

Heavy-Tailed Distribution and Risk Management of Gold Returns

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Abstract: Gold has been recognized as the most important precious metals in the human society. Other than as a medium of exchange, gold has been a consumption and investment product for a long history. It has been recognized a well-positive role in portfolio performance by many financial market practitioners. During the recent financial crisis, gold spot prices have exhibited significant volatility. Thus, effective risk management of gold spot prices play a crucial role for the industry. In this paper, we consider several types of heavy-tailed distributions and compare their performance in risk management of gold spot prices. Our results show the Skewed t distribution has the best goodness-of-fit in modelling the distribution of daily gold spot returns and generates suitable Value at Risk measures.

Keywords: Skewed t Distribution, Goodness of Fit, Risk Management

1. Introduction

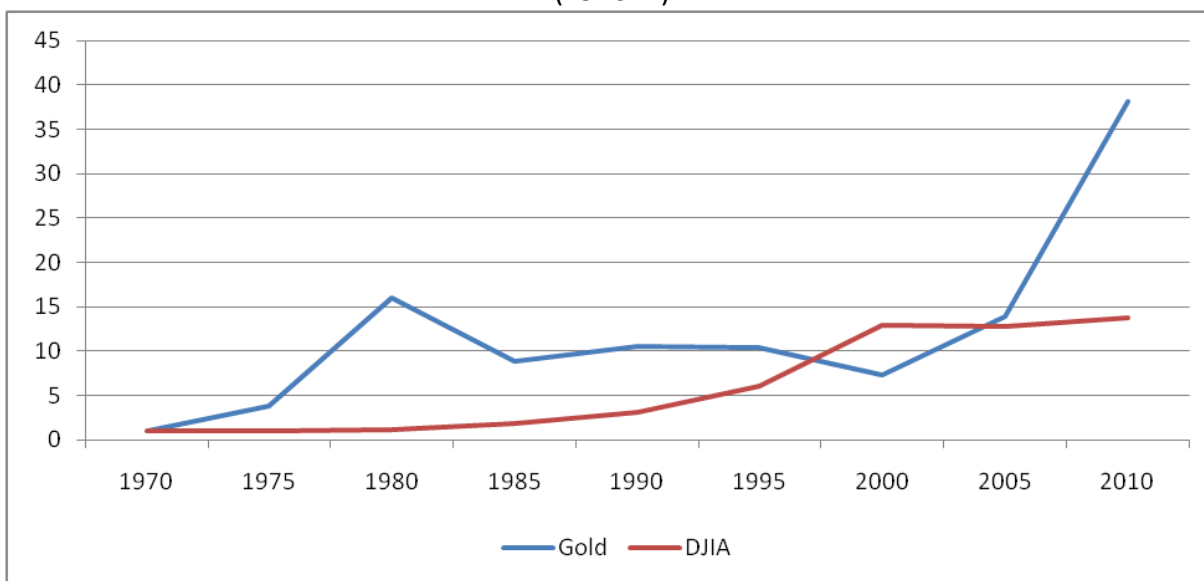
Perhaps, gold has been in a longer history as money than any other types of currency, including commodities. From 3600 BC to the present day, from ancient past to the present day, gold has played a major role in the world's development and economy. Nowadays, very few people still use gold as a medium of exchange, but gold is still one of the most popular investment products. According to the World Gold Council (WGC), as of the end of 2014 even after discounting jewellery, total market capitalization of the gold market stands around \$3 trillion. Gold ranks higher than all European sovereign debt markets, and trails only US Treasuries and Japanese government bonds. If one sets both gold price and Dow Jones Industrial Average Index to be one in 1970, one could see actually returns of investment in the gold market is even higher than returns of investment in the equity market as shown in Figure 1 (yearly data).

Investors also buy gold as a way of diversifying risk, especially through the use of futures contracts and derivatives. The world gold market is subject to speculation and volatility as other financial markets. Although gold is considered risk immunized in face of political risk and

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sovereign risk, risk management of the gold market is still very important for investors and market practitioners. According to Hammoudeh, Malik and McAleer (2011), most of industry participants apply some risk management tools based on the concept of Value at Risk (VaR) for risk management of precious metals. In this paper, we follow the newly developed method by Guo (2017a). We compared several widely-used heavy-tailed distributions and discuss how these statistical distributions perform in risk management, especially in VaR calculation.

Figure 1: Gold price vs. DJIA Index (1970=1)



Literature Review

The finance literature has been focusing on studies of heavy-tailed distributions for a long time. In 1994, Hansen introduced the Skewed t distribution, as a generalized version of the Student's t distribution into the finance literature, and the Skewed t distribution gained its popularity quickly. Guo (2017a) considered the Skewed t distribution, and compared it with four other types of statistical distributions in fitting the SP 500 index returns: normal, Student's t , Skewed t , normal inverse Gaussian (NIG), and generalized hyperbolic (GH) distributions. Guo showed the Skewed t distribution has the best goodness of fit and generates suitable hypothetical rare scenarios. Although the four heavy-tailed distributions have existed in the literature for a while, to the best of our knowledge Guo (2017a) is the first one who empirically compare them for regulatory risk management practice. In 1977, Barndorff-Nielsen developed the generalized hyperbolic (GH) distribution into the finance literature, and it soon becomes popular. Since then different types of subclass of the GH distribution have been investigated, including the very popular normal inverse Gaussian (NIG) distribution (see Figueroa-Lopez, et al., 2011, for a survey). In this paper, we follow Guo (2017a) and reconsider these five distributions but focus on risk management of the world gold market.

There are quite extensive researches on statistical distributions and risk management of the gold commodity. Hammoudeh, et al. (2011) examined volatility and correlation dynamics in price returns of gold, silver, platinum and palladium, and explores the corresponding risk management implications for market risk and hedging. Hammoudeh, et al. used the concept of Value-at-Risk (VaR) and showed the best approach for estimating VaR based on conditional and unconditional statistical tests. Hammoudeh, et al. focused on volatility modeling instead of statistical distributions fitting, which differ from our paper. Similarly, Hammoudeh, Santos and Al-Hassan (2013) considered the market downside risk associated with investments in six key individual assets including four precious metals, oil and the S&P 500 index, using VaR, but focused on volatility modeling instead of statistical distributions fitting. Batten, Ciner and Lucey (2010) modeled the monthly price volatilities of four precious metals, including gold, silver, platinum and palladium prices, and investigated the macroeconomic determinants of these volatilities. Batten, et al. showed that gold volatility can be explained by monetary variables, but not true for silver. Again, Batten, et al. focused on volatility modeling instead of statistical distributions fitting. Tully and Lucey (2007) investigated macroeconomic influences on gold using the asymmetric power GARCH model. Reboredo (2013) studied whether gold could be used as hedging tool for the US dollar for risk management purpose. Almost all of the above cited papers focus on volatility modeling, while our main interest is statistical fitting and risk measures calculation, which differ from the existing literature.

The paper is structured as follows. In Section 2, we introduce the heavy-tailed distributions. Section 3 summarizes the data. The estimation results are in Section 4. Finally, we conclude in Section 5.

2. Heavy-tailed Distributions

We consider four types of widely-used heavy-tailed distribution in addition to the normal distribution: (i) the Student's t distribution; (ii) the Skewed t distribution; (iii) the normal inverse Gaussian distribution (NIG); and (iv) the generalized hyperbolic distribution (GH). All the distributions have been standardized to ensure that their mean and standard deviation equal to zero and one respectively. Their probability density functions are given as follows.

(i) Student's t distribution:

$$f(e_t | \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^2}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \tag{1}$$

where ν indicates degrees of freedom and e_t is daily equity market index return.

(ii) Skewed t distribution:

$$f(e_i | \nu, \beta) = \begin{cases} bc \left(1 + \frac{1}{\nu-2} \left(\frac{be_i + a}{1-\beta} \right)^2 \right)^{-(\nu+1)/2} & e_i < -a/b \\ bc \left(1 + \frac{1}{\nu-2} \left(\frac{be_i + a}{1+\beta} \right)^2 \right)^{-(\nu+1)/2} & e_i \geq -a/b \end{cases}, \quad (2)$$

where e_i is the standardized log return, and the constants a , b and c are given

by $a = 4\beta c \left(\frac{\nu-2}{\nu-1} \right)$, $b^2 = 1 + 3\beta^2 - a^2$, and $c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The density function has a mode

of $-a/b$, a mean of zero, and a unit variance. The density function is skewed to the right when $\beta > 0$, and vice-versa when $\beta < 0$. The Skewed t distribution specializes to the standard Student's t distribution by setting the parameter $\beta = 0$.

(iii) Normal inverse Gaussian distribution (NIG):

$$f(e_i | \mu, \alpha, \beta, \delta) = \frac{\alpha \delta K_\lambda(\alpha \sqrt{\delta^2 + (e_i - \mu)^2})}{\pi \sqrt{\delta^2 + (e_i - \mu)^2}} \exp(\delta \sqrt{\alpha^2 - \beta^2} + \beta(e_i - \mu)), \quad (3)$$

where $K_\lambda(\cdot)$ is the modified Bessel function of the third kind and index $\lambda = 0$ and $\alpha > 0$. The NIG distribution is specified as in Prause (1997). The NIG distribution is normalized by setting

$\mu = -\frac{\delta\beta}{\sqrt{\alpha^2 - \beta^2}}$ and $\delta = \frac{(\sqrt{\alpha^2 - \beta^2})^3}{\alpha^2}$, which implies $E(e_i) = 0$ and $Var(e_i) = 1$.

(iv) Generalized hyperbolic distribution:

$$f(e_i | p, b, g) = \frac{g^p}{\sqrt{2\pi} (b^2 + g^2)^{\frac{1}{2}(p-\frac{1}{2})}} d(p, b, g) K_p(g) q\left(\frac{e_i - m(p, b, g)}{d(p, b, g)}; p, b, g\right), \quad (4)$$

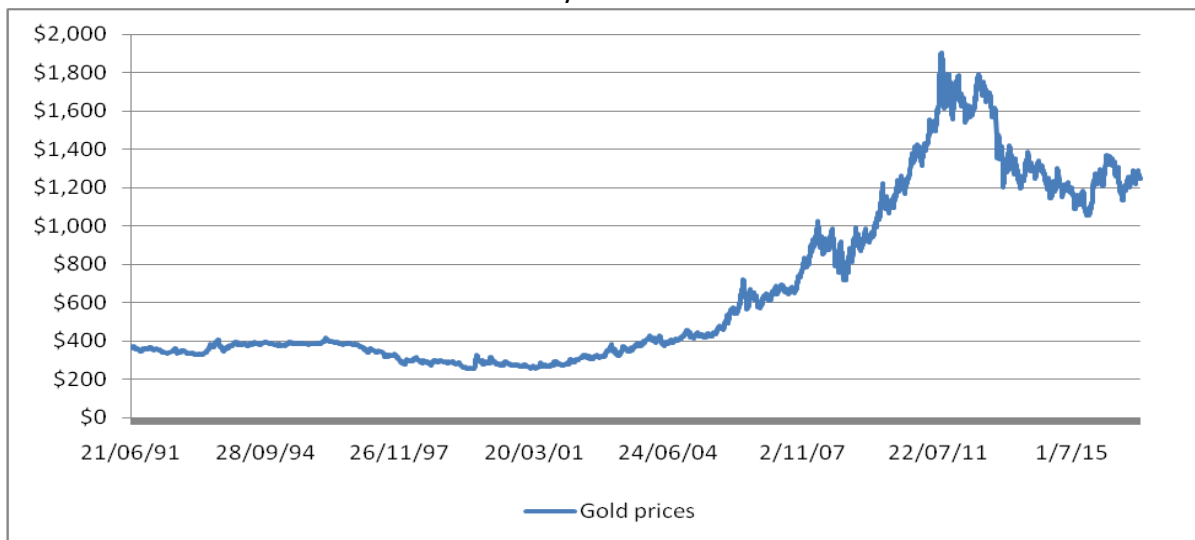
where $\tilde{R}_n \square \frac{K_{n+p}(g)}{g^n K_p(g)}$, $d(p, b, g) \square [\tilde{R}_1 + b^2 \{\tilde{R}_2 - \tilde{R}_1^2\}]^{-\frac{1}{2}} \geq 0$, and $m(p, b, g) \square -b d(p, b, g) \tilde{R}_1$.

p, b and g are parameters. The generalized hyperbolic distribution is a standardized version of Prause (1997).

3. Data

We collected the daily gold spot prices from Yahoo Finance for the period from June 27, 1991 to June 30, 2017, covering all the available data in Yahoo Finance. There are in total 7503 observations. Figure 2 indicates gold price increased quite significantly since 1991 except the Great Recession period.

Figure 2: Gold spot prices dynamics



We fit the heavy tailed distributions with the normalized gold spot returns. The world gold market is the most important precious metal markets. In this paper, Yahoo Finance originally collected the gold prices data from the Chicago Mercantile Exchange (CME). Currently, CME is the largest commodity exchange in the world. It merged with the Chicago Board of Trade in July 2007 to become the largest commodity derivative exchange. Figure 3 illustrates the dynamics of the gold spot returns. There are significant volatility clustering phenomenon and high volatilities are observed in the Great Recession period.

Figure 3: Gold spot returns

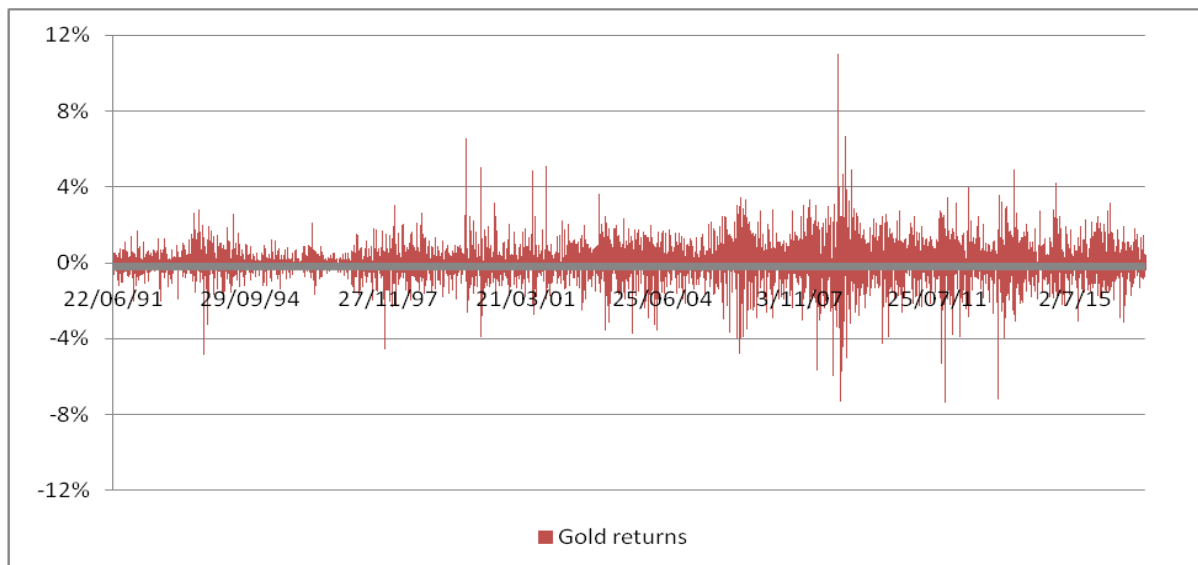


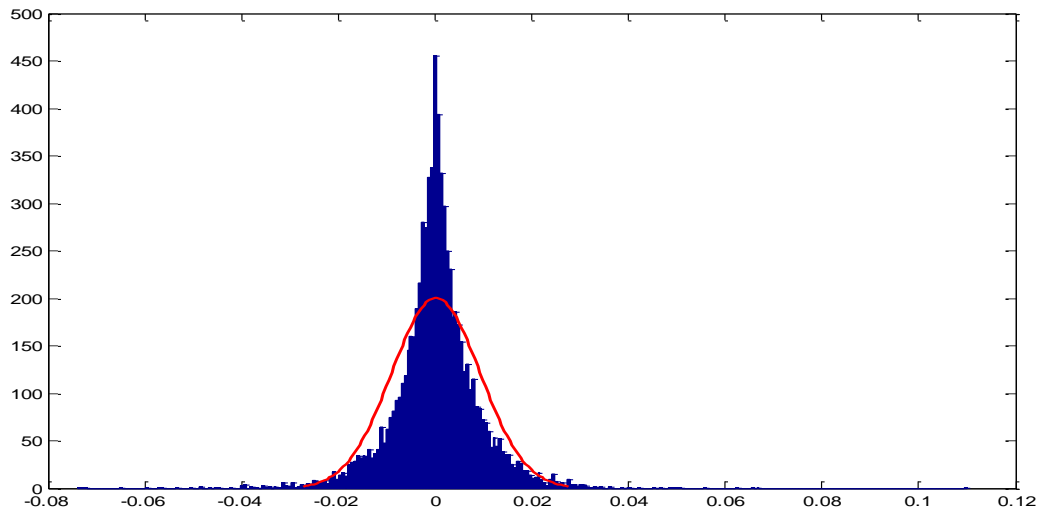
Table 1 exhibits basic statistics of the gold spot returns. The results show the gold daily returns are leptokurtotic and marginally negatively skewed. The extreme downside move is slightly less than the extreme upside move, which is at odds with most of major financial assets over the world.

Table 1: Descriptive statistics

min	max	mean	std	skewness	kurtosis
-7.38%	10.99%	0.02%	0.92%	-0.02	9.14

Figure 4 is the histogram of the raw data. We fit the returns by the Gaussian distribution and the figure clearly exhibits heavy tails.

Figure 4: Histogram of Gold returns



4. Empirical Results

4.1 Parameters Estimation

The raw return series is normalized to allow zero mean and unit standard deviation. We use the maximum likelihood estimation (MLE) method to fit the series and the estimation results of the key parameters are given in Table 2. All the parameters are significantly different from zero at 10% significance level except the parameter beta in the Skewed *t* and NIG distributions and *b* in the generalized hyperbolic distribution.

Table 2: Estimated values of key parameters

	Normal	Student's t	Skewed t	NIG	Generalized Hyperbolic
Symmetric	Y	Y	N	N	N
Fat-tailed	N	Y	Y	Y	Y
Estimated Parameters		Nu=2.96	Nu=3.01; beta=-0.005	alpha=1.13; beta=-0.011	p=-1.21; b=-.013; g=0.08

4.2 Goodness of Fit

As discussed in Huber-Carol, et al. (2002) and Taeger and Kuhnt (2014), we compare the four heavy-tailed distributions and the benchmark normal distribution in fitting the gold daily spot returns through four different criteria: (i) Kolmogorov-Smirnov statistic; (ii) Cramer-von Mises criterion; (iii) Anderson-Darling test; and (iv) Akaike information criterion (AIC) in Akaike (1973). (i) Kolmogorov-Smirnov statistic is defined as the maximum deviation between empirical CDF (cumulative distribution function) $F_n(x)$ and tested CDF $F(x)$:

$$D_n = \sup_x |F_n(x) - F(x)|, \tag{5}$$

where $F_n(x) = \frac{1}{n} \sum_{i=1}^n I_{[-\infty, x]}(X_i)$.

(b) Cramer-von Mises criterion is defined as the average squared deviation between empirical CDF and tested CDF:

$$T = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 dF(x) = \frac{1}{12n} + \sum_{i=1}^n \left[\frac{2i-1}{2n} - F_n(x_i) \right]^2, \tag{6}$$

(c) Anderson-Darling test is defined as the weighted-average squared deviation between empirical CDF and tested CDF:

$$A = n \int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1 - F(x))} dF(x),$$

and the formula for the test statistic A to assess if data comes from a tested distribution is given by:

$$A^2 = -n - \sum_{i=1}^n \frac{2i-1}{n} [\ln(F(x_i)) + \ln(1 - F(x_i))]. \tag{7}$$

(d) Akaike information criterion (AIC) is defined as:

$$AIC = -2k - 2\ln(L), \tag{8}$$

where L is the maximum value of the likelihood function for the model, and k is the number of estimated parameters in the model.

The comparison results are showed in Table 3, indicating the Skewed t distribution has the best goodness of fit compared with other selected types of distribution, followed by the generalized hyperbolic distribution, and the Student's t distribution.

Table 3: Comparison of selected types of distribution

	Normal	Student's t	Skewed t	NIG	Generalized Hyperbolic
K-S Test	0.021	0.016	0.015	0.017	0.016
Cv-M Test	0.062	0.058	0.056	0.058	0.057
A-D Test	1.79	1.66	1.61	1.64	1.62
AIC	24667	23584	23185	23795	23420

4.3 Hypothetical Rare Scenarios

To properly manage the market risk of gold commodity, we are interested in how the market performs under extreme conditions. Similarly as in Hammoudeh, et al., we adopt the concept of Value at Risk (VaR), which has been widely used in the industry. In quantitative risk management, VaR is defined as: for a given position, time horizon, and probability p , the p VaR is defined as a threshold loss value, such that the probability that the loss on the position over the given time horizon exceeds this value is p . with the estimated parameters in Section 4.1, we calculate VaRs for different confidence levels:

$$VaR_{\alpha}(e_t) = \inf\{e \in \mathbb{R} : P(e_t > e) \leq 1 - \alpha\}, \tag{9}$$

where $\alpha \in (0,1)$ is the confidence level. We select the following levels for downside moves: {99.99%, 99.95%, 99.9%, 99.5%, 99.0%}, and for upside moves: {0.01%, 0.05%, 0.1%, 0.5%, 1.0%}. From Equation (9), the hypothetical rare scenarios based on the VaR levels are given as in Table 4. Table 4 indicates that the Skewed t distribution has the closest VaRs to the nonparametric historical VaRs compared with other types of distributions.

Table 4: Scenarios for Gold return

Left Tail					
Confidence	99.99%	99.95%	99.90%	99.50%	99.00%
Empirical	-7.32%	-6.76%	-5.55%	-4.48%	-3.84%
Normal	-4.61%	-4.19%	-3.98%	-3.63%	-3.41%
T	-7.05%	-6.40%	-5.92%	-5.00%	-4.35%
Skewed T	-7.25%	-6.80%	-5.37%	-4.55%	-3.95%
NIG	-6.35%	-5.94%	-5.28%	-4.91%	-4.31%
GH	-7.07%	-6.50%	-5.69%	-4.74%	-4.07%
Right Tail					
Confidence	0.01%	0.05%	0.10%	0.50%	1.00%
Empirical	9.53%	8.45%	8.06%	7.25%	6.77%
Normal	4.61%	4.19%	3.98%	3.63%	3.41%
T	7.05%	6.40%	5.92%	5.00%	4.35%
Skewed T	9.73%	8.62%	8.21%	7.32%	6.84%
NIG	10.04%	9.41%	8.60%	8.04%	7.51%
GH	10.46%	9.52%	8.75%	7.99%	7.21%

5. Conclusions

The world gold market has grown very rapidly in the past several decades. In 1971, the gold mine production was just 1,518 tones, but in 2016 it had reached 3,169 tones. With so many individuals, financial institutions and government regulators participate in the world gold market, risk management plays a crucial role in facilitate the gold market development. In this paper, we investigate several widely-used heavy-tail distributions and their performance in fitting daily gold spot returns. Our results show the Skewed *t* distribution has the best empirical performance and provides suitable risk measures of VaR.

As observed in Figure 3, the daily gold spot returns exhibit quite striking volatility clustering effects, and thus if one could combine the fat-tailed distributions with the generalized autoregressive conditional heteroskedasticity (GARCH) framework as in Guo (2017b, 2017c), it might be another interesting contribution to the literature.

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