ABSTRACT

Foreign trade is indispensable for any economy because of the country's needs to import a variety of products and on the other hand to export their products to other countries for earning foreign exchange reserves. So, it is necessary to evaluate the balance of trade and examine thoroughly historical data of BoT to figure out the main reasons behind BoT deficit and to forecast BoT on its bases. This will assist in overcoming the deficit in BoT through avoiding past mistakes and planning according to the future estimation. For analysis purpose the data was taken from 1972-2015 and different econometric techniques has been used. To avoid the trend or time effect we used Holt-Winter model. To check the stationarity, ADF test was used and then ARIMA Model has been used to forecast the trade Balance of Pakistan. The results of this research study showed that the situation will become chronic in this issue seriously by the micromanagers and policy makers.

Keywords: Balance of Trade, ARIMA, Trend analysis, Holt-Winter.

1.1. Background of the Study

At the time of independence Pakistani economy was completely relied on agricultural sector. Most of the export at that time consists of raw material like jute from East Pakistan and cotton from West Pakistan to India which accounts about sixty percent of the total exports from Pakistan, while, most of imports consisted of manufactured consumer goods from India. But soon in 1949 when Pakistan refuses to devalue its currency India suddenly cut down its imports from Pakistan. To overcome the demand problem Pakistan starts diversification of market and export its products to Germany, Italy, U.K., Belgium, and France. However, Pakistan still is facing a chronicle deficit in balance of trade since decades. Imports have surpassed exports in almost every year since 1950 and thus, each year from FY 1973 through FY 1992, Pakistan had to face a deficit on its balance of trade.

Monetary value of merchandise net exports (exports minus imports of services and goods) of an economy in a specific time period is known as trade balance. Balance of trade shows the situation and relationship of exports and imports during a specific time span in a country. It is the largest constituent of the balance of payment of any nation. Imports, foreign
aid, domestic investment and spending abroad are constituent of debit. While exports, foreign investment and spending in domestic economy are included in credit items of BOP.

In general point of view trade deficit is not a weird phenomenon. It appreciates the living standard of the resident in an economy. As the country has access to wider range of commodities and services for a much competitive and reasonable price hence the risk of the inflation can be reduced by lower price products. Trade deficit also signify the confidence and purchasing power of the nationals because they have more to consume than they produce.

On the other hand trade deficit can cause increase in unemployment ratio. As a country increase imports, demand for the domestic products reduces and thus local companies have to face higher competition from international market and most of the time start to go out of business. In response local residents faces lake of job opportunities and lower living standards.

Economists due to this reason suggest a lower trade deficit to boost the employment rate. Most of the time trade agreements are considered to be the cause of trade deficits. To reduce trade deficit imports are often restricted by qualitative (tariff) and quantitative (quotas) measures or by other types of trade protection. But these measures seldom work, for the reason that usually the industry is already on the way out from the market and loss the capability even before the policies are implemented.

1.2. Objective of the Study
The main objectives of this study are

- To examine historical trends in the trade balance of Pakistan.
- To forecast the future trends in balance of trade of Pakistan.
- To device policies and strategies for macro managers and micro managers in view of opportunities and capacity in balance of trade in Pakistan.

2. A Brief Overview of Balance of Trade of Pakistan

Pakistan was trapped in a vicious cycle of deficit in balance of trade since 1958. While in 1953-54 and 56 the foreign trade performance was rationally good. The average exports were 161 million US$ more than its imports and it had a surplus balance of trade up to 1956-57. After fifteen years of trade deficit in 1973 Pakistan had a favorable trade balance, yet that was the last year for the surplus trade balance in Pakistan. Since then Pakistan is facing a trend of deficit balance of trade and balance of payment. Its current account deficit was at record level of US$ 4575 million in 1996 claiming -4.96% of GDP and at its highest surplus in fiscal year 2003 with US$ 3,165 million that is 4.77% of the GDP. On the other side, the situation was totally changed in fiscal year 2005 where the current account had a favorable position, while the trade deficit was at US$ -6,185 million that is -8.2% of GDP, the highest ever in the trade history of Pakistan (MPS, 2006). In 1971 separation of East Pakistan has greatly affected balance of trade. This dissertation covers the quantitative analysis of Pakistan, so the data is taken beyond 1971.
In the 1980s incentives were offered to the industrial sectors to increase the export of their manufactured products. However, in order to control imports, the government sustained lists of allowable imports and used quantitative regulations and constraints on foreign exchange. The most widespread list of permissible imports covers capital goods as well as raw materials and consumer merchandises and only commercial and industrial users were allowed to import these products. The second and third list generally covers raw materials and commodities that can only be imported by industrial users and public sector respectively.

In the early 1990s the export set up was as mostly reliant on two agrarian products, namely rice and cotton, which was usually exposed to great fluctuation in yield and demand. In 1992, cotton products including raw cotton, cotton yarn, cotton cloth, and cotton waste comprises 37% of all exports. Readymade clothes (15 %), rice (6 %) and synthetic textiles (6 %) were among the other important exporting items. Between FY 1986 and FY 1993 there was some diversification in manufacturing sector’s production pattern, as share of manufactured goods rose from 49 percent to 64 percent and share of primary goods fell from 35 % to 16 %.

Pakistan's balance of trade remained mainly susceptible to variations in the world economy in the early 1990s. Country’s import bill rose significantly due to upsurges in crude oil prices, as for instance in 1979-81 and 1990. On the other hand, country’s export being sensitive to agricultural production was seriously affected by the deterioration in cotton production in 1993.
Sources of trade market are broadly disseminated, and they vary on yearly biases. In early 1990s, Japan and United States were most important trading partners of Pakistan. In 1993, United States traded for 11.2 % in term of imports and 13.7 % in term of exports. While Japan traded with Pakistan for 6.6 % of exports and 14.2 % of imports. Saudi Arabia, Germany and U.K., were too among the leading trading parties, whereas, Hong Kong and China were significant export market and supplier of imports respectively.

Pakistan, in 1990s, started adopting and developing new trading initiatives with nearby nations. To develop its economy Pakistan became the part of two regional organizations, the Economic Co-operation Organization (ECO) and the South Asian Association for Regional Cooperation (SAARC).

Figure: 2-3: Trends of Pakistan’s Foreign Trade from 1992-2001

Surrounded by the challenging and complicated international economic conditions, like decelerating situation of the international trade, decrease in global product prices, and nationwide severe energy crisis, exports from Pakistan persisted relatively better off by US$ 14.0 million during 2011-12 over the previous year and stood at $ 20,474 million. The growth of imports remained about 14.5 % continued to show almost similar growth in the corresponding period’s growth. Exports remained declining and imports continued to grow stressing the importance of external developments. Pakistan’s exports growth would have been in much improved situation, had there been stabilization in global predictions during the period. In 2011-12, workers’ remittances grew by $ 1.83 billion over the last year which provided great assistance in financing import bills and reducing the foreign exchange shortage created from trade deficit.
Pakistan Bureau of Statistics reported the Balance of Trade in 2015, was a record trade deficit of 169621 PKR Million. Over all, from 1957 till 2015, the trade balance of Pakistan averaged -17710.85 PKR Million. In June 2003 trade balance reached all-time high of 6457 PKR Million and a record low of -215020.49 PKR Million in December 2011. One of the prime reason due to which Pakistan runs consistent trade deficits is great importations of energy. About 40% of total export consists of fuel products, machinery and transport equipment constitute about 18% and chemicals contribute almost 16 % to total imports. While on export side relatively low value added products are in fashion like, cotton and knitwear share almost 28% of total exports, bed wear, carpets and rugs makes about 8% and rice contribute almost 8% to the total exports for Pakistan. Direction of foreign trade is limited to few countries like 10 % of total exports and 17 % of imports are from United Arab Emirates and trade with China comprise 9% of exports and 15% imports. Other foreign trading partners for Pakistan are United Kingdom, United States and Germany.

3. Trend Analysis and Balance of Trade

Trend Analysis is the exercise of gathering data and trying to spot a trend, or an arrangement, in the statistics. While trend analysis is frequently used to forecast future events, it could be used for the assessment of previous uncertain events. In statistics, trend analysis often refers to methods for removing a fundamental pattern of behavior in a time series which would then be partially or almost entirely concealed by noise. In simple words trend analysis is a calculation technique that uses past results to forecast future consequence.

3.1. Forecasting Trends

Yogi Berra’s one of the famous quote is "It's tough to make predictions, especially about the future." Before explaining the forecasting trend, forecasting can be explained as; Application of variety of tools in order to predict future situations or results is termed as ‘Forecasting’. "Fore" means a forward predication, e.g. a company may use historical data or information to determine the future trends. Similarly if we want to move backward, trying to predict a foregoing situation, or in other words we want to reverse-forecasting i.e. starts with a specific future result and then move backwards to the present conditions, the term back
casting can be used. If we have to deal with such a data series that contains some holes then we can estimate those missing values between known data points, that procedure is known as interpolation.

3.2. Time-Series Data and Trends

Some data is certainly supportive in quantitative forecasting. This is known as time-series data, where the numerical quantities are obtained for numerous points at regular different time periods. Such data may be observed at regular period of time such as daily, weekly, monthly, quarterly or annually etc. Time series plot is a very helpful tool to study the trend of the data. This plot is constructed by plotting time along the horizontal axis, and the variable to be forecasted is plotted along the vertical axis of the graph. Whereas the term trend refers to the pattern of values of the measured variable over time, and if it is suspected that trend occurs, it might be capable of projecting that trend to forecast data for a time period in future.

3.3. Some Common Types of Trends

Trends in the data set can be better explained if data is presented graphically usually line graphs are used for this purpose. Graphs are constructed with the time period on the x-axis and the level of concerned variable on the y-axis. Following are some trends that are studied in time series analysis.

1). Constant trends
2). Linear trends
3). Exponential trends
4). Damped trends
5). Polynomial trends

These trends are presented in graphical forms below. Graphs are constructed using 100 points of hypothetical data associated with blue line, and the trend overlay with black line.

3.3.1. Constant trends

The constant trends are those trends which show no net increase or decrease. However, there may be seasonality, or periodic fluctuations in the data.

3.3.2. Linear trends

If the plotted data attain the shape of a straight line then it may be said that there exist linear trend. This trend shows a steady increase or decrease along a straight line and the formed angle may be flat or steep.

3.3.3. Exponential trends

When formation of trend line appears to rise or fall with an increasing but not at a steady rate, or if the data plot makes the form of curve either downward or upward, then the existing trend is termed as exponential trend. The horizontally plotted x-value is an exponent of the trend line formula to derive the y-value.

3.3.4. Damped trends

Damped trends are formed when the plotted data points forms a trend line that approach a horizontal asymptote.

3.3.5. Polynomial trends
In this type of trend the formed trend line changes direction more than once. When data points are increased the trend tends to become less linear and show more fluctuations. The polynomial trends are best displayed by a polynomial equation. The data may exhibit a parabolic shape, mathematically follow second-order equations of the form $y = ax^2 + bx + c$.

3.4. Quantitative Forecasting using Trend Extrapolation

The most usual method for forecasting is extrapolation and trend extrapolation is used to locate trend that is visible in the data over time. Many tools and methods are available for using trend extrapolation and these methods check and study for trends and cycles in the past data. Algebraic techniques are one of the more precise, varied, and powerful techniques of trend extrapolation.

The three common algebraic techniques for trend line extrapolation are the use of mathematical formulations, use of specialized function with the help of computer programming and the third technique is to make a graph with a trend line, and extend it to future time periods. All the trend extrapolations methods assume that current trend in the data will continue in future. However, less certain will be the forecasting if we attempt to forecast for large time period.

4. Literature Review

Balance of trade of Pakistan over the year has been affected in both short run and the long run. But the impact of the short run is more than the long run. It also showed that the persistent or long run deficit in balance of trade will create chronic situation over the years.

Akhtar (2003) the study investigated the presence of regular trends in the exports and imports of goods in Pakistan for quarterly data over the period of 1982:1 to 2002:1. To check stochastic non-stationarity in seasonal component in each variable, unit root tests were applied. Empirical results showed an uncertain finding that stochastic effects are relatively less significant than deterministic. But with respect to forecasting, integrated models such as ARIMA, mixed ARIMA, and ARIMA-GARCH, beat deterministic models.

Ghafoor et al., (2005) studied the historical trends of trade (BOT) in Pakistan from 1971 to 2003, and investigated future projections for BoTh exports and imports. Log linear model is used for estimating growth rate of past trends whereas for the purpose of forecasting of future trends in exports and imports Auto Regressive Integrated Moving Average (ARIMA) model was used. This study showed an increasing tendency for BoTh exports and imports.

Maravall (2006) investigated time series analysis with seasonal adjustment and trend-cycle estimation of Japanese balance of trade and its determinants using automatic form of TRAMO and SEATS programs based on the ARIMA model methodology. Results of the study showed that SEATS can be considered as best technique among different econometric models for estimation. The study also analyzed selection of choice between direct and indirect aggregate adjustments. It is concluded that, because aggregation has a strong effect on the supernatural shape of the series, and because seasonal adjustment is a non-linear transformation of the original series, even at the cost of demolishing identities between the original series, direct adjustment is preferable. Kumar et al., (2010) attempted to investigate the element of the balance of trade. Their results showed that a deep decline in the exports of the industrial products is the major cause of the trade deficit. They construct a Uni-variate time
series model by employing Box-Jenkin’s methodology of constructing Autoregressive Integrated Moving Average model to forecast the exports of industrial goods from Punjab for period 2010-2020, whereas model was estimated on the basis of data collected from 1974 to 2008. For statistical significance of the accurate model different analytical tests were used and the results showed a sharp decline in growth of exports of industrial products from Punjab in the period selected for forecasting.

Haque et al., (2006) investigated econometric models (Box–Jenkins type Autoregressive Integrated Moving Average & Deterministic type Growth models) for efficient forecasting of shrimp and frozen food export earning of Bangladesh. The results revealed that the ARIMA and the Quadratic Deterministic Type model were best forecasting technique for both shrimp and frozen food export earnings. While for short term forecasting ARIMA model is more efficient than the Quadratic model.

Fullerton et al (2004) investigated international merchandise trade flow and trade growth through the United States - Mexico border region. The study analyzed short term time series characteristics of cross border trade flows all the way through Texas, El Paso for the period January 1995 to December 2002, to know if border trade flows can be effectively modeled or not. The assumption was tested by applying a transfer function ARIMA econometric methodology. The outcome of the study indicated that in determination of monthly fluctuations in border area trade flows, economic activity in the United States and Mexico, along with relative prices adjusted for exchange rates plays an important role.

5. Methodology and Descriptive Analysis
5.1. Theoretical Modeling
Mercantilism is the first economic-based theory of international trade. Mercantilists advocated that trade was a win-lose game; the benefits of trade which a country experiences correspond with loses for another country (Jager and Jepma, 2011). Adam Smith laid the foundations for the next trade theory: absolute advantage. After Smith’s ideas, Ricardo’s theory of comparative advantage explains how a country can still benefit from trade even if it does not have an absolute advantage. Ricardo depicted the scenario between two countries which exchange two goods. Its core concept is the opportunity cost: the possibility of producing more of one good in the detriment of another.

Economists afterwards started to think more about the determinants of trade composition and the source of comparative advantages. The factor proportions theory answers those queries. In conclusion, the theories briefly explained above although have different starting points, they all mainly assume the flow of trade for greater production and consumption opportunities, the widening of international markets, more variety, especially for developing economies, lower prices and increases in income. This research study more importantly investigate on the chronic issue of the balance of trade of a country and how to forecast balance of trade with the help of most sophisticated techniques of trend analysis.

5.2. Hypothesis for Investigation
If balance of trade progress at the same pace Pakistan has to face deficit in future Trade Balance and reduction in its foreign reserves; with the passage of time rate of exports are
increasing but not at a progressive or required rate i.e. exports are increasing at a decreasing trend. The following hypotheses are to be tested:

- **H₀**: Historical data predicted the trend of Balance of trade from FY 1972 to 2015.
- **H₁**: Historical data has not predicted the trend of Balance of trade from FY 1972 to 2015.
- **H₀**: Balance of trade has been forecasted on the basis of ARIMA model from FY 1972 to 2015.
- **H₂**: Balance of trade has not been forecasted on the basis of ARIMA model from FY 1972 to 2015.

### 5.3. Data Collection and Sampling
Secondary data on Balance of Trade (1972-2015) of Pakistan was used for the study. Data was collected from various issues of Economic Surveys, Annual Reports of State Bank of Pakistan and Ministry of Finance.

The quantitative analysis was being conducted from 1971-72 due to the immense effects of the partition of East Pakistan (Bangladesh) on both trade volume and pattern. Due to these economic fluctuations the result of the model may not support the actual situation so to avoid the influence of extreme values on the outcome of the final result data under consideration, empirical analysis starts from 1971.

### 5.4. Method Used for Trend Analysis for Balance of Trade
Various econometric techniques of forecasting were used for predicting future trend of foreign trade balance. But the study showed that ARIMA was most efficient and suitable model for forecasting such series.

### 5.5. The Simple Moving Average
Generally, a simple or un-weighted moving average is the easiest technique to smooth a time series calculations. Where the smoothed statistic \( s_t \) is the mean of the final \( k \) observations:

\[
s_t = \frac{1}{k} \sum_{n=0}^{k-1} x_{t-n} = \frac{x_t + x_{t-1} + x_{t-2} + \ldots + x_{t-k+1}}{k} = s_{t-1} + \frac{x_t - x_{t-k}}{k} \quad \ldots \ldots (5.1)
\]

And the choice of a digit \( k > 1 \) is random. A small value of \( k \) will have a smaller amount of a smoothing result and is more reactive to new variation in data, while a higher \( k \) will have a larger smoothing outcome, and generate more obvious lag in the smoothed series. One drawback of this method is that it cannot be used on the first \( k - 1 \) terms of the time series.

### 5.6. The Weighted Moving Average
A little more complicated technique for smoothing an unprocessed time series \( \{x_t\} \) is to compute a weighted moving average by selecting a set of weighting factors in first step

\[
\{w_1, w_2, \ldots, w_k\} \quad \ldots \ldots (5.2a) \quad \text{Such that} \quad \sum_{n=1}^{k} w_n = 1 \quad \ldots \ldots (5.2b)
\]

Now applying the above weights to determine the smoothed statistics \( \{s_t\} \):
\[ s_t = \sum_{n=1}^{k} w_n x_{t-1-n} = w_1 x_t + w_2 x_{t-1} + W_3 x_{t-2} + \ldots + w_k x_{t-k+1} \] (5.3)

The weighted factors are usually selected to grant more weight to the most up-to-date terms in the time series and lesser weight to older data. This method of smoothing data has the same inconvenience as the simple moving average procedure (i.e., it cannot be used until at least \( k \) observations have been made), and that it involves a more intricate estimation at each step of the smoothing process.

Adding to this shortcoming, if the data from each period of the averaging is not accessible for analysis, it may be complicated if not impracticable to rebuild a varying indication correctly (as older samples may be given less weight). If the number of stages missed is known however, the weight age of values in the average can be adjusted by giving equivalent weight to all missed samples to evade this problem.

5.7. HOLT- WINTERS Model

To take into account a possible linear trend, Holt Winters model expand exponential smoothing can be observed. The equations constitute of two smoothing constants \( \alpha \) and \( \beta \):

\[ L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + b_{t-1}) \] (5.4a)
\[ b_t = \beta (L_t - L_{t-1}) + (1 - \beta)b_{t-1} \] (5.4b)
\[ F_{t+m} = L_t + b_t m \] (5.4c)

Here \( L_t \) and \( b_t \) are respectively (exponentially smoothed) calculate approximately the level and linear trend of the series at time \( t \), whilst \( F_{t+m} \) is the linear prediction from \( t \) onwards.

5.7. ARIMA Model

Box and Jenkins (BJ) or ARIMA (\( p, q \)) refers to the model with \( p \) autoregressive terms and \( q \) moving-average terms. The model contains the AR (\( p \)) and MA (\( q \)) models,

\[ X_t = c + \varepsilon_t + \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \]

A stationary time series is assumed in the Box-Jenkins model. To achieve stationarity it recommends differencing non-stationary series one or more times. Such methodology produces an ARIMA model, with the "I" denoting "Integration".

ARIMA is one of the most efficient and advance techniques of forecasting. The acronym ARIMA, is a combination of auto regressive and moving average models, stands for ‘Auto Regressive Integrated Moving Average’. Lags of the differenced series and lags of forecast errors appearing in the forecasting equation are called, ‘Auto Regressive’ and ‘Moving Average’ terms respectively.

As mentioned earlier that stationary time series is assumed in the BJ model so a time series, which needs to be differentiated, in order to make stationary is said to be integrated version of stationary series. A non-seasonal ARIMA model is denoted by ARIMA (\( p, d, q \)), according to Box-Jenkins. Here \('d'\) is the number of times series is differenced to make stationary. ARIMA or BJ methodology consists of sequence of four steps: Identification, Estimation, Diagnosing and forecasting.
6. Methodology, Results and Discussion

For forecasting, one has to decide which method shall be used in order to get a good forecast for the underlying data series. Here both model based and model free procedures will be analyzed. For their relative simplicity the study first discusses the model free forecasting methods which do not explicitly suspect any particular model to be responsible for having generated the original data series. For the choice of a particular forecasting method one should certainly look at the characteristics of the observed data series. Apart from many statistical criteria used for these purposes also a pure visual inspection of the time plotted data series might offer a valuable insight into its actual form and dynamics. Figure below represents the series of balance of trade for past 45 years of Pakistan. The data is shown through histogram below.

Figure: 6.1. Trends in BOT of Pakistan for Past Forty Five Years

6.1. HOLT-WINTERS Method

The simplest model free method is the exponential smoothers, e.g. simple and double exponential smoothing. According to time plot of the Balance of Trade series in above figure these smoothing methods are not expected to provide a good forecast for this data series. Even a quick look at the time plot points out an obvious seasonality of the series, at least for the first 24 years. After 1971 the regularity of the fluctuations seems to be widely reduced, though the fluctuations from the mean level are quite high anyway. This change in balance of trade might be explained because of partition of the country happened in 1971 and before that Pakistan was the big agricultural industrial economy and reduce the effect of the deficit in the economy due to huge exports as seen in the first figure (6.1) above. Neither simple nor double exponential smoothing are able to capture seasonal elements of the analyzed data series, therefore none of them can actually provide an accurate fit to the underlying trade balance series.

Though to see if our expectations are right and also for the purpose of comparison to the later used procedures we estimated the optimal smoothing parameter $\alpha$ for sample of trade balance series. The calculation was provided by the statistical program E-views which had used no seasonal holt-winter method. Since the optimal $\alpha$ equal’s 0.01and beta is 0.07 which actually points out a very week smoothing, the smoothed series is not similar to a flat line.
Because of the form of the smoothing equation simple exponential smoothing only offers a constant forecast while extrapolating any k-step estimation into the future. Since by applying the simple exponential smoothing the value of the alpha goes down to 0.005 with no trend and while using the double exponential smoothing method its goes the same value of alpha but with trend as shown in the graph below.

Since the double exponential smoothing might only be advantageous in the way that it might enable to capture a potential trend in a data series. It might be much more plausible to apply a method which could also exploit the information of the series. Holt-Winters method seems to offer a good alternative to the previously discussed simplest exponential smoothers. There are two variants of the Holt-Winters procedure, one multiplicative, the other one additive. They only differ in the way how to enter the smoothing equation. Either it adds up or multiplies with the smoothed variable in order to get the forecast. Unfortunately the multiplicative and additive version is unable to treat data series which takes both positive and negative values.

From the results it is concluded that the Small values of alpha means that the forecasted value will be stable (show low variability) Low alpha increases the lag of the forecast to the actual data if a trend is present. Large values of alpha means that the forecast will more closely track the actual time series. So, in both cases alpha value is lower and trends, which actually points out a very strong smoothing. To evaluate the accuracy of the forecasting method let us now look at our objective criterion, the mean squared error. It seems as if the fluctuations of the forecast series caused by the factor γ often went in the opposite direction than the actual BOT development out of sample. The speed at which the older responses are dampened is a function of alpha and beta. When alpha and beta is close to 1, the dampening effect is quick; when alpha and beta is close to 0, the dampening effect is slow. (Appendix-1)

6.2. Estimation of Stationarity {Augmented Dicky-Fuller (ADF) Unit root Test}

A new insight into the forecast ability of the trade balance series might be gathered through the application of some model based procedures. In such a case one tries to find a model which might be responsible for having generated the data or at least to be as similar as possible to the original data generating process of the series. Each model coincides with some data series which might fulfill some specific characteristics. Thus, before the proper selection of a potential model let us first look more closely at our existing trade balance series.

First of all the stationarity of the data (the independence of the first two moments on time) and the autocorrelation (dependencies between the lags of the series) are to be discussed in order to get some insight into the data generating process. E.g. autoregressive or moving average models require stationary data with specific autocorrelation or partial correlation functions if considered as responsible for the generating process. To check the stationarity one usually applies the Augmented Dickey–Fuller (ADF) test to the underlying data series, which is a test for a unit root in a time series sample. The null hypothesis simply assumes that there is a unit root.

This assumption may or may not be rejected for some chosen confidence level. The ADF value (lagged differences of 4) of the in sample trade balance series has a value of -5.49. Since
the respective 1% critical value equals -3.61, we can clearly reject the presence of a unit root at a confidence of over 99%. Regarding the first moment of the series this actually is not much surprising since the mean of the series even visually seems to stay constant over time, which also was supported by a trend variable of 0 estimated within the previously executed Holt-Winters procedure. Aggregately however the augmented Dickey-Fuller test has demonstrated that the analyzed in sample BOT series does not have a unit root and therefore implies a stationary process with a quite high level of confidence. Hence we will treat it as stationary for the purposes of model selection.

6.3. Correlation Function & Partial Auto Correlation Function Testing

Another important property of the data series which can be very useful in order to select an accurate forecasting model is the autocorrelation of the observed variable, thus the correlation between its different lags is observed. The autocorrelation function (ACF) and the partial autocorrelation function (PACF) both offer a valuable insight into the linear dependencies between the lags of the variable which might simplify the inclusion of independent variables within the model equation. Both, ACF and PACF for the in sample trade balance series are portrayed in figure below. It seems some pattern at the level shown in the figure below.

Table: 6.1. Auto-correlation and Partial Correlation Test Results

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| Sample: 1972 2016 |
| Included observations: 45 |

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To see whether first difference can get level-stationary time series or not, the first difference series "DBoT" becomes stationary as showing in line graph and is white noise as shown no significant patterns in the graph of correlogram. And the unit root test also confirms the first-difference becomes stationary. The evident supports that ARIMA (0, 1, 0) is suitable for the time series. Then, it can construct the ARIMA model.

Table: 6.2. Unit Root Test Results

| Date: 05/21/16 | Time: 13:36 |
| Sample: 1972 2016 |
| Included observations: 44 |

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6.4. ARMA Model

Autoregressive (AR) process along with Moving Average (MA) constitutes autoregressive and moving average (ARMA) process, which is used only to model stationary time series. In order to extend this model into non-stationary time series model, the concept of Autoregressive Integrated Moving Average (ARIMA) has been developed. Thus an ARIMA model is a combination of an autoregressive process
and moving average (MA) process followed by an integration term, which make the non-stationary time series into stationary time series. The general non-seasonal model is known as ARIMA (p, d, q):

**AR:** P = order of the auto regression part

**I:** d = degree of differencing mixed up

**MA:** q = order of the moving average part

The equation for the ARIMA (p, d, q) model is as follows:

\[ Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q} \]

Or in back shift notation

\[(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p) Y_t = c + (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q) \varepsilon_t \]

Where,

- \(C\) = Constant term,
- \(\phi_i\) = i the autoregressive parameter,
- \(\theta_j\) = j the moving average parameter,
- \(\varepsilon_t\) = the error term at time t,
- \(B^k\) = the k the order backward shift operator.

To identify a perfect ARIMA model for a particular data series, Box and Jenkins proposed a methodology that consists of three phases is known as Box-Jenkins methodology, or in short BJ methodology. The total process of selecting a model is nothing but an iteration procedure that contains the following phases, namely Identification, Estimation & Diagnostic Checking and Application.

Before proceeding, to the empirical part of this research study it will also narrate briefly about an autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average model. These models are fitted to time series data either to better understand the data or to predict future points in the series. The model is generally referred to as an ARIMA (p,d,q) model where p, d, and q are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average parts of the model respectively. Since we already discussed the AR and MA part in precise, the integrated aspect of the data leads to differencing of the series in order to achieve a subsequent stationary series fit ARMA process and together they are mentioned as ARIMA.

We tested the data for many basic kind of autoregressive and moving average models, but the best fit on the basis of aforementioned objective functions is achieved for ARMA (0,1,0) with autoregressive and moving average component of order 0, depending upon all the mentioned objective functions namely minimized square sum or errors, Akaike info criterion and Schwarz criterion. ARIMA (0,1,0) shows the data is constant with white noise and the suitable model will be random walk model. Eviews 9 provide automatic ARIMA forecasting model selection criteria, which also support the aforementioned outcome.

**Table: 6.3. Automatic ARIMA Forecasting**

Selected dependent variable: D(BOT)

Date: 05/22/16   Time: 00:12

Sample: 1972 2016

Included observations: 44

Forecast length: 0
Number of estimated ARMA models: 25  
Number of non-converged estimations: 0  
Weighting method: Smoothed AIC

### Table 6.4: Model Selection Criteria Table

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<th>LogL</th>
<th>AIC*</th>
<th>BIC</th>
<th>HQ</th>
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6.5. The RANDOM WALK MODEL

Random walk model is among the easiest and continent as well as one of the most significant models in time series forecasting. The model presumes that in every time period an observed variable will have a random walk away from its prior step, and the trend has autonomous and identical scattered steps in range independent and identically distributed (i.i.d.). In other words it can be stated that variable at first difference is a series to which the mean model can be applicable. Therefore, a time series that walk all over the plot, but its first difference seems to be an i.i.d. series, then the series will be appropriate for a random walk model.

Random walk (without constant) without

A random walk model might be “drift” or “without drift” with respect to the division of step ranges either having a non-zero mean or a zero mean. The forecast for variable \( Y \) in \( k \) steps forward at time period \( n \) then expected random walk model without drift for the variable \( Y \) is:

\[
\hat{Y}_{n+k} = Y_n
\]

Or it can be stated that, it estimates that all future values will be equivalent to the most recent observed value. It doesn’t support the idea that all the values will be identical, rather they will be equally likely higher or lower, and as one goes far from the prediction point the bound or limits will increase in value or distance. Extrapolation forecast of far off future values by random walk model will produce a horizontal line, similar to that of the mean model forecasts.

Table: 6.5. Random walk (without constant) without drift
Dependent Variable: BOT
**Method: Least Squares**  
Date: 05/31/16   Time: 00:42  
Sample (adjusted): 2 45  
Included observations: 44 after adjustments

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<th>Variable</th>
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<th>t-Statistic</th>
<th>Prob.</th>
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<td>31.03176</td>
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R-squared           0.922858  
Adjusted R-squared  0.922858  
S.E. of regression  1746.095   
Sum squared resid   1.31E+08   
Log likelihood      -390.3936   
Durbin-Watson stat  1.708299   

The model shows that there is no Random walk with drift
Random-walk-with-drift model has k-step forward forecast from time period n is:

\[ \hat{Y}_{n+k} = Y_n + k \hat{d} \]

where \( \hat{d} \) represents expected drift, i.e., the average increase from one time interval to the next. Thus, forecasting in the long run by the random-walk-with-drift seems as a trend line with slope \( \hat{d} \), but it is usually attached to the most recent observed value.

**Table: 6.6. Random walk with drift**

Dependent Variable: BOT  
Method: Least Squares  
Date: 05/31/16   Time: 00:49  
Sample (adjusted): 2 45  
Included observations: 44 after adjustments

<table>
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R-squared           0.925653  
Adjusted R-squared  0.923882  
S.E. of regression  1734.458   
Sum squared resid   1.26E+08   
Log likelihood      -389.5817   
Durbin-Watson stat  1.708299   

F-statistic 522.9159   
Prob(F-statistic) 0.000000
The results integrated in the above table obtained from regression analysis of the model shows that seasonal coefficients of autoregressive part and moving average part are insignificant at critical level and are having no impact on the changes in BoT. that shows that BOT series is independent of its past values (lags of BOT) as well as its indifferent with past values of its error term (lags of error term). It is also suggested that the model sufficiently provides a fit as Akaike info criterion and Schwarz criterion are decreasing for the model with time and this in result suggest absence of over estimation as these criterions provide a penalty for estimating each parameter and in terms of over estimation of the model the value for these criterions tend to increase. Also the Durbin-Watson statistic is roughly 1.711367 for the fitted ARMA model. It means that BOT series wander around the mean, this phenomenon might be due to the uncertain production, export and henceforth imports of Pakistan.

Figure: 6.6: Residuals Test Results
We examine the residuals and it suggests that there aren’t any interdependencies in the residuals and is white noise. The ACF and PACF both remain within the bound ,of two standard errors computed as ±2/ (√T) , implying that the auto correlation is not significantly different from zero at (approximately) 5% significance level. The decay in both functions doesn’t appear
exponentially as per the theoretical suggestion of true ARMA as data generating model but might it be the strong impact that leads to the distortion in the actual behavior of ACF and PACF but otherwise the residuals appear to be clean and white noise. Also as figure above shows that the model appears to track the impact in the right direction.

7. Conclusion

Balance of trade is the monetary value of exports minus imports in a given period of time for any economy. So, it is an important barometer to measure the health of any economy. Pakistan facing the problem of trade deficit for the past decade and the deficit in balance of trade becomes more chronic over the years. So, it is necessary to evaluate the balance of trade and examine thoroughly historical data of BoT to figure out the main reasons behind BoT deficit and to forecast BoT on its bases. This will assist in overcoming the deficit in BoT through avoiding past mistakes and planning according to the future estimation. The objective of this research study was to find out firstly, historical trend in the trade balance of Pakistan, secondly, forecast the future trend in balance of trade of Pakistan, and lastly device policies and strategies for macro managers and micro managers in view of opportunities and capacity in balance of trade in Pakistan.

Different techniques were applied to check the trend and forecast of the balance of trade series. The results showed that based on the experience and the results of our analysis we would like to conclude this research emphasizing two important points. First, there might arise great difficulties in the model fitting and forecasting procedure if the data series itself is somehow divided into or in a way differently behaving sets (downward steep) thus not having a certain level of homogeneity for the aggregate sample. Usually external effects are responsible for such properties and if these cannot be captured by the information used the prediction gained might be very poor. We assume that this effect might have been caused by the political and economic situation of the country. Second, at least for the shorter underlying BOT series, which we analyzed, it has been shown that their introduction into the modeling process doesn't necessarily improve the accuracy of the forecasts. On the basis of the above discussion, we can conclude that, to forecast BOT data, one can easily use ARIMA model.

From the pattern of the graphical representation of the models (Figure) we can conclude that on the whole the trend of BOT rose downward through all around the years. It should also be borne in mind that a good forecasting technique for a situation may become inappropriate technique for a different situation. The validation of particular model must be examined as time changes. So, the first two objectives achieved from the above discussion and lastly, government should take serious action at both macro and micro level in order to boost up the exports of the country and on the other hand government should reduce the imports by all means. As from the analysis (trend/ forecast) it is understood that the situation of balance of trade become more chronic. Political official and macro manager should take it seriously and devices all polices to the capacity of the economy in order to enhance the exports and decreases the imports and which will shows a healthy impact on the balance of trade.

REFERENCES


Appendix-1

Sample: 241
Included observations: 40
Method: Holt-Winters No Seasonal
Original Series: BOT
Forecast Series: BOTSM

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<tr>
<td></td>
<td>Trend</td>
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</tbody>
</table>

Sample: 241
Included observations: 40
Method: Single Exponential
Original Series: BOT
Forecast Series: BOTSM

Parameters: Alpha 0.0050
Sum of Squared Residuals 1.33E+08
Root Mean Squared Error 1822.555

End of Period Levels: Mean -172.6668

Sample: 2 41
Included observations: 40
Method: Double Exponential
Original Series: BOT
Forecast Series: BOTSM

Parameters: Alpha 0.0420
Sum of Squared Residuals 1.39E+08
Root Mean Squared Error 1866.053

End of Period Levels: Mean -630.7583
Trend -0.212793

Null Hypothesis: BOT is a martingale
Date: 05/21/16  Time: 14:18
Sample: 1972 2016
Included observations: 44 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

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<table>
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<th>Std. Error</th>
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</table>

*Probability approximation using studentized maximum modulus with
parameter value 4 and infinite degrees of freedom
Martingale series shows whether the data is random walk or not? Variance ratio=1 depicts random walk, var>1 shows positive autocorrelation and mean averting and var<1 is negative autocorrelation and is mean reverting. If p-value <0.05% then we reject null that the series is martingale.

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