrial good is unity. Following Zhang (2012), we deal with a small-open economy. This study introduces education and tourism. Domestic households consume both goods and foreign tourists consume only services. Both human capital and physical capital are depreciated at constant exponential rates. Domestic households hold wealth and obtain income from wages and interest payments of wealth. All markets are perfectly competitive. Factor inputs of production include capital and qualified labor force. Capital and labor are completely mobile between the sectors. Capital is also perfectly mobile in international markets. There is no emigration or immigration. The constant population is homogeneous. As the economy is too small to affect the rate of interest in world markets. We introduce following variables

\[ i, s, e \] - subscript indexes fixed population and time-dependent national labor force;
\[ r^* \] and \[ w(t) \] - rate of interest and wage rate;
\[ p(t) \] and \[ p_e(t) \] - price of service and price of education per unit of time;
\[ \bar{N} \] and \[ N(t) \] - fixed population and time-dependent national labor force;
\[ K(t) \] and \[ \bar{K}(t) \] - capital stocks employed and owned by the country, respectively;
\[ K_i(t) \] and \[ N_i(t) \] - capital stocks and labor force employed by sector \( j \), \( j = i, s, e \);
\[ F_j(t) \] - output level of sector \( j \), \( j = i, s, e \);
\[ T(t), \bar{T}(t) \] - and \[ T_e(t) \] - work time, leisure time, and education time of the representative household;
\[ T_0 \] - total available time for work, leisure, and education; and
\[ H(t) \] - level of human capital;
\[ \delta_k \] and \[ \delta_H \] - fixed depreciation rates of physical capital and human capital.

The national labor force is defined as follows

\[ N(t) = H^m(t)T(t)\bar{N}, \quad (1) \]

where \( m \) is the human capital utilization efficiency parameter. We call \( H^m(t) \) the level of effective human capital.

**Industrial sector**

The industrial sector uses capital and labor as inputs. The production function of the industrial sector is taken on the following form

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\[ F_i(t) = A_i K_i^\alpha \beta_i^\beta_i(t), \quad \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \]  

(2)

where \( A_i, \alpha_i, \) and \( \beta_i \) are parameters. Labor and capital are paid at marginal products. The marginal conditions are

\[ r_\delta = \alpha_i A_i k_i^{-\beta_i}(t), \quad w(t) = \beta_i A_i k_i^\alpha(t), \]  

(3)

where \( k_i(t) \equiv K_i(t)/N_i(t) \) and \( r_\delta \equiv r^* + \delta_i \).

**Service sector**

The production function of the service sector is

\[ F_s(t) = A_s K_s^\alpha \beta_s^\beta_s(t), \quad \alpha_s, \beta_s > 0, \quad \alpha_s + \beta_s = 1, \]  

(4)

where \( A_s, \alpha_s, \) and \( \beta_s \) are parameters. The marginal conditions are

\[ \frac{r_\delta}{\alpha_s A_s} = p(t)k_s^{-\beta_s}(t), \quad \frac{w}{\beta_s A_s} = p(t)k_s^\alpha(t), \]  

(5)

where \( k_s(t) \equiv K_s(t)/N_s(t) \).

**Education sector**

The service sector employs capital and labor force to conduct education. The production function of the education sector is

\[ F_e(t) = A_e K_e^\alpha \beta_e^\beta_e(t), \quad \alpha_e, \beta_e > 0, \quad \alpha_e + \beta_e = 1, \]  

(6)

where \( A_e, \alpha_e, \) and \( \beta_e \) are parameters. The marginal conditions are

\[ \frac{r_\delta}{\alpha_e A_e} = p_e(t)k_e^{-\beta_e}(t), \quad \frac{w}{\beta_e A_e} = p_e(t)k_e^\alpha(t), \]  

(7)

where \( k_e(t) \equiv K_e(t)/N_e(t) \).

**Full employment of capital and labor**

The total capital stocks employed by the country is belongs to domestic residents and the rest of the world. Full employment of labor force implies
\[ N_1(t) + N_2(t) + N_3(t) = N(t). \] (8)

Full employment of physical capital implies
\[ K_1(t) + K_2(t) + K_3(t) = K(t). \] (9)

**Demand function of foreign tourists**

We use \( y_f(t) \) to stand for incomes in foreign countries. We apply and generalize the iso-elastic tourism demand function by Schubert and Brida (2009)
\[ D_f(t) = a(k(t), H(t), t) y_f^\phi(t) p^{-\varepsilon}(t), \] (10)

where \( \phi \) and \( \varepsilon \) are respectively the income and price elasticities of tourism demand. The variable, \( a(t) \), is dependent on many conditions, such as national infrastructures (airports and transportation systems) and environment (like criminal rates, pollutants and congestions), and cultural capital (e.g., Throsby, 1999; Beerli and Martin, 2004). This study also considers that the country’s attractiveness for foreign tourists is dependent on the country’s human capital and wealth per household. In our study foreign tourists pay the same price as domestic households. This is a strict requirement as tourism industry actually charges prices differently (e.g., Marin-Pantelescu and Tigu, 2010; Stabler, et al., 2010; and Krauz, 2017).

**Optimal decision of domestic households**

In our approach households decide education time, leisure time, consumption levels of industrial goods and services, and saving. We apply the utility function by Zhang (1993, 2005). The current income of the household is
\[ y(t) = r^* k^*(t) + H^m(t)T(t)w, \] (11)

where \( r^* k^*(t) \) is the interest payment and \( H^m(t)T(t)w \) the wage total payment. It should be noted that \( y(t) \) is called the disposable income in traditional neoclassical growth theory. It is obvious that in a modern society the so-called “disposable” income is not equal to what one (such as a retired worker or no-working rich man) earns today. One can also dispose negatively or positively a part of his wealth. In Zhang’s approach, disposable is the sum of the current income and the value of wealth that the household owns. It is assumed that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income is
\[ \hat{y}(t) = y(t) + k(t). \] (12)

The disposable income is used for saving, education and consumption.

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The household distributes the disposable income between services, industrial goods, education, and saving. This implies

\[ p_c(t)T_c(t) + p(t)c_s(t) + c_i(t) + s(t) = \bar{y}(t). \]  

(13)

Budget constraint (13) states that consumption and saving exhaust the disposable income.

The time constraint implies

\[ T(t) + \bar{T}(t) + T_c(t) = T_0. \]  

(14)

Insert (14) in (13)

\[ H^m(t)\bar{T}(t)w + \bar{p}_c(t)T_c(t) + p(t)c_s(t) + c_i(t) + s(t) = \bar{y}(t), \]  

(15)

where

\[ \bar{p}_c(t) = p_c(t) + H^m(t)w, \quad \bar{y}(t) = (1 + r^*)\bar{k}(t) + H^m(t)T_0 w. \]

We specify utility function \( U(t) \) as follows

\[ U(t) = \theta \bar{T}^{\sigma_0}(t)T_{c}^{\eta_0}c_s^{\gamma_0}(t)c_i^{\xi_0}(t)s^{\lambda_0}(t), \quad \sigma_0, \eta_0, \gamma_0, \xi_0, \lambda_0 > 0, \]

in which \( \sigma_0, \eta_0, \gamma_0, \xi_0, \) and \( \lambda_0 \) are the representative household’s elasticity of utility with regard to leisure time, education, services, industrial goods, and saving. We call \( \sigma_0, \eta_0, \gamma_0, \xi_0, \) and \( \lambda_0 \) propensities to consume the leisure time, to receive education, to consume services, to consume industrial goods, and to hold wealth, respectively. We maximize \( U(t) \) subject to the budget constraint. The marginal conditions imply

\[ \bar{T}(t) = \frac{\sigma \bar{y}(t)}{H^m(t)w}, \quad T_c(t) = \frac{\eta \bar{y}(t)}{\bar{p}_c(t)}, \quad c_s(t) = \frac{\gamma \bar{y}(t)}{p(t)}, \quad c_i(t) = \xi \bar{y}(t), \quad s(t) = \lambda \bar{y}(t), \]  

(16)

where

\[ \sigma = \rho \sigma_0, \quad \eta = \rho \eta_0, \quad \gamma = \rho \gamma_0, \quad \xi = \rho \xi_0, \quad \lambda = \rho \lambda_0, \quad \rho = \frac{1}{\sigma_0 + \eta_0 + \gamma_0 + \xi_0 + \lambda_0}. \]
Change in wealth is saving minus dissaving. According to the definition of $s(t)$, we have the following wealth accumulation

$$\dot{k}(t) = s(t) - \bar{k}(t).$$ \hspace{1cm} (17)

**Demand and supply for services**

The equilibrium condition for services is the demand of domestic and foreign tourists

$$c_s(t)\bar{N} + D_F(t) = F_s(t).$$ \hspace{1cm} (18)

**Balance in education market**

The total demand for education service in group $j$ is $T_s(t)N_0$. The demand and supply for education balances at any point in time

$$T_s(t)\bar{N} = F_s(t).$$ \hspace{1cm} (19)

**Accumulation of human capital**

We follow Uzawa (1965) in modelling human capital accumulation. We apply a generalized Uzawa’s human capital accumulation as follows

$$h(t) = \nu_c \left( F_c(t) / \bar{N} \right)^{\pi_c} \left( H^m(t)T_e(t) \right)^{\beta_e} - \delta_h H(t),$$ \hspace{1cm} (20)

where $\nu_c$, $\pi_c$, and $\beta_e$ are non-negative parameters. If $\pi_c$ is positive (negative), we say that learning through education exhibits decreasing (increasing) returns to scale. The equation implies that human capital rises in education service per capita and in the (qualified) total study time, $\left( H^m(t)T_e(t) \right)^{\beta_e}$.

The national wealth is given by

$$\bar{K}(t) = \bar{k}(t)\bar{N}.$$ 

The trade balance is given by

$$E(t) = r^* \left( \bar{K}(t) - K(t) \right).$$ 

We built the dynamic growth model. The economy is composed of three sectors with endogenous wealth, human capital. The model is a synthesis of the well-known Solow one-sector growth
model, Uzawa two-sector model, Uzawa-Lucas education-based growth model, and some studies on tourism.

3. The dynamics of the economy

We now show that the movement of the economic system is described by two differential equations. The following lemma demonstrates how to follow a computational program to plot the motion of all the variables.

Lemma

The variables, \( k_i, k_s, w, p, \) and \( p_e \) are uniquely determined as functions of \( r^* \). The movement of wealth and human capital is given by the following two differential equations

\[
\begin{align*}
\dot{k}(t) &= \Omega_s(k(t), H(t)), \\
\dot{H}(t) &= \Omega_H(k(t), H(t)),
\end{align*}
\]

where \( \Omega_s \) and \( \Omega_H \) are functions of \( k(t) \) and \( H(t) \) defined in the Appendix. We decide all the variables as functions of \( k(t) \) and \( H(t) \) as follows: \( k_i \) and \( w \) by (A1) \( \to k_s \) by (A2) \( \to k_e \) by (A3) \( \to p \) by (5) \( \to p_e \) by (7) \( \to \bar{y}(t) \) by (15) \( \to \bar{F}(t), \ bar{c}_i(t), \ bar{c}_s(t), \ bar{s}(t) \) by (16) \( \to D_i(t) \) by (10) \( \to N_i(t) \) by (A6) \( \to N_s(t) \) by (A7) \( \to N_e(t) \) by (A6) \( \to T(t) \) by (A8) \( \to N(t) \) by (1) \( \to N_i(t) \) by (A10) \( \to K_j(t) = k, N_j(t) \) \( \to \dot{y}(t) \) by (12) \( \to F_i(t) \) by (2) \( \to F_s(t) \) by (4) \( \to F_e(t) \) by (6).

The lemma implies if we know initial values of wealth and human capital at any point in time, we can follow the motion of all the variables of the economy. As there are many nonlinear relations between the variables, it is difficult to get explicit analytical results. We simulate the model. First we specify the function which describes relations between income from tourism and other conditions

\[
D_i(t) = 0.5k(t)^{0.3} \bar{y}^\phi p^{-e}.
\]

It states that the income is positively related to international economic conditions, negatively related to the price of services. The wealth has a positive impact on attracting tourists. With given prices a wealthy economy tends to be well behaved and environment-friendly. The rest parameters are specified as follows

\[
\begin{align*}
r^* &= 0.06, \ \delta_k = 0.05, \ \bar{N} = 10, \ T_0 = 24, \ L = 20, \ A_i = 1.2, \ A_s = 1, \ \alpha_i = 0.3, \ \alpha_s = 0.31, \\
\alpha_e &= 0.35, \ \beta_s = 0.6, \ \lambda_0 = 0.6, \ \xi_0 = 0.15, \ \gamma_0 = 0.06, \ \eta_0 = 0.02, \ \sigma_0 = 0.2, \ a = 1, \ y_f = 4, \\
\phi &= 1.5, \ \varepsilon = 1.5, \ m = 0.6, \ \delta_h = 0.04, \ \nu_s = 0.7, \ \alpha_e = 0.2, \ b_s = 0.4, \ \pi_e = 0.3.
\end{align*}
\]

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The rate of interest is 6 per cent and the population is 10. Although the specified values are not based on empirical observations, the choice is referred to some studies. For instance, some empirical studies on the US economy use the value of the parameter, $\alpha$, in the Cobb-Douglas production around 0.3. It is estimated that income elasticity of tourism demand is larger than unity (Syriopoulos, 1995). Lanza et al. (2003) identify the price elasticity between 1.03 and 1.82 and income elasticities between 1.75 and 7.36. There are many other studies on elasticities (e.g., Garin-Müños, 2007).

We specify the initial conditions as follows

$$\bar{K}(0) = 255, \quad H(0) = 15.$$ 

The dynamics is plotted in Figure 1. In Figure 1, we have

$$Y(t) = F_y(t) + p F_r(t) + p_e F_e(t).$$

Except the work time, the variables rise in time in the long term. The national income, national labor force, capital employed by the economy, and output and labor input of the industrial sector fall initially and rise in the long term. As the household becomes richer, it spends more time on leisure and education.

Figure 1 The Motion of the National Economy

Figure 1 demonstrates convergence of the economic system to an equilibrium point. Simulation identifies the following equilibrium values of the variables
The eigenvalues are 
\[ \{-0.393, -0.04\} \]
This confirms that the unique equilibrium point is stable. This also guarantees the validity of following comparative dynamic analysis.

4. Comparative dynamic analysis

The previous section illustrates the movement of the variables. This section is concerned with how economic development path is affected if there are some exogenous changes. On the basis of the previous section we can easily conduct comparative dynamic analysis. We introduce a variable, \( \Delta t \), to present the change rate of the variable, \( x(t) \), in percentage due to changes in the parameter value.

4.1. The propensity to receive education rises
First, we are interested in how the economy and tourism industry are affected if the population has a higher propensity to received education as follows: \( \eta_0 = 0.02 \Rightarrow 0.025 \). It is straightforward to see that the time-independent variables are not affected by the preference change, \( \Delta p = \Delta p_e = \Delta w = 0 \). We plot the rest of the simulation results in Figure 2. The rise in the propensity to receive education leads to the increases in education time and decreases in leisure and work hours. The redistribution of time results in an initial falling in the total labor supply. As human capital is increased, the total labor supply rises after a very short-run falling. Initially, most variables fall. This occurs as the household spends more time on education and human capital growth takes time. After a very period of time, the variables are enhanced due to enhanced human capital. Tourist consumption is also increased due to augmented wealth which makes the economy more attractive. The trade balance is initially improved and deteriorated in the long term.
4.2. The propensity to enjoy leisure time rises
We are now concerned with the effects on the economy due to the following rise in the propensity to enjoy leisure time: $\sigma_0 = 0.2 \Rightarrow 0.21$. The time-independent variables are not affected by the preference change. We plot the rest of the simulation results in Figure 3. The rise in the propensity to have leisure time leads to the increases in leisure time and decreases in education time and work time. Human capital falls. The trade balance is improved. The three sectors employ less labor and capital inputs. The economy shrinks. The three sectors shrink. The economy has less tourist consumption.

Figure 3. The Propensity to Enjoy Leisure Time Rises
4.3. The rate of interest is increased in global markets
We now study effects of the following rise in the rate of interest: \( r_b = 0.06 \Rightarrow 0.062 \). It is straightforward to see that the time-independent variables are affected as follows

\[
\Delta w = -0.77, \quad \Delta p = 0.026, \quad \Delta p_x = 0.129.
\]

The wage rate falls in association with the rise in capital. The service price and price of education are enhanced. We plot the rest of the simulation results in Figure 4. The total capital stock employed by the economy and capital stocks employed by the three sectors are all reduced. The household works less hours and spends more time on education and leisure. The total output of the education sector and human capital are enhanced. The net result of reduction in work hours and increase in human capital results in fall in the total labor supply. The industrial sector employs less labor input but the other two sectors more labor inputs. The industrial and service sectors shrink. The household consumes less goods and services and owns less wealth. The total tourist consumption falls.

![Figure 4. The Rate of Interest is Increased in Global Markets](image)

4.4. The total factor productivity of the industrial sector rises
We now study effects of the following rise in the total factor productivity of the industrial sector: \( \lambda_i = 1.2 \Rightarrow 1.3 \). The time-independent variables are changed as follows

\[
\Delta w = 12.1, \quad \Delta p = 8.2, \quad \Delta p_x = 7.7.
\]

The wage rate rises in association with the rise in the total factor productivity. The service price and price of education are enhanced. We plot the rest of the simulation results in Figure 5. The time distribution is slightly affected. The total capital stock employed by the economy and
capital stocks employed by the three sectors are all increased. The national output and wealth are increased. Human capital falls initially but rises in the long term. The industrial sector expands. The other sectors shrink initially but expand in the long term. The trade balance is deteriorated. Tourist consumption falls.

Figure 5. The Total Factor Productivity of the Industrial Sector Rises

4.5. The propensity to save is increased

We now study effects of the following rise in the propensity to save: \( \lambda_s = 0.6 \Rightarrow 0.62 \). The time-independent variables are not affected. We plot the rest of the simulation results in Figure 6. In the long term the household spends more time on education and leisure and less time on work. Human capital falls slightly initially and rises in the long term. In the long term the household consumes more goods and services and owns more wealth. The nation employs less capital and has lower national output. The industrial sector shrinks but the other sectors expand in the long term. The trade balance is improved. Tourist consumption rises.
4.6. The human capital utilization efficiency is improved

We now study effects of the following improvement in the human capital utilization efficiency: $m = 0.6 \Rightarrow 0.62$. The time-independent variables are not affected. We plot the rest of the simulation results in Figure 7. The household spends less hours on education initially but more hours. Human capital rises. The national wealth and capital employed by the economy are augmented. The household consumes more goods and services and owns more wealth. The national output rises. The trade balance is deteriorated. Tourist consumption rises. The education sector shrinks initially but expands in the long term. The industrial and service sectors expand.

5. Conclusions

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The purpose of this study is to build an economic growth of a small-open economy with endogenous education and tourism. The growth machines of the economy are endogenous human capital, wealth and tourism. The national economy is composed of one industrial sector, one service sector, and one education sector. All the markets are perfectly competitive. Following the traditional literature of small open economies, we consider the rate of interest fixed. The production side is based on Solow’s one-sector growth model and Uzawa’s two-sector growth model. The education sector is based on the Uzawa-Lucas two-sector growth model. We applied Zhang’s utility function to describe household behavior. The household chooses consumption of goods and services, education time, and saving at each point of time. We simulate the nonlinear dynamic system. With the chosen parameter values the dynamic system is characterized of a unique stable equilibrium point. We carried out comparative dynamic analysis to demonstrate the effects of changes in different parameters. As the model is built on the basis of a few well-known models and each of these models has resulted a large literature of extensions and generalization, our model can be obviously extended and generalized on the basis of the vast literature. We can also study the economic dynamics different forms of utility and production functions. Domestic households may also travel to other countries. There are some other important variables, such as exchange rates and inflation policies, which should be analyzed in further research. Different forms of tariffs and taxes are important for understanding modern economies.

Appendix: Proving the Lemma
As $r^\ast$ is fixed, from (3) we have

$$k_i = \left( \frac{\alpha_i A_i}{r_g} \right)^{1/\beta_i}, \quad w = \beta_i A_i k_i^{\alpha_i}. \quad \text{(A1)}$$

Hence, we can treat $k_i$ and $w$ as functions of $r^\ast$. From (5) we solve

$$k_s = \frac{\alpha_s w}{\beta_s r_g}. \quad \text{(A2)}$$

Hence, we treat $k_s$ and $p$ as a function of $r^\ast$. From (7) we solve

$$k_e = \frac{\alpha_e w}{\beta_e r_g}. \quad \text{(A3)}$$

Hence, we treat $k_e$ and $p_e$ as a function of $r^\ast$. We already showed that $k_i, k_s, k_e, w, p$ and $p_e$ are determined as functions of $r^\ast$, which is fixed in the international market. We rewrite (9) as
\[ k_i N_i + k_s N_s + k_e N_e = K. \]  \hspace{1cm} (A4)

From (16) and (19) we get
\[
\frac{\eta \bar{y} \bar{N}}{\bar{p}_e} = F_e. \]  \hspace{1cm} (A5)

Insert (6) in (A5)
\[
N_e = \frac{\eta \bar{y} \bar{N}}{A_e k_e^{\alpha_e} \bar{p}_e}. \]  \hspace{1cm} (A6)

From (18), (16) and (4) we solve
\[
N_s = \left( \frac{\gamma \bar{y} \bar{N}}{p} + D_r \right) \frac{1}{A_s k_s^{\alpha_s}}. \]  \hspace{1cm} (A7)

From (14) and (16) we get
\[
T = T_0 - \frac{\sigma \bar{y}}{H^m w} - \frac{\eta \bar{y}}{\bar{p}_e}. \]  \hspace{1cm} (A8)

By (1) and (A8) we have
\[
N = \left( T_0 - \frac{\sigma \bar{y}}{H^m w} - \frac{\eta \bar{y}}{\bar{p}_e} \right) H^m \bar{N}. \]  \hspace{1cm} (A9)

By (8) we have
\[
N_i = N - N_s - N_e. \]  \hspace{1cm} (A10)

By the following procedure we can determine all the variables as functions of \( \tilde{k} \) and \( H \): \( k_i \) and \( w \) by (A1) \( \rightarrow k_s \) by (A2) \( \rightarrow k_e \) by (A3) \( \rightarrow p \) by (5) \( \rightarrow p_e \) by (7) \( \rightarrow \bar{y} \) by (15) \( \rightarrow \bar{T}, T_e, c_i, c_s, s \) by (16) \( \rightarrow D_r \) by (10) \( \rightarrow N_e \) by (A6) \( \rightarrow N_s \) by (A7) \( \rightarrow N_e \) by (A6) \( \rightarrow T \) by (A8) \( \rightarrow N \) by (1) \( \rightarrow N_i \) by (A10) \( \rightarrow k_j = k_j N_j \) \( \rightarrow \hat{y} \) by (12) \( \rightarrow F_i \) by (2) \( \rightarrow F_s \) by (4) \( \rightarrow F_e \) by (4) \( \rightarrow F_e \) by (6). From this procedure, (17) and (20), we have
\[ \tilde{k} = \Omega_s (\tilde{k}, H) \equiv s - \tilde{k}, \]
\[ \mathbf{R}^{k} = \Omega_{k}(y^{k}, H) \equiv \nu_{k}(F_{c}/\mathbf{N})^{\nu_{k}}(H^{\mu}T_{v})^{\nu_{k}} - \delta_{h}H. \]  

We thus proved the Lemma.

References


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