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The Forward Premium Anomaly in the Foreign Exchange Markets

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ABSTRACT
The purpose of this paper is to address the issue of the forward premium anomaly by using two different approaches: the long memory process and Multivariate GARCH. Initially, through the ARFIMA model, we study the properties of the forward premium on foreign exchange markets, including the presence of any long memory. Since the univariate study framework obscures the effect of conditional covariances in the measurement of risk, the transition to a more parsimonious multivariate framework is required. Therefore, we estimate, in a second time, the DCC-MVGARCH model to capture the dynamic links between forward premium series and the spot exchange return. The estimation results argue in favor of a forward premium that exhibits a phenomenon of long memory. In addition, they reveal the existence of a significant correlation sensitivity to shocks following a process of mean reversion and the detection of a strong correlation between these forward premiums and low correlation between the forward premium and the spot exchange return.

Keywords: Forward premium anomaly, ARFIMA, DCC-MVGARCH, long memory, multivariate approach.

1. INTRODUCTION

The forward premium anomaly was generally considered as one of the most important unresolved puzzles in the field of International Finance since it leads to a prominent empirical result which is often enigmatic. Indeed, in an influential paper, Fama (1984) attributes the attitude of spot and forward exchange rates to a time varying risk premium. In addition, it shows that a negative estimation of the coefficient from the regression of the uncovered interest rate parity implies that the risk premium should be negatively correlated with the expected rate of depreciation, and should have a higher variance.

In view of this, various explanations have been presented to address this anomaly but none of them has proved entirely satisfactory. In addition, a second line of research has affirmed the presence of a "peso problem" or even released the assumption of "rational expectations" in order to arrive at a reconciliation between the theory and the puzzle. It is only a few other studies of the forward premium puzzle that eventually were able to link the exchange risk premium to interest rates differentials (Carlson and Osler (2003), including the work of Obstfeld and Rogoff (1998), and Hierce Hagiwara (1999), Mark and Wu (1998), Meredith and Ma (2002) and Driskill and McCafferty (1982)). Moreover, Boudoukh, Richardson and Whitelaw (2005) attribute much of the forward premium anomaly to abnormal attitude of short-term interest rate, and not to the analysis of the relationship between fundamentals and exchange rates.

Exchange rates and their volatility are important determinants of international capital flows, relative prices of foreign direct investment, trade in goods and services and macroeconomic performance (especially small open economies). Although some currency risk can be hedged in the derivatives markets, fluctuations in the longer term are more difficult and quite expensive to be able to speculate the distorted relative prices and the optimal allocation of resources. Given this, researchers are often interested in the univariate properties of the exchange rate, with a recent interest in the behavior of the shocks towards with respect to the process of income. However, it should be checked whether these exchange rates tend to decline rapidly, as with non-integrated processes, or rather to decline more slowly, as with fractionally integrated process. In the latter case, the exchange rates show a long memory character.

Several conflicting findings on the forward premium nature allow suggesting that either a short memory or unit root models are not appropriate to model the data. In particular, Maynard and Phillips (2001) and Baillie and Bollerslev (1994) find that a fractionally integrated model can adjust properly with the forward premium while providing an explanation for the dichotomy that exists in the literature model. It is obvious that the attitude of long memory or unit root in the forward premium implies persistence in the forecast error, then allowing it to be predictable from past values. This can only lead a rejection of the hypothesis of no bias in the forward rate. Therefore, Maynard and Phillips (2001) suggest that the literature should be interested in the study of the reasons why the forward premium can demonstrate such characteristics of time series. As recognized Maynard (2003), any rejection of non bias should not be particularly problematic.

Several recent studies suggest that the forecast horizon is an important element in understanding the forward premium puzzle. We cite, for example, Chaboud and Wright (2005) who have provided some empirical validation showing that the coefficient of the regression slope is close to unity, and for a very short horizons (at a frequency of 5 minutes for the spot interest rate differentials). On the other hand, Alexius (2001) and Chinn and Meredith (2004) used quarterly data for the yields of long-term government bonds. In total, these papers suggest that in extreme cases of the distribution, the role of the risk premium or other factors causing the forward premium anomaly could be less important than in the case of a median horizon. In addition, Yang and Shintani (2006) analyze the regression of the Forward Rate Unbiased Hypothesis by varying time horizons from one day to one year. Through panel data, they offer the possibility to obtain a slope coefficient that is positive for short horizons and negative at longer horizons and improving forecast performance coefficient. Thus, their approach is less prone to the problem of potential bias caused by a mixture of different sources, periods of time or frequencies.

Our empirical study is in the same line of this work. A first empirical part is interested in the study of the dynamics of the forward premium through ARFIMA modeling *(Auto Regressive Fractionally Integrated Moving Average)* to reflect the phenomena of long memory present in the time series of forward premiums. Thereafter, in a second empirical part, we will identify any correlation between the forward premium series and the spot exchange return expressing the forward premium anomaly via a Multivariate GARCH modeling. Apart from the obvious advantage to confront the specifics of the latter, our study has the merit of wear on the parity of the Euro against the U.S. Dollar with the aim to analyze the degree of substitutability of the new single currency "the euro "against the U.S. dollar in the eyes of investors.

The present paper is organized as follows: section 2 presents the analysis and modeling of the forward premium on foreign exchange markets. Section 3 provides a specification of fractional integration process. Section 4 will be devoted to the study of volatilities and correlations of forward premiums and the spot exchange return following a multivariate approach. Section 5 concludes with the implications of our findings.

2. ANALYSIS AND MODELING OF THE FOREIGN EXCHANGE FORWARD PREMIUM

To analyze the forward exchange premium, we specify the difference between the forward exchange rate and the spot exchange rate $(f_t^{t+1} - s_t)$ as the forward premium, we denote by:

s_t : represents the natural logarithm of the spot exchange rate at time t

f_t^{t+1} : represents the natural logarithm of the forward exchange rate at time t

E_t (.): the expectations operator conditional on the information available at that date
ε_t: a white noise error term.

The main objective is to identify the best model to be used for the EUR / USD forward premium for the three-month, six-month and one-year horizons. To do this, we adopt the methodology of Box and Jenkins. This approach proposes to choose from the wide class of models AR (I) MA the model that reproduces the most the chronic. However, we note that the approach of Box and Jenkins applies only to stationary series or series may be stationary. Therefore, we apply the unit root test.

2-1- The Data

Our study focuses on the parity of the Euro against the U.S. Dollar. We examine daily observations, end of period, which are the spot and three-month, six-month and one-year forward exchange rates. We have 2408 observations covering the period from 04/01/1999 to 26/03/2008. All time series are obtained from the Datastream base and are expressed in logarithmic form to avoid the Siegel's paradox (Baillie and McMahon, 1989).

2-2- Descriptive Statistics

The Descriptive statistics relating to daily EUR/USD 3, 6 and 12-month forward premiums are shown in table (1.1).

Tab 1.1. Descriptive statistics of forward premium series

	Forward premium (3 months)	Forward premium (6 months)	Forward premium (12 months)
Nb.observations	2407	2407	2407
Mean	-4.13^{e-06}	-8.26^{e-06}	-1.59^{e-05}
Median	0.0000	1.51^{e-06}	0.0000
Std.Dev	0.003086	0.003074	0.003067
Skewness (Sk)	0.042575	0.029015	0.052139
Kurtosis (Ku)	8.622270	7.569343	7.637303
Jarque-Bera (J-B)	3170.938	2094.317	2157.821
Prob	0.0000	0.0000	0.0000
Q(12)	560.44	543.97	520.37
Q(24)	564.46	550.87	526.10

Statistics provided by Eviews 5.0

Inspection of Table (1.1) shows that the distributions of EUR/USD forward premiums (whatever the 3, 6 and 12-month horizon) are asymmetric showing skewness coefficients which are positive, then inducing thicker right series. We also note that there are indeed extreme values for all premiums eventually studied, since the skewness and their respective averages have opposite signs. This shows in particular that the Euro met phases of sudden depreciation and appreciation respectively.

About the kurtosis coefficient of 3, 6 and 12-month forward premium series, it is higher than the reference value of the normal distribution equal to 3. We then deduce that the distribution of the forward premium of the euro against the dollar is leptokurtic, then having a thicker tail than that of the normal distribution.

Given the analysis above - mentioned, it is not surprising that the null hypothesis of normality is strongly rejected by the asymptotic Jarque-Bera (1980) test for the EUR/USD forward premiums. Indeed, the JB statistic is much higher than the critical value given by the Chideux table with two degrees of freedom equal to 5.99 at the 5% level significance. Eventually, these normality tests have helped us to prove some heteroscedasticity materialized by leptokurtic distributions, and thereby confirming that it is of volatile variables.

Regarding the Q statistic, it is distributed asymptotically as a Chideux (at 12 and 24 degrees of freedom). We note clearly, from this table, all Q Ljung-Box statistics are above $\chi^2(20)$ read in the table at 5% level significance and with a value of 31.41. Also, they clearly indicate, by their critical zero probabilities, series of forward premiums unrepresentative of white noise. They also indicate that these series demonstrate significantly from a phenomenon widely known as the volatility clustering, which is ultimately linked to the notion of heteroscedasticity.

At this stage, it is important to note that the existence of non-linearity can be explained either by the presence of ARCH effect, or by the existence of a long memory.

2-3- The unit root tests

In order to test the stationarity of the Euro / U.S. Dollar three-month, six-month and one-year forward premiums, we have used the unit root tests of Dickey and Fuller test (noted ADF) (1979, 1981), Elliot, Rothenberg and Stock (denoted ADF-GLS) (1996) and Kwiatkwski and al. test (denoted KPSS)(1992). The choice depended on testing ADF and ADF-GLS tests is based on the fact that they can test the validity of the null hypothesis of a unit root against the alternative hypothesis of no unit root. At this level, the disadvantage is that they show through due to the acceptance of the null hypothesis of unit root. As for the KPSS test procedure, it helps to overcome this problem by imposing the condition of stationarity under the null hypothesis. In addition, the combined use of such tests can draw conclusions about the nature of the processes they are short memory and long memory.

We note that the ADF and ADF-GLS tests were conducted in the presence of levels of delay from 1 to 40 in the first differences of the series of the variables studied. Concerning the KPSS test, it was conducted in the window Newey-West (respectively that of Bartlett). In addition, the assumption about the presence or absence of a constant and a trend was also taken into consideration.
The results of the stationarity tests are reported in Table (1.2).

Tab. 1.2. The unit root tests

	ADF Test H_0 : unit root		ADF-GLS Test H_0 : unit root		KPSS Test H_0 : stationarity	
	In level	In 1st difference	In level	In 1st difference	In level	In 1st difference
Forward premium (3 months) EUR/USD						
Test statistic	-2.4461*** (10) [1]	-61.5077 (1) [1]	-2.3980*** (6) [1]	-19.3664 (1) [1]	1.1146*** [2]	0.1161 [2]
Critical value(1%)	-2.565927	-2.565927	-2.565926	-2.565926	0.216	0.216
Forward premium (6 months) EUR/USD						
Test	-2.2368***	-60.4702	-2.0598***	-20.0416	1.0813***	0.1419

statistic	(5) [1]	(1) [1]	(3) [1]	(1) [1]	[2]	[2]
Critical value(1%)	-2.565925	-2.565924	-2.565924	-2.565924	0.216	0.216
Forward premium (12 months) EUR/USD						
Test statistic	-2.0528*** (2) [1]	-60.4044 (1) [1]	-1.9832*** (1) [1]	-21.0929 (1) [1]	1.021*** [2]	0.1498 [2]
Critical value(1%)	-2.565924	-2.565924	-2.565923	-2.565924	0.216	0.216
Spot exchange return						
Test statistic	-34.3060 (1) [1]	-58.8128 (1) [1]	-18.1373 (1) [1]	-53.5199 (1) [1]	0.1365 [2]	0.0393 [2]
Critical value(1%)	-2.565924	-2.565924	-2.565924	-2.565924	0.216	0.216

Note: Values in parentheses denote the number of lags used.
*, **, *** indicate that corresponding statistics are significant respectively at 10%, 5% and 1% levels.
Values in brackets indicate the type of model used for knowing the ADF test: The model (1): without constant. The model (2): with constant. The model (3): Constant and trend.

We note, in light of the results of unit root tests, that the EUR/USD forward premium series at 3 months, 6 months and 12 months horizons are not stationary at the 1% level significance; then we reject the hypothesis H_1 of stationarity of series. Moreover, referring to

the calculated values of ADF, ADF-GLS and KPSS tests, we reject unambiguously the null hypothesis of a unit root in differentiated forward premium series whatever the model considered. The stationary nature of differentiated once series allows us to conclude an integration order equal to one. However, the spot exchange return series show a stationarity which is maintained for different levels of delays of up to 20, in particular for the ADF test.

$$(f_{t,3} - s_t) \rightarrow I(1), (f_{t,6} - s_t) \rightarrow I(1), (f_{t,12} - s_t) \rightarrow I(1)$$

$$d(f_{t,3} - s_t) \rightarrow I(0), d(f_{t,6} - s_t) \rightarrow I(0), d(f_{t,12} - s_t) \rightarrow I(0)$$

The series considered are non-stationary, then they should be stationnarised (remove the deterministic component) by the method of Ordinary Least Squares (OLS).
We will be based in our empirical investigation on stationary series.
At present, we can apply the Box-Jenkins technique to forward premium series expressed in first differences[1].

2-4- The Box and Jenkins method

We recall that any such procedure ARMA requires three steps:
i. Identification process
ii. Parameter estimation and model selection
iii. Validity check

The identification phase is the most important and the most difficult: it consists in determining the appropriate model in the family of ARIMA models. It is based on the study of simple and partial correlograms. Therefore, it is to choose the candidate models by detecting the possible existence of autocorrelation and trying to determine the orders of p and q delays to estimate. The most widely used method is the analysis of the autocorrelation functions (ACF) and partial autocorrelation functions (FAP) of stationary series of forward premiums expressed in first differences.

We find, for each series of forward premiums that it's only the first term of the simple correlogram is different from zero while the partial correlogram shows a damped decay of its terms. Thus, we can identify a priori process type MA (1) (moving average of order 1), we will check through regressions by OLS of the series on the candidate models. Indeed, the identification step led us to identify three candidate models representing the three-month, six-month and one-year forward premiums. These are the models AR (1), MA (1) and ARMA (1,1).
The estimation results of these models are shown in Table (1.3).

Tab. 1.3. Estimation results of the Box-Jenkins method

		AR(1)	MA(1)	ARMA(1,1)	ARIMA(p,d,q)
Forward premium (3months) EUR/USD	Coefficient	-0.473132 (-26.34356)*	-0.935627 (-130.2358)*	0.024545 (1.126576) -0.938259 (-125.3009)*	(0,1,1)
	Prob $_{Q(12)}$	0.0000**	0.530	0.543	

[1] Wold (1954) shows that the ARMA models are used to represent most of the stationary process.

	TR^2	272.9393 [0.0000]***	5.289156 [0.021459]***	10.37418 [0.005588]***	
Forward premium (6months) EUR/USD	Coefficient	-0.468676 (-26.02457)*	-0.890882 (-96.24824)*	0.029685 (1.296582) -0.895918 (-88.43161)*	(0,1,1)
	$Prob_{Q(12)}$	0.0000**	0.564	0.681	
	TR^2	194.8789 [0.0000]***	12.31154 [0.00045]***	24.50990 [0.000005]***	
Forward premium (12 months) EUR/USD	Coefficient	-0.455128 (-25.07114)*	-0.825566 (-71.77872)*	0.028098 (1.136768) -0.833076 (-61.08696)*	(0,1,1)
	$Prob_{Q(12)}$	0.0000**	0.277	0.304	
	TR^2	204.7852 [0.0000]***	13.66995 [0.000218]***	27.51187 [0.000001]***	

Estimates made on EVIEWS software (version 5.0)

Note : The values in parentheses are the t-Student statistics.
The superscript * indicates that the model is significant and is to be used for the test on the residues.
Prob is the probability assigned to the autocorrelations obtained from the Box-Pierce test compared with 0.01 (significance level of 1%).
Exhibitors (**) indicate that there is error autocorrelation of order greater than 1.
TR^2 is the test statistic obtained from the ARCH LM test on high squared residuals with a lag order specified by the appearance of the autocorrelations and partial autocorrelations.
Values in brackets denote the probabilities associated with the statitistique test TR^2.
Exhibitors (***) indicate the presence of heteroscedasticity of errors at 1% level significance.

Parameter estimation can thus be carried out on the series in first differences. A first model estimation with a constant term has shown that they are not significant. In contrast, the coefficients of the explanatory variable MA (1) are significantly different from 0.The other statistics DW and empirical F suggest a good fit. It is now necessary to analyze the residue from its autocorrelation function. We see clearly from the correlogram (Box-Pierce test) no term is outside the two confidence intervals, and the Q statistic has a critical probability greater than 0.05 regardless of the delay k and which approximates unit. The residue can be likened to a white noise process. Since there is no autocorrelation of the residuals, the model is well specified.

Therefore, the estimation of ARIMA (0,1,1) model is validated and the EUR/USD forward premium series may be validly represented by a process of ARIMA (0,1,1).

3. SPECIFICATION OF FRACTIONAL INTEGRATION PROCESSES

The ARFIMA models are long memory processes and identify the phenomena of persistence. These models were developed by Granger and Joyeux (1980) and Hosking (1981) and they are a generalization of ARIMA processes of Box and Jenkins in which the exponent of differentiation *d* was an integer.

3-1- Definition of long memory process

There are two types of definition of long memory process, as provided and presented in Mignon and Lardic (2002):

"In the time domain, the long memory processes are characterized by an autocorrelation function which decreases hyperbolically as and as the delay increases, while that of short-term memory decreases exponentially. In the frequency domain, the long memory processes are characterized by a spectral density increasing without limit when the frequency tends to zero. "(Mignon and Lardic [2002], p.324).

In the case of ARFIMA process, *d* may take the actual values, and not only integer values. A fractionally integrated series has the characteristic of a dependence between remote observations as we can see in the autocovariance function or in the spectral density function. We note that the introduction of fractional integration process helps to reduce the constraints on the autoregressive and moving average coefficients of parametric models.

An ARFIMA (*p,d,q*) process where $d \in \left]-\frac{1}{2},\frac{1}{2}\right[$ is defined by:

$$\Phi(L)X_t = \Theta(L)\varepsilon_t$$

Où

$$\varepsilon_t = \nabla^{-d}u_t, u_t: BB(0,\sigma^2)$$

$\Phi(L)$ et $\Theta(L)$ are delay polynomials of degree p and q respectively.

$$\nabla^{d} = (1-L)^{d} = 1 - dL - \frac{d(1-d)}{2!}L^{2} - \frac{d(1-d)(2-d)}{3!}L^{3} - \cdots$$

$$\nabla^{d} = \sum_{j=0}^{\infty} \pi_j L^j$$

$$\boldsymbol{\pi}_j = \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} = \prod_{0<k\leq j} \frac{k-1-d}{k}$$

and *j= 0,1,…..*

Γ corresponds to the gamma function.

The processes ARFIMA (p, d, q) are long memory processes when $d \in \left]-\frac{1}{2},\frac{1}{2}\right[$ and $d \neq 0$. They are invertible if $d > -\frac{1}{2}$ and stationary if $d < \frac{1}{2}$.

More specifically, three cases can be distinguished according to the values of the parameter *d* :

- If 0 <d <1/2, the ARFIMA process is a long memory stationary process. Autocorrelations are positive and decreases hyperbolically to zero as the delay increases. The spectral density is concentrated around low frequencies and tends to infinity when the frequency tends to zero.
- If d = 0, the ARFIMA process reduces to the standard ARMA process.
- If -1 / 2 <d <0, the process is anti-persistent: the autocorrelations decreases hyperbolically to zero and the spectral density is dominated by high-frequency components (it tends to zero as the frequency tends to zero).

3-2-The estimation methods of long memory parameter

The semi-parametric methods

Among these, we mention the method of Geweke and Porter-Hudak (1983) and the method of Robinson (1995).

Geweke and Porter-Hudak (GPH) were the pioneers of the development of methods for semi-parametric estimation in the early 1980s. These methods are based on the expression of the spectral density function of the process ARFIMA (p, d, q) when the frequencies tend to zero. As heuristic methods, these methods can only estimate the long memory parameter (d).

To illustrate the method of Geweke and Porter-Hudak (1983), we will present first, the expression of the spectral density function of the stationary process.

This method relies on the behavior of the spectral density around zero. It is simply to estimate the coefficients *b* and *d* by the least-squares on the following simple equation of linear regression :

$$Y_j = a + bZ_j + \delta_j$$

Where is the periodogram of the time series and b = d[8].
The estimation $\hat{d}$ follows a normal distribution when T$\rightarrow \infty$.

Parametric methods

The maximum likelihood methods are considered among the most effective methods in estimating the long memory parameter (*d*). These methods are used to estimate all parameters simultaneously, including the method of exact maximum likelihood and the method of approximated maximum likelihood of Whittle (1951).

The approximated maximum likelihood estimator of Whittle proves to be a good estimator since it is asymptotically and normally distributed (Fox and Taqqu (1986), Dahlhaus (1989)). Indeed, given the complexity of the implementation of the exact maximum likelihood parameter of fractional integration (developed by Sowell (1992) later in the time domain), Fox and Taqqu (1986) proposed an approximation of the log-likelihood function given by Whittle (1951).
The Whittle procedure is part of the parametric estimation methods using maximum likelihood which occupy an important place among the methods for estimating the parameters of a process ARFIMA (p, d, q).
We recall that the application of this method requires the prior choice of initial values for the parameters representing the ARFIMA model (p, d, q).

3-3- The estimation of long memory parameter

After specifying the ARFIMA model , we will apply , in what follows , the GPH method and the estimation technique of the approximated maximum likelihood of Whittle to the daily EUR/USD three-month, six month and one-year forward premiums in order to detect the possible presence of long memory. The implementation of these estimation techniques requires prior stationarity of the series studied. To do this, we propose to work with series of forward premiums expressed in first differences.

The estimation of long memory parameter for 3, 6 and 12-month forward premiums by the method of Geweke and Porter- Hudak (1983) requires at first fixing the power to specify the width of the band (m) of the periodogram . To do this, we found it useful to work with powers equal to 0.45 , 0.5, 0.55 and 0.8 in order to follow the evolution of the estimates obtained from the variation in the number of periodogram ordinates . The results for the estimation of the ARFIMA model by the method of Geweke and Porter- Hudak are shown in Table (1.4).

The estimation results of Fractional integration parameter parameter *d* by GPH procedure indicate that EUR / USD forward premiums at 3 months, 6 months and 12 months horizons are characterized by long-term memory . Indeed, the values of the estimates are positive and statistically significant for all premiums studied with an ordered number *(m)* of the periodogram limited to ($T^{0.45}$ and $T^{0.5}$) for the forward premiums at 3 months and 6 months horizons. Regarding the forward premium at 12 months horizon, the results are in favor of the presence of long memory as the values (m) , where the estimated parameter *d* is positive and significantly different from zero, extend to ($T^{0.45}$, $T^{0.5}$ et $T^{0.55}$) .

Tab. 1.4. ARFIMA estimation by the GPH method

m		$T^{0.45}$	$T^{0.5}$	$T^{0.55}$	$T^{0.8}$	(p,q)
Forward premium(3 months) EUR/USD	d_{GPH}	0.34370 (0.13208)	0.21806 (0.10428)	0.02224 (0.08354)	-0.68826 (0.02971)	(0,1)
	t-Student	2.6022*	2.0911*	0.26622	-23.1659*	
Forward premium (6 months) EUR/USD	d_{GPH}	0.34370 (0.13208)	0.21806 (0.10428)	0.02224 (0.08354)	-0.68826 (0.02971)	(0,1)
	t-Student	2.6022*	2.0911*	0.26622	-23.1659*	
Forward premium (12 months) EUR/USD	d_{GPH}	0.43730 (0.13208)	0.39180 (0.10428)	0.16804 (0.08354)	-0.58012 (0.02971)	(0,1)
	t-Student	3.3108*	3.7571*	2.0115*	-19.526*	

Estimates made on the RATS software (version 7.0)

Note: T is the number of observations.
m represents the width of the strip of the periodogram (with m = T^{power}).

The values in parentheses are asymptotic standard deviations.
The superscript * indicates that the fractional integration coefficient is statistically significant.
The last column indicates the order (p, q) of the estimated ARFIMA model.

These results portend a phenomenon of long memory present in a huge way for the forward premium series studied on different horizons (3 months, 6 months and 12 months). Therefore, the ARFIMA process we are studying is a stationary long memory process.

Since the application of the method of approximated maximum likelihood of Whittle (1951) requires prior setting initial values for the model parameters. This choice is crucial since the log-likelihood function is not globally concave. Thus, we evaluated all possible combinations to retain the model minimization algorithm converges, the parameters are significant and that the values of information criteria AIC, SC and HQIC are the lowest possible .

The estimation results of the ARFIMA model by Whittle procedure are shown in Table (1.5).

Tab. 1.5. ARFIMA estimation by the method of approximated maximum likelihood of Whittle (1951)

	Forward premium 3 months EUR/USD	Forward premium 6 months EUR/USD	Forward premium 12 months EUR/USD
d_{WHIT}	0.0103	0.0023	0.0020
t-Student	31.0057*	6.0254*	14.76723*
σ^2	9.3309E^{-07}	9.4099 E^{-07}	1.3665 E^{-06}
AIC	-33416.6385	-33396.3368	-32498.2825
SC	-33405.0662	-33384.7645	-32486.7102
HQIC	-33412.4283	-33392.1266	-32494.0723

Estimates made on the RATS software (version 7.0)

Note: The superscript * indicates that the fractional integration coefficient is statistically significant.
σ^2 is the estimated variance.

AIC, SC, HQIC respectively represent the Akaike information criterion (1973), Schwarz information criterion and Hannan-Quinn information criterion.

We note that all forward premiums studied are characterized by a long memory because the estimated fractional integration parameters are positive and statistically significant. This confirms the presence of a long memory.

These results are consistent with those generated from the estimation of ARFIMA process by the GPH procedure.

Following these results, we can conclude then that the EUR/USD forward premium series exhibit a long memory phenomenon whatever the horizon (3, 6 and 12 months). In addition, we deduce the existence of some persistence in the forward premium series which induces the presence of persistent shocks.

In summary, the ARFIMA modeling has contributed to highlighting the long-term dynamics and the strong dependence of the EUR/USD 3, 6 and 12-month forward premiums. The results that we have reached are consistent with the work of Kellard and Sarantis (2008) who detected the same phenomenon using the procedure GPH.

4. VOLATILITIES AND CORRELATIONS OF THE FORWARD PREMIA AND THE SPOT EXCHANGE RETURN: A MULTIVARIATE APPROACH

In this section, we propose to submit the question of the forward premium anomaly on the foreign exchange market to empirical test using a multivariate GARCH. The use of multivariate ARCH models proves intuitive since such models can capture the dynamic links between the forward premium series and the spot exchange return.

The proposed empirical application is then based on the DCC methodology in the family of multivariate ARCH models, the choice is based on its superiority over other specifications. Indeed, the DCC model is very flexible, has the advantage of being limited to a reasonable number of parameters to be estimated taking into account the time variation of the correlations between variables and the possible effect of asymmetric shocks the conditional variance

4-1- Presentation of the DCC model MVGARCH:

In this section, we propose to continue the work of Engle (2002) by exploring the conditional covariance that may exist in the relationship characterizing the forward premium anomaly.

Through DCC-MVGARCH modeling, we intend to model both variances and conditional correlations of forward premiums and the spot exchange return jointly. In this context, the DCC model (Dynamic Conditional Correlation) proposed by Engle (2002) is best suited for this purpose. The choice of this model is mainly based on comparative advantage demonstrated by the DCC specification compared to other multivariate GARCH models such as BEKK, CCC and VEC. Indeed, such a model reduces the number of parameters to be estimated.

The DCC_E model proposed by Engel (2002) is written as follows :

$$\begin{cases} H_t = D_t R_t D_t \\ D_t = \mathrm{diag}(\sqrt{h_{11t}}, \sqrt{h_{22t}}, \dots, \sqrt{h_{NNt}}\,) \\ R_t = (\mathrm{diag}\, Q_t)^{-1/2} Q_t (\mathrm{diag}\, Q_t)^{-1/2} \end{cases}$$

Where Qt is a matrix of size (N x N), symmetric and positive. It is given by:

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 u_{t-1} u'_{t-1} + \theta_2 Q_{t-1}$$

The term $\overline{Q_t}$ is the unconditional variance-covariance matrix of dimension (N x N), symmetric and positive definite while $u_t = (u_{1t}, u_{2t}, \dots u_{Nt})'$ is a column vector of standardized residuals of N assets portfolio at time t: $u_{it} = \varepsilon_{it}/\sqrt{h_{iit}}$ for i = 1, ..., N. The coefficients θ_1 and θ_2 are parameters to be estimated. The sum of these coefficients must be less than 1 to satisfy the positivity of the matrix Q. If $\theta_1 = \theta_2 = 0$ and $\overline{q_{ii}} = 1$, then we get the CCC model.

4-2- Estimation of the DCC-MVGARCH model:
We focus our analysis on the forward premiums at 3 months, 6 months and 12 months horizons and on the spot exchange return or changes in the exchange rate.
First, we present the unconditional correlation matrix and the variance-covariance matrix of the DCC model whose results are reported in Tables (1.6) and (1.7).

Tab. 1.6. Unconditional correlation matrix

	Premium 3 months	Premium 6 months	Premium 12 months	Spot exchange return
Premium 3 months	1.0000	0.934902	0.924249	0.185518
Premium 6 months	0.934902	1.0000	0.951937	0.176004
Premium 12 months	0.924249	0.951937	1.0000	0.179006
Spot exchange return	0.185518	0.176004	0.179006	1.0000

Extracted from the software Eviews 5.0

Table (1.6) exhibits remarkable unconditional correlation coefficients between the forward premiums for different horizons. Indeed, the three-month, six-month and one-year forward premiums exhibit strong unconditional correlations highlighted by the coefficients of the order of 95%, 93% and 92%. In contrast, reading this table clearly shows that the EUR/USD forward premiums are weakly correlated with the spot exchange return with levels almost close. In fact, the highest correlation between premiums is attributed to the pair (6 months premium, 12 months premium), followed by the pair on the horizon (3 months, 6 months), and the lower pair (3 months premium, 12 months premium). On the other side, the correlations of these premiums with the spot exchange return does not exceed 18.55% for an horizon of 3 months.

Tab. 1.7. Variance-covariance matrix of the DCC model

	Fwd premium 3 months	Fwd premium 6 months	Fwd premium 12 months	Spot exchange return
Fwd premium 3 months	0.672078	0.93687	0.92683	0.17943
Fwd premium 6 months	0.660981	0.740629	0.95272	0.17244
Fwd premium 12 months	0.649196	0.700539	0.730018	0.17420
Spot exchange return	0.145385	0.146669	0.147106	0.976815

Extracted from the software RATS 7.0

The majority of conditional correlation coefficients between forward premium series and the spot exchange return are high, which leads us to infer the correlation of forward premiums for the EUR / USD parity between them. Relating to the correlation between forward premiums and the spot exchange return, it is weak.

We note that DCC-MVGARCH modeling seems to be appropriate to capture the dynamic evolution of the unconditional correlation matrix. In addition, it seems to incorporate more flexibility in the specification of the variance-covariance matrix. Graphic illustrations of these results are shown in Figures 4 and 5.

Through the estimation of DCC-MVGARCH model, we try to examine the correlation between the variable conditional correlation between the forward premiums and the spot exchange return. The estimation results are presented in Table (1.8).

Tab. 1.8. Estimation results of DCC model MVGARCH

	Forward premium 3 months	Forward premium 6 months	Forward premium 12 months	Spot exchange return
Constant (M)	1.8921E^{-04} (2.33958)	1.7533E^{-04} (2.59153)	1.7835 E^{-04} (2.61338)	2.7455 E^{-04} (2.80363)
Constant (V)	2.5601E^{-05}	9.1507 E^{-06}	6.6309 E^{-06}	9.2605 E^{-08}

	(91.69026)	(5.43630)	(5.82203)	(1.78918)
Arch	0.0340 (12.16608)	0.0498 (8.17592)	0.0484 (9.32011)	0.0217 (5.36213)
Garch	-0.8887 (-48.86900)	0.1978 (1.37204)	0.4145 (4.34554)	0.9764 (217.85312)
α_{DCC}	0.1323 (28.38812)			
β_{DCC}	1.2893E^{-15} (9.31859E$^{-14)}$			

Note: The values in parentheses are t-Student statistics.

Considering the results shown in Table (1.8) relative to DCC-MVGARCH model estimations, we find that these tests conclude that the dynamic conditional correlations have a relatively small and insignificant autoregressive effect. On the other side, the coefficient α is positive and significant, it demonstrates the existence of a significant correlation sensitivity to shocks. In addition, in the bivariate estimation DCC (1,1), the sum of the parameters α and β being less than unity, shows that the process described by the model is a process of mean reversion. This finding implies that, following the occurrence of a shock, the correlations converge to the unconditional long-term level.

However, the amount of Arch and Garch parameters for each univariate GARCH estimation is very close to unity only for the case of the spot exchange return. Such a result confirms the strong persistence in conditional variances, and therefore indicates the effect of regime change that contain the series.

In addition, considering the average values of conditional correlation coefficients generated from the estimates of the DCC-MVGARCH (1.1) model, we derive high value and relatively low values. The first three mean values of Table (1.9) are very high, reflecting the strong unconditional correlation between the forward premiums for different horizons. However, this is not the case of the unconditional correlation between forward premium series and the spot exchange returs, which is rather low.

The statistical significance of the parameter α at the 5% level significance partially explains the advantage of using a multivariate modeling DCC relative to the CCC specification essentially based on the constancy of the correlation.

Tab. 1.9. Conditional correlation test

Pair (i,j)	Fw3-Fw6	Fw3-Fw12	Fw6-Fw12	Fw3-Ret	Fw6-Ret	Fw12-Ret
$COR_{i,j}$	0.93574** (0.04533)	0.92536** (0.04462)	0.95218** (0.03711)	0.185014** (0.10207)	0.175414** (0.10249)	0.178489** (0.10211)

Note : Fw3, Fw6 and Fw12 sont respectivement les primes à terme à 3 mois, à 6 mois et à 12 mois. Ret. is the spot exchange return. $COR_{i,j}$ is the conditional correlation between the studied series (i) and (j) of the pair (i, j). The values in parentheses are standard deviations. The exponent (**) indicates that the coefficient is significantly different from zero at the 5% level significance.

In summary, we can conclude that this MVGARCH modeling avoids overestimating the persistence and ensures a better measure of the transmission of volatility shocks. It also leads to a more adequate understanding of the co-movement of the markets as measured by the conditional correlation.

5. CONCLUSION

In this paper, we have analyzed the forward exchange premium anomaly given its remarkable persistence among the puzzles which characterized the foreign exchange markets. In this context, a review of the related literature clearly reports the sources of the volatility of exchange rates, the dynamics of long memory, and fractional dynamics in the financial time series. Moreover, we have studied the properties of the forward premium on the foreign exchange markets.

Given the relevance of ARFIMA processes in time series modeling characterized by a structure of long run dependence, they distance from ARMA processes by their joint perception of the dynamics of the short and long run of the studied series. Indeed, the fractional integration parameter allows to relate the dynamics of long run which is not detected by the autoregressive parameters and of moving average.

The objective being to look for the presence of a possible long memory in the forward premiums at various horizons. Accordingly, it is necessary to implement various methods of estimate the coefficient of fractional integration. The results report the relevance of ARFIMA models to recall the dynamics of long run of the exchange forward premium, corroborating the results of Kellard and Sarantis (2008) and Choi and Zivot (2007).

A second empirical shutter is carried out in this paper and whose the contribution proves enriching in the study of the forward premium anomaly. It is the multivariate GARCH modeling which presents the advantage to avoid the overestimation of persistence and guarantees a better measurement of the transmission of the shocks of volatility. It also allows a more adequate apprehension of the Co-movement of the markets measured by the conditional correlation. With this intention, we applied a rather intuitive methodology by using the DCC-

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MVGARCH model in order to capture the dynamic links between the EUR/USD 3, 6 and 12-month forward premiums and the spot exchange return of the same parity. The empirical application is based on the DCC methodology proposed by Engle (2002) due to its superiority over other specifications. Indeed, the DCC model is very flexible and has the advantage of being limited to a reasonable number of parameters to be estimated taking into account the time variation of the correlations between variables and the possible effect of asymmetric shocks on the conditional variance. In addition, this specification takes into account any changes in the conditional correlation over time.

The analysis of the unconditional correlation matrix and the variance-covariance matrix of the estimated model confirms, on the one hand, the presence of high correlations between the unconditional EUR/USD forward exchange premiums at 3, 6 and 12 months horizons, and on the other hand, a low correlation between them and the spot exchange return. The estimation results show that the dynamic conditional correlations have a relatively small and insignificant autoregressive effect, in addition to the existence of significant correlation sensitivity to shocks.

Fig.1. Graph of the differentiated EUR/USD 3-month forward premium

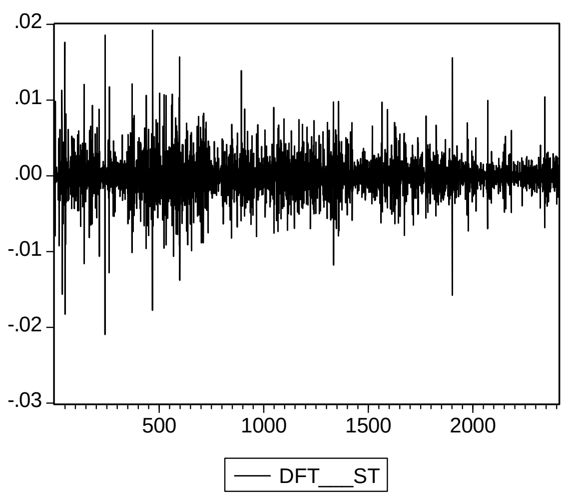


Fig.2. Graph of the differentiated EUR/USD 6-month forward premium

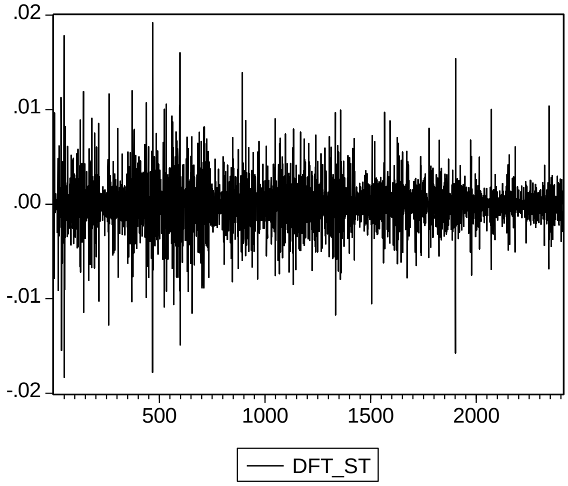


Fig.3. Graph of the differentiated EUR/USD 12-month forward premium

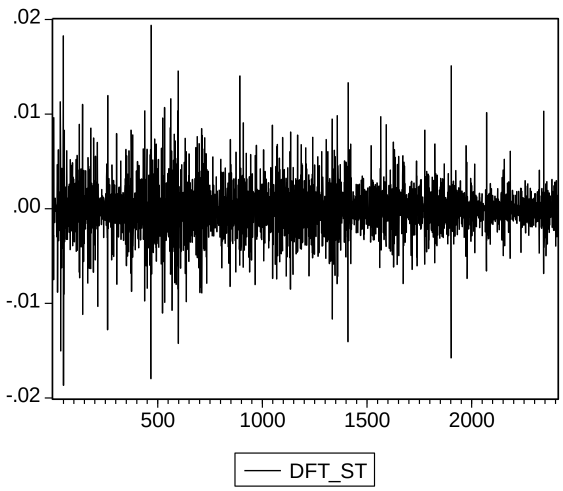
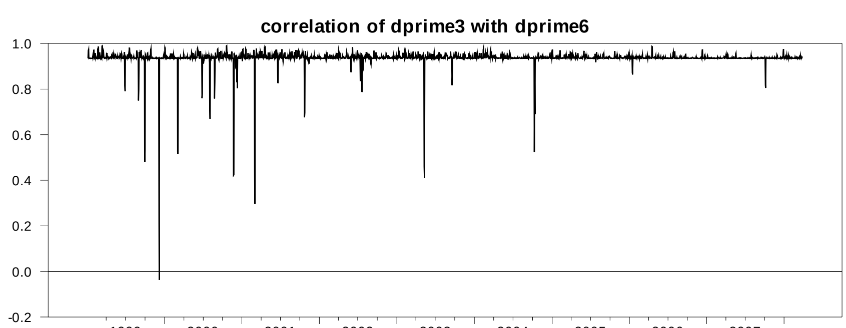
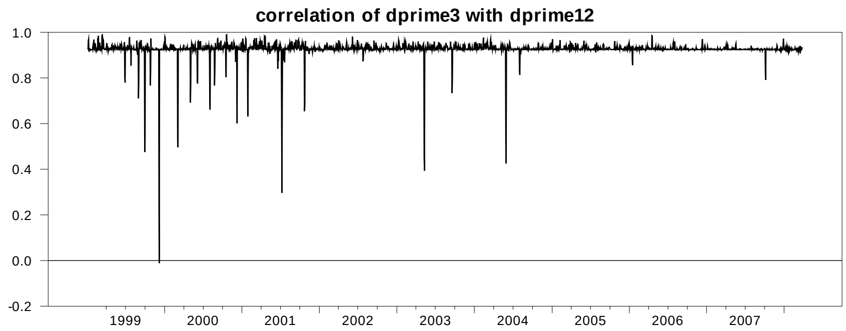


Fig. 4. The conditional correlations of the DCC model




correlation of dprime3 with dprime12
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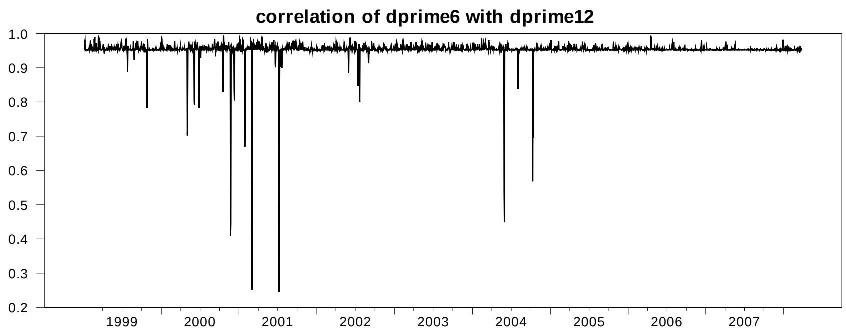
correlation of dprime6 with dprime12
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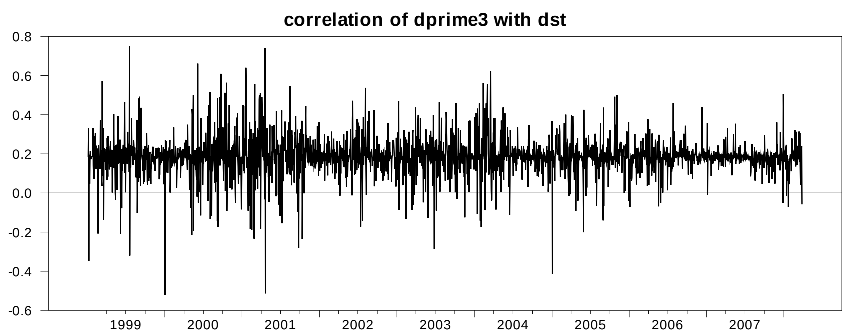
correlation of dprime3 with dst
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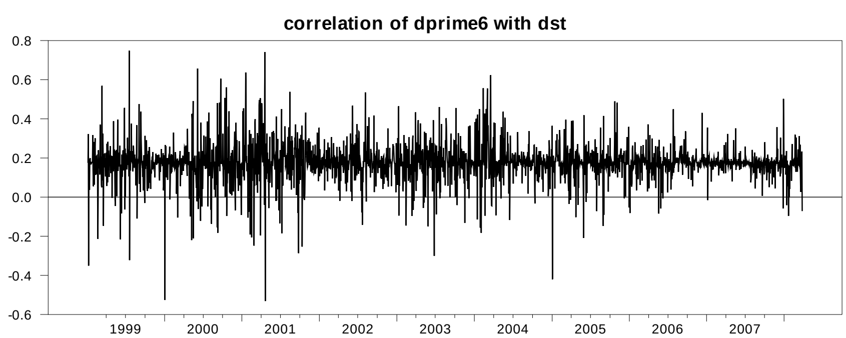
correlation of dprime6 with dst
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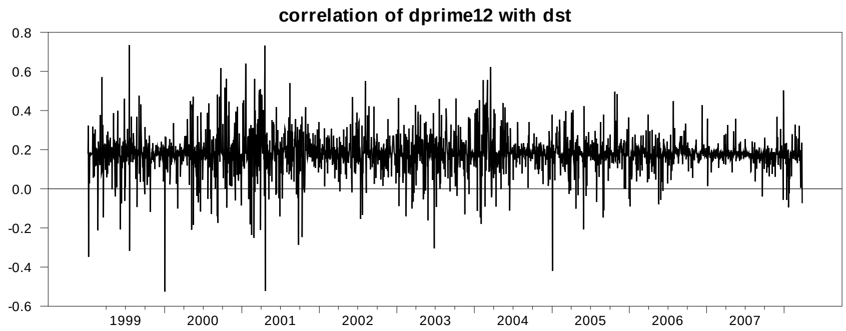
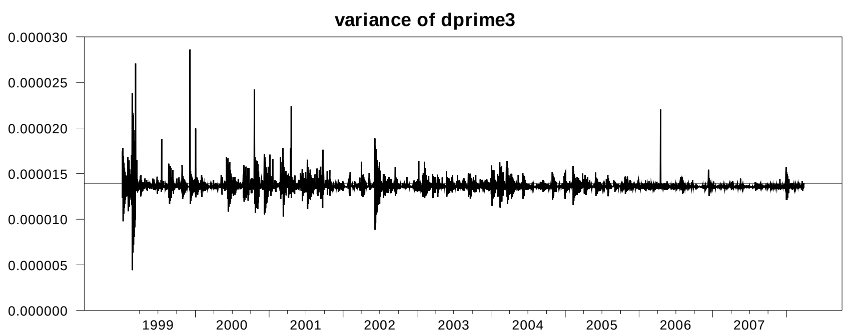
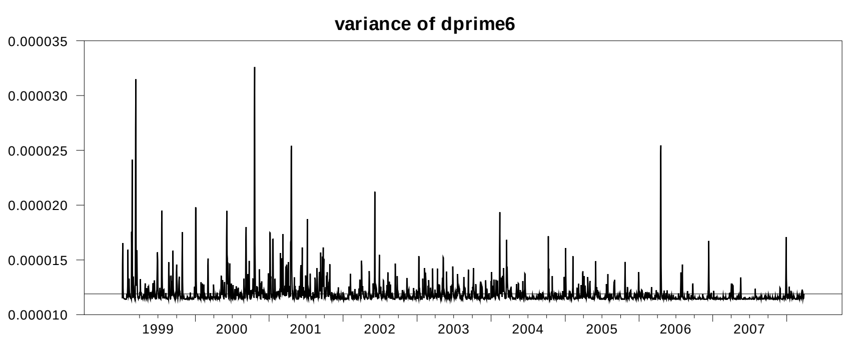
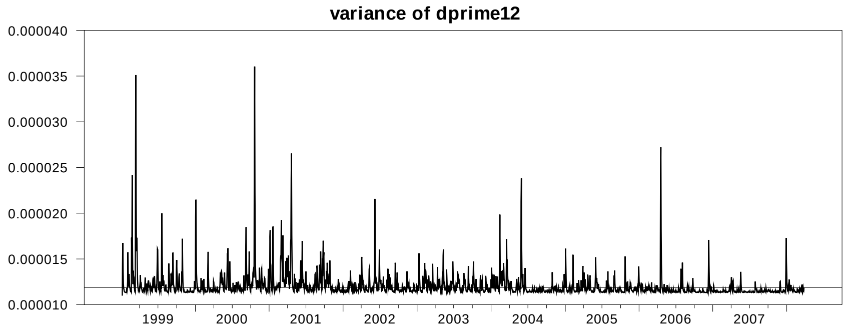
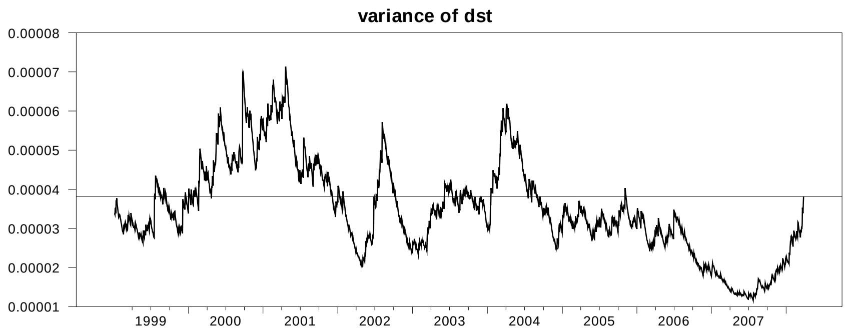


Fig. 5. The conditional variances of the DCC model









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