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Optimal Prices and Inventories Decisions on Returns Policy with Practical Examples Thorough a Stackelberg Game

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Abstract
This study explores a Stackelberg game that consists of a manufacturer who is a leader manufacturing newsvendor-type products, and two retailers who are two followers selling the products in a stochastic demand market that is divided into two various prices sub-markets allowing demand leakage from a high-priced market to a low-priced market. The objective of the game is that the manufacturer offers a returns policy contract in an effort that not only to maximize its expected profit by determining wholesale price and buy-back price, but also to improve the two retailers' expected profits by determining their prices and order sizes. We develop a simple solution procedure to the case of uniformly distributed demand, and thereby conduct a string of examples incorporating with the factors of demand leakage rate, consumers' price-sensitivity and demand variability. Many significant contributions of this study include: the chain should give up some sales opportunity in high price-sensitive markets and then offset back from low price-sensitive ones; the two retailers jointly bear the entire risk of demand uncertainty; and the returns policy contract indeed outperforms a price-only contract although it is not the Pareto improvement to low-priced market segment.
Keywords: Inventory, Newsvendor, Returns Policy, Demand Uncertainty Stackelberg Game.

Introduction

A manufacturer is always in need of many cooperative retailers to help sell its products unless it itself can do so. In that case, the manufacturer would likely offer some incentives for large order stimulation in that both sides profit performances improve, especially in the presence of market with demand uncertainty. Two most widely used incentives in practice could be a contractual commitment of returns policy accepting unsold products at the end of selling season and a guaranty of better profit in comparison with original one.

A supply chain is said to be coordinated if it creates a possible maximal channel profit as a whole by means of contractual terms negotiating among chain members. A traditional price-only contract is a trade whereby the manufacturer does not offer any incentive to retailers, and wholesale price is the only decision variable between channel partners (see Lariviere and Porteus, 2001). Lariviere (1999) reported that the price-only contract fails to coordinate a supply chain. A returns policy contract, however, is a commitment provided by an upstream channel member to accept unsold stock of a downstream channel member, and in practice it actually helps ease the retailer's nervousness on being overstocked in an uncertain demand market. A benchmark paper by Pasternack (1985) expressed the belief that channel coordination could be accessible via an implementation of returns policy. Therefore, there are two types of returns policy which are extensively discussed in the extant literature: the first is a complete returns policy that promises to refund the retailer a buy-back price (smaller than the wholesale price to avoid an arbitrage opportunity) for every returned unit; the second is a partial returns policy which reimburses retailer wholesale price only for part of unsold stock, usually a certain percentage of order size known as the quantity flexibility (see Tsay, 1999). Accordingly, Lariviere (1999) announced a mathematical equivalence between the two returns policies. Bose and Anand (2007) also dealt with the related issue by first adopting the partial returns policy and later extending to the complete returns policy with the same approach in a framework of a single-period problem.

Many studies have termed revenue management as one of the most important subjects in the fields of management science and operation research since large revenue generally yields large profit (see Bell, 1998). Thus, how to increase revenue has become a pressing topic in dealing with the optimization problem of inventory management, of which a commonly used strategy is to differentiate a single market into multiple sub-markets through various prices, mainly drawing more potential buyers. This explains why many firms offer discount prices for earlier purchases and online purchases as well. Gerchak et al (1985) ever tackled the problem whether a limited supply of bagels should be sold as a single item at a higher price or as part of another combination at a lower price. Zhang and Bell (2007) and Zhang et al (2010) initiated a model with two demand classes associated with price-sensitivity and demand dependency allowing leakage across two market segments with a linear function of price difference between the segments. Also, consumers are partitioned into two distinct sub-groups: price-sensitivity and price-insensitivity by (Pfeifer, 1989).

Whitin (1955) first investigated the newsvendor problem taking price into account and derived an explicit formula for optimal decisions for the case of uniformly distributed demand. Mills (1959) proposed a stochastic demand form in additive case with a random variable, whose result then was expanded by Petruzzi and Dada (1999) to both additive and multiplicative demand cases where the existence of a unique optimal solution was derived. Yao et al (2008) also addressed the newsvendor coordinating chain in additive demand form embracing impacts of price-sensitivity and random demand with normal distribution. Recently, Zhang et al (2010) jointly determined price and inventory replenishment when facing a distinct market segment with demand leakage and uniformly distributed demand.

In a competitive newsvendor environment, ordering decisions are assumed to be set simultaneously – there is no time sequence or priority among competitors; each competes with others for satisfying an uncertain demand market with substitutable products. Parlar (1988) verified the existence of a unique Nash solution for two vendors in such an environment. Netessine and Rudi (2003) sequentially generalized his results to the case of any number of vendors. Aside from the competitive problem, a Stackelberg game includes a leader and other follower(s); the leader first sets contractual terms and then makes his optimal decisions after knowing the follower's responses, aiming to earn higher profit than that in a competitive game. Bose and Anand (2007) studied the game stressing on one manufacturer and one vendor with exogenously fixed wholesale price, where the manufacturer maximizes its expected profit subject to the participation constraint of the retailer's optimal profit. With aid of a numerical example, they claimed that the constrained Nash equilibrium is channel coordinating. Yao et al (2008) also conducted a number of studies of the game in light of the price-only contract and the returns policy contract, ending with a conclusion that the returns policy contract actually improves channel profit. As well, in the case of high price-sensitivity and demand variability, they suggested that the manufacturer, who is a leader, needs to split some profit to the retailer, who is a follower, in order to continue the game – a similar result ever appeared in Lau and Lau (1999); Lau et al (2000) and Tsay (2001). Our numerical examples in Section 3 will also present this kind of result. Serin (2007) analyzed a related two-vendor problem with initial and reallocated demands consideration, a core value of which is that, under some conditions on profit function, problems of two-player type (Nash game) and leader-follower type (Stackelberg game) share common optimal solutions in inventory decisions. In contrast with the above-mentioned papers, Serel (2008) investigated a single-period inventory problem assuming that retailer could place his order from a reliable supplier, a risky supplier or both, in which the retailer will face a problem on how to divide his order between the reliable but high-cost supplier and the unreliable but low-cost supplier.

As stated by Zhang et al (2010), total expected profit of two variously priced sub-market segments always outstrips those from a single market; thus, this study is mainly devoted to a 1-leader, 2-follower type of decentralized channel game in association with the complete returns policy effects on channel profit performances compared to the price-only contract. Besides, the impacts of parameters such as demand leakage rate, consumer's price-sensitivity and demand variability will also be respectively emphasized. The purpose of this study is to maximize the manufacturer's expected profit by determining its wholesale price and buy-back price after knowing the two retailers' responses to the contractual commitment, and at the same time to improve the two retailers' expected profits by determining their prices and order sizes.

The remainder of this work is organized as follows. Assumptions and notation are made in Section 2 where we propose the pertinent models, along with corresponding theoretical analyses. In Section 3, we develop an approximation solution procedure towards the optimization problem such that a series of examples are conducted from which many managerial insights are accordingly acquired. Finally, remarks on this work and directions for further research are presented in Section 4 to close the study.

The Model

The problem of this study is defined as follows. A newsvendor-type item is sold in a monopoly market with a decentralized channel, including one manufacturer and two retailers. The manufacturer offers a complete returns policy contract to the two retailers before selling season, who face random demands. Unit production cost c and unit shortage cost s for unsatisfied demand are respectively incurred by the manufacturer and the two retailers. Following Petruzzi and Dada (1999) who assumed demand leakage as a linear function of two segments' price difference, the deterministic parts of demand for the two retailers are given by:

$$D_1(p_1,p_2) = \alpha_1 - \beta_1 p_1 - \gamma(p_1 - p_2)$$

$$D_2(p_1,p_2) = \alpha_2 - \beta_2 p_2 + \gamma(p_1 - p_2) \tag{1}$$

Where for segment I = 1,2, $\alpha_i \rangle 0$ is a primary demand; $\beta_i \rangle 0$ is a consumer's price-sensitivity; $p_i \rangle 0$ is a selling price; $\gamma \rangle 0$ represents a leakage rate, a marginal demand leaking from high-priced market to low-priced one. Note that, rather than the stricter regulation of $p_1 \rangle p_2$ in Zhang and Bell (2007) and Zhang et al. (2010), we are not imposing such conditions on p_1, p_2. In other words, the following three cases could happen: $p_1 \rangle p_2$ means demand leakage from segment 1 to segment 2; $p_1 \rangle p_2$ implies from segment 2 to segment 1, and $p_1 = p_2$ stands for zero demand leakage (illustrative examples in the next section will show a decisive factor that determines which segment should be classified in the high - or low - priced market). Based on Mills (1959), the two retailers' stochastic demands then are extended to an additive case below.

$$X_i(p_1,p_2,\varepsilon_i) = D_i(p_1,p_2) + \varepsilon_i \tag{2}$$

Where for segment I = 1,2, ε_i is a random variable defined on a range $\lfloor A_i, B_i \rfloor$ with density function $f_i(\varepsilon_i)$ and cdf $F_i(\varepsilon_i)$; also the mean μ_i is assumed to be zero. For convenience, we abbreviate $D_i(p_1,p_2) = D_i, X_i(p_1,p_2,\varepsilon_i) = X_i$, hereafter. Ultimately, the manufacturer optimizes its expected profit by determining the unit wholesale price *w* and the unit buy-back price *b* for returns; in response to the offered contractual terms, the two retailers jointly determine their optimal prices p_1 and p_2 as well as optimal order quantities Q_1 and Q_2. According to previous assumptions, profit of segment *i* is calculated by

$$\pi_i(p_i,p_j,Q_i) = \begin{cases} p_i X_i - wQ_i + b(Q_i - X_i) & X_i \le Q_i \\ \\ (p_i - w)Q_i - s(X_i - Q_i) & X_i > Q_i \end{cases} \tag{3}$$

for *i*=1,2, *j*=3-*i*

Applying the same scheme in Thowsen (1975) and Petruzzi and Dada (1999), we substitute $X_i = D_i + \varepsilon_i$ and define $z_i = Q_i - D_i$ into equation (3), then

$$\pi_i(p_i,p_j,z_i) = \begin{cases} p_i(D_i+\varepsilon_i) - w(D_i+z_i) + b(z_i - \varepsilon_i) & \varepsilon_i \le z_i \\ \\ (p_i - w)(D_i + z_i) - s(\varepsilon_i - z_i) & \varepsilon_i > z_i \end{cases} \tag{4}$$

for *i*=1,2, *j*=3-*i*. And expected profit for segment *i* is

$$E[\pi_i(p_i,p_j,z_i)] = \int_{A_i}^{z_i} (p_i(D_i+\varepsilon_i) - w(D_i+z_i) + b(z_i-\varepsilon_i))\, f_i(\varepsilon_i)\, d\varepsilon_i$$

$$+ \int_{z_i}^{B_i} ((p_i - w)(D_i+z_i) - s(\varepsilon_i - z_i))\, f_i(\varepsilon_i)\, d\varepsilon_i \tag{5}$$

Thus total expected profit for the two retailers is

$$E[\pi_r] = \sum_{i=1}^{2}(\int_{A_i}^{z_i}(p_i(D_i+\varepsilon_i) - w(D_i+z_i) + b(z_i-\varepsilon_i))f_i(\varepsilon_i)d\varepsilon_i$$
$$+\int_{z_i}^{B_i}((p_i-w)(D_i+z_i) - s(\varepsilon_i - z_i))f_i(\varepsilon_i)d\varepsilon_i) \quad (6)$$

Next, we let $\Lambda_i(z_i)=\int_{A_i}^{z_i}(z_i-\varepsilon_i)f_i(\varepsilon_i)d\varepsilon_i$ and $\Theta_i(z_i)=\int_{z_i}^{B_i}(\varepsilon_i-z_i)f_i(\varepsilon_i)d\varepsilon_i$ such that equation (6) can be re-written as

$$E[\pi_r] = \sum_{i=1}^{2}((p_i-w)D_i - (w-b)\Lambda_i(z_i) - (p_i-w+s)\Theta_i(z_i)) \quad (7)$$

To prove the concave property of $E[\pi_r]$ is rather complicated due to its function of p_1, p_2, z_1 and z_2, the following is the statement of our first theorem.

Theorem 1 $E[\pi_r]$ is concave in p_1, p_2, z_1 and z_2 as long as $(p_i+s-b)\beta_i f_i(z_i) > \frac{1}{2} \quad for \quad i=1,2$

Now that the concavity of $E[\pi_r]$ is obtained, we now can find optimal solutions z_i by solving first-order necessary condition $\frac{\partial E[\pi_r]}{\partial z_i}=0$, leading to $F_i(z_i)=\frac{p_i-w+s}{p_i-b+s}$ or $z_i = F_i^{-1}(\frac{p_i-w+s}{p_i-b+s})$, and the optimal inventory level $Q_i = D_i + F_i^{-1}(\frac{p_i-w+s}{p_i-b+s})$.

Likewise, optimal prices can be gained from $\frac{\partial E[\pi_r]}{\partial p_i}=0$, thus the following results are therefore drawn.

Theorem 2 For *i*=1,2, *j*=3-*i*, optimal solutions of z_i, Q_i and p_i satisfy

$$F_i(z_i)=\frac{p_i-w+s}{p_i-b+s} \text{ or } z_i = F_i^{-1}(\frac{p_i-w+s}{p_i-b+s}) \quad (8)$$

$$Q_i = D_i + F_i^{-1}(\frac{p_i-w+s}{p_i-b+s}) \quad (9)$$

$$p_i = \frac{(\beta_j+\gamma)(\alpha_i+w\beta_i)+\gamma(\alpha_j+w\beta_j)}{2(\beta_i\beta_j+\gamma(\beta_i+\beta_j))} - \frac{(\beta_j+\gamma)\Theta_i(z_i)+\gamma\Theta_j(z_j)}{2(\beta_i\beta_j+\gamma(\beta_i+\beta_j))} \quad (10)$$

Notice that the above equations (8), (9) and (10) are the participant constraints that the manufacturer should take into account when making optimal decisions. One particular result worth mentioning is that, from equation (10), the optimal p_i is composed of two irrelevant terms that are respectively induced by demand certainty and demand uncertainty; as a result, it implies that optimal prices in an uncertain demand market are always lower than those in certain demand market. Zhang et al. (2010) reported the same result as ours.

As for the manufacturer, its profit is computed by

$$\pi_m(w,b)=\begin{cases}(w-c)(Q_1+Q_2)-b(Q_1-X_1+Q_2-X_2) & X_1\le Q_1, X_2\le Q_2\\ (w-c)(Q_1+Q_2)-b(Q_1-X_1) & X_1\le Q_1, X_2>Q_2\\ (w-c)(Q_1+Q_2)-b(Q_2-X_2) & X_1>Q_1, X_2\le Q_2\\ (w-c)(Q_1+Q_2) & X_1>Q_1, X_2>Q_2\end{cases}$$

(11)

Using the same manipulations as in the retailer's problem, it can be re-expressed by

$$\pi_m(w,b)=\begin{cases}(w-c)(D_1+z_1+D_2+z_2)-b(z_1-\varepsilon_1+z_2-\varepsilon_2) & \varepsilon_1\le z_1, \varepsilon_2\le z_2\\ (w-c)(D_1+z_1+D_2+z_2)-b(z_1-\varepsilon_1) & \varepsilon_1\le z_1, \varepsilon_2>z_2\\ (w-c)(D_1+z_1+D_2+z_2)-b(z_2-\varepsilon_2) & \varepsilon_1>z_1, \varepsilon_2\le z_2\\ (w-c)(D_1+z_1+D_2+z_2) & \varepsilon_1>z_1, \varepsilon_2>z_2\end{cases} \quad (12)$$

Theorem 3 The manufacturer's expected profit is in the form of

$$E[\pi_m]=(w-c)\sum_{i=1}^{2}D_i-(c+b-w)\sum_{i=1}^{2}\Lambda_i(z_i)-(w-c)\sum_{i=1}^{2}\Theta_i(z_i)$$

(13)

Thus the manufacturer's objective is to determine *w* and *b* so as to maximize $E[\pi_m]$ subject to the equations (8) and (10).

We note that the above proposed models are applicable to any type of distribution of the random variable. However, because of difficulty in theoretically analyzing this problem, we plan to handle this optimization problem under the assumption that the random demand is uniformly distributed, and thereby provide an approximation procedure towards analytical solutions to conduct serial numerical examples, aiming at more managerial insights of the game.

The Numerical Examples

As mentioned earlier, we assume the random variable ε_i is uniformly distributed with $f_i(\varepsilon_i)=\frac{1}{\xi_i}$ where $\varepsilon_i\in[-\frac{\xi_i}{2},\frac{\xi_i}{2}]$ and $\xi_i>0, i=1,2$; then based on previous analyses, we obtain $\Lambda_i(z_i)=\frac{\xi_i(p_i-w+s)^2}{2(p_i-b+s)^2}$ and $\Theta_i(z_i)=\frac{\xi_i(b-w)^2}{2(p_i-b+s)^2}$, also optimal solutions z_i and p_i satisfy

$$z_i=\frac{\xi_i(p_i-w+s)}{p_i-b+s}-\frac{\xi_i}{2}$$

(14)

$$p_i=\frac{(\beta_j+\gamma)(\alpha_i+w\beta_i)+\gamma(\alpha_j+w\beta_j)}{2(\beta_i\beta_j+\gamma(\beta_i+\beta_j))}$$

$$-\frac{(b-w)^2}{4(\beta_i\beta_j+\gamma(\beta_i+\beta_j))}(\frac{(\beta_j+\gamma)\xi_i}{(p_i-b+s)^2}+\frac{\gamma\xi_j}{(p_j-b+s)^2}) \quad (15)$$

And the corresponding manufacturers expected profit is:

$$E[\pi_m]=(w-c)\sum_{i=1}^{2}D_i-(c+b-w)\sum_{i=1}^{2}\frac{\xi_i(p_i-w+s)^2}{2(p_i-b+s)^2}-(w-c)\sum_{i=1}^{2}\frac{\xi_i(b-w)^2}{2(p_i-b+s)^2} \quad (16)$$

Now the manufacturer's problem under the condition of uniform distribution turns into

$$\underset{w,b}{Maximize}\ E[\pi_m] \quad (17) \qquad s.t.\ \text{Equations (14) and (15)}$$

Unfortunately, its complexity prevents us from obtaining a closed-form solution; thus, the software MATHEMATICA is applied to solve the problem, but it experimentally fails to do so. After many times of trial-and-error runs, we find that the participant constraint (15), whose unknown prices p_i and p_j in this implicit expression get entangled with each other, is a main obstacle that hinders the computer from proceeding the optimization problem – which at the same time inspires us to attempt an assumption of first giving two values of p_i and p_j at the right-hand side of (15) and then solving the problem. Therefore, the following solution procedure is developed.

For *i*=1,2, *j*=3-*i*

Step 1 Let $p_i^c=\frac{(\beta_j+\gamma)(\alpha_i+w\beta_i)+\gamma(\alpha_j+w\beta_j)}{2(\beta_i\beta_j+\gamma(\beta_i+\beta_j))}$

Step 2 Clear *w,b*

Step 3 Find $p_i=\frac{(\beta_j+\gamma)(\alpha_i+w\beta_i)+\gamma(\alpha_j+w\beta_j)}{2(\beta_i\beta_j+\gamma(\beta_i+\beta_j))}$

$$-\frac{(b-w)^2}{4(\beta_i\beta_j+\gamma(\beta_i+\beta_j))}(\frac{(\beta_j+\gamma)\xi_i}{(p_i^c-b+s)^2}+\frac{\gamma\xi_j}{(p_j^c-b+s)^2})$$

Step 4 Find $\underset{w,b}{Maximize}\ E[\pi_m]$ and corresponding *w* and *b* by MATHEMATICA

Sep 5 Repeat *Step* 1~4 until absolute value of the difference between present and prior values of $E[\pi_m]$ is not exceeding a tolerant error (usually 3 repeats will reach three decimal places)

Step 6 Find optimal z_i and p_i via (14) and (15) with the above final-obtained *w* and *b* in *Step* 4. A surprising finding is that the difference between the solution p_i and the final-obtained p_i^c in *Step* 1 is to the first or even to the second decimal place, a consequence supporting the fact that the solutions z_i and p_i are approximations to analytical solutions of the problem.

Step 7 Output. We now utilize the above procedure to conduct serial numerical studies whose results will be tabulated as follows. Table 1 shows optimal solutions with respect to a variety of demand variability and demand leakage rates; Table 2 illustrates price-sensitive impacts on channel profit performances; and a comparison between the returns policy contract and the price-only contract in face of increasing demand uncertainty is listed in Table 3, where b=0 is set in the proposed models in response to the price-only contract, and products remaining at the end of selling season are assumed to be disposed of at zero for the two retailers. For convenience, let M, M_1 and M_2 respectively represent optimal expected profits for the manufacturer , the retailers in the first and the second market segment; channel profit $M_c = M + M_1 + M_2$ and channel quantity $Q_c = Q_1 + Q_2$.

Table 1.

α_1=100,

Mc
1823.09
1807.71
1792.00
1852.70
1837.26
1821.49
1858.10
1842.64
1826.84

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	ξ	w	b	P_1	P_2	Q_1	Q_2	Qc	M	M_1	M_2
γ=5	5	23.6	11.8	33.99	32.8	26.93	41.33	68.26	1256.43	256.21	310.45
	10	23.4	11.4	33.79	32.7	27.76	42.46	70.22	1258.95	244.71	304.05
	15	23.2	10.9	33.59	32.5	28.57	43.57	72.14	1261.59	232.94	297.47
γ=20	5	23.6	11.9	33.44	33.1	26.91	41.35	68.26	1256.24	242.06	354.40
	10	23.4	11.5	33.25	32.9	27.72	42.51	70.23	1258.57	230.91	347.78
	15	23.2	11.1	33.06	32.7	28.50	43.64	72.14	1261.01	219.50	340.98
γ=35	5	23.6	12.0	33.35	33.1	26.90	41.36	68.26	1256.21	239.57	362.32
	10	23.4	11.5	33.16	32.9	27.71	42.52	70.23	1258.50	228.48	355.66
	15	23.2	11.1	32.96	32.7	28.49	43.65	72.14	1260.91	217.13	348.80

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Table 2. Optimal results with respect to β_1 and β_2

$\alpha_1=100$, $\alpha_2=200$, $c=5$, $s=6$, $\gamma=5$, $\xi_1=\xi_2=\xi=5$											
	β_2	w	b	P_1	P_2	Q_1	Q_2	M	M_1	M_2	Mc
$\beta_1=2$	3	32.19	19.68	45.05	46.67	18.52	52.50	1894.20	212.80	802.44	2909.44
	4	27.23	15.21	38.54	38.54	23.43	46.20	1521.51	242.14	500.64	2264.29
	5	23.69	11.85	33.99	32.89	26.93	41.33	1256.43	256.21	310.45	1823.09
$\beta_1=3$	3	27.23	15.25	36.62	40.47	9.71	59.93	1521.34	71.43	920.33	2513.10
	4	23.69	12.20	31.99	34.12	14.99	53.28	1255.91	106.20	623.52	1985.63
	5	21.03	9.74	28.63	29.55	18.94	47.97	1057.85	126.68	429.09	1613.62
$\beta_1=4$	3	23.68	11.79	30.92	36.24	3.05	65.21	1256.28	5.43	994.64	2256.35
	4	21.03	9.66	27.41	30.99	8.33	58.57	1057.84	37.67	705.10	1800.61
	5	18.97	7.84	24.81	27.12	12.43	53.12	904.41	57.77	509.40	1471.58

Table 3. Optimal results comparing the returns policy contract and price-only contract

α_1=100, α_2=200, c=5, s=6, β_1=2, β_2=5, γ=5, ξ_1=ξ_2=ξ

	w	b	P_1	P_2	Q_1	Q_2	M	M_1	M_2	Mc
ξ=5	23.69	11.85	33.99	32.89	26.93	41.33	1256.43	256.21	310.45	1823.09
	23.45	0	33.80	32.72	26.53	41.30	1251.97	255.21	314.38	1821.56
ξ=10	23.45	11.42	33.79	32.71	27.76	42.46	1258.95	244.71	304.08	1807.74
	22.99	0	33.43	32.37	27.01	42.39	1250.60	242.67	311.37	1804.64
ξ=20	22.96	10.54	33.39	32.34	29.34	44.46	1264.33	220.90	290.72	1778.95
	22.17	0	32.77	31.76	27.97	44.50	1249.85	216.68	303.35	1769.88
ξ=30	22.46	9.64	32.98	31.95	30.82	46.76	1270.15	195.96	276.68	1742.79
	21.45	0	32.17	31.21	28.97	46.50	1251.51	189.60	292.75	1733.86
ξ=40	21.96	8.72	32.56	31.56	32.18	48.78	1276.42	169.86	261.90	1708.18
	20.86	0	31.63	30.71	29.99	48.40	1255.37	161.53	279.80	1696.70

From these results, we have the following managerial insights.

1. The γ represents internal demand-exchange from a high-priced market to a low-priced one. So, from manufacturer's point of view, it would not alter overall channel demand; thus Table 1 displays a nearly unchanged demand quantity Q_c that allows the manufacturer to set a constant wholesale price w regardless of γ, thereby resulting in an almost constant

profit *M* for the manufacturer. Also, on fears of huge demand leakage, Table 1 suggests that p_1 decreases but p_2 increases with the increase of γ in purpose of a small price gap $p_1 - p_2$, respectively leading to a decreasing profit M_1 for the retailer in segment 1 and an increasing profit M_2 for the retailer in segment 2 as γ increases. Zhang et al. (2010) reported a conforming consequence in this respect. Meanwhile, the amount of increases in M_2 overshadows the amount of decreases in M_1, eventually resulting in a higher channel profit M_c. Thus, we conclude that higher γ has slight impact on the manufacturer, negative impact on segment 1, and positive impact on segment 2 and the channel profit.

2. As anticipated, Table 1 indicates the decreasing values of *w, b,* p_1 and p_2 responding to higher demand uncertainty ξ, whereas the order sizes Q_1 and Q_2 are unexpectedly increasing. This could be interpreted as the manufacturer attempts to allure the two retailers' aggressive orders via lowering wholesale price and this movement subsequently urges the two retailers to lower prices because of that the cheaper purchasing cost as well as for consumption-stimulating concerns. However, at the moment, higher profit is not always accompanied by larger order. Table 1 show that, during an uncertain demand environment, only the manufacturer receives higher profit, but the two retailers earn less ones. This could also be understandable from the case of $\gamma=5$, $\xi=5$ in Table 1, showing that the loss of cost from each unsold product for the two retailers is $w-b=11.84$, while the manufacturer still makes a profit of $w-b-c=6.84$ from it even though it is returned, which is tantamount to the two retailers jointly bearing the entire risk of demand uncertainty during the game.

3. Intuitively, the two retailers are supposed to set cheaper prices for sales boosting purpose when dealing with high consumer price-sensitivity. Table 2 confirms this presumption by demonstrating that p_1 and p_2 respectively decrease with the increases of β_1 and β_2. Yao et al. (2008) presented a consistent result to ours. Also, to optimize channel profit, Table 2 advises the chain should give up some of the sales opportunity in a high price-sensitive market and then offset back from a low price-sensitive one, showing that Q_1 and M_1 increase, but Q_2 and M_2 decrease as β_2 increases; Q_1 and M_1 decrease, but Q_2 and M_2 increase as β_1 increases. Table 2 further points out higher β_1 and β_2 whichever would be detrimental to the manufacturer's profit, and channel profit M_c likewise. As a result, Table 2 summarizes price-sensitive impact on each game member is significantly negative.

4. From Table 2, we also disclose that p_1 is not necessary to be larger than p_2, which totally differs from the assumptions made by Zhang and Bell (2007) and Zhang et al. (2010) regulating $p_1>p_2$ must hold. Taking $\beta_1=2$ in Table 2 for example, it shows three different outcomes of $p_1<p_2$, $p_1=p_2$ and $p_1>p_2$ when $\beta_2 = 3$, 4 and 5. Making a closer inspection on Table 1-3, we realize that it is the ratio of primary demand α_1, α_2 over price-sensitivity β_1, β_2 to determine which segment should be categorized as in a high -or low- priced market. Specifically, we utilize the example above to explain that $\frac{\alpha_1}{\beta_1}(=\frac{100}{2})<\frac{\alpha_2}{\beta_2}(=\frac{200}{3})$ implies p_1 (=45.05) < p_2 (=46.67); $\frac{\alpha_1}{\beta_1}(=\frac{100}{2})<\frac{\alpha_2}{\beta_2}(=\frac{200}{4})$ implies $p_1(=38.54)=p_2(=38.54)$; $\frac{\alpha_1}{\beta_1}(=\frac{100}{2})<\frac{\alpha_2}{\beta_2}(=\frac{200}{5})$ implies $p_1(=33.99)> p_2(=32.89)$. No published works have ever discovered this particular phenomenon, and it still remains unclear from managerial or economic perspectives.

5. Table 3 shows the returns policy contract indeed renders a better channel profit M_c, which might be due to the buy-back commitment that encourages the two retailers' orders and rises in prices without hesitation. Nonetheless, Table 3 shows that profit in segment 2 from the returns policy contract is always worse than that from the price-only contract, an analogous result appearing in Bose and Anand (2007) and Yao et al. (2008). Thus, as a leader, the manufacturer should appropriate some extra profit for game continuation − at least 279.80-261.90=17.90 in the case of ξ=40 in Table 3. Even so, the manufacturer still earns higher profit compared with the price-only contract. Furthermore, Table 3 also reveals a growingly important impact of the policy on channel profit performances, from 0.08% of increment as ξ=5 to 0.68% as ξ=40.

Conclusions

In this study, we explored a one-leader, two-follower type of supply chain where the manufacturer sells a newsvendor-type item in a monopoly market with two variously priced sub-market segments for revenue increase purposes. In order to maximize its profit, a complete returns policy is offered before the selling season in anticipation of creating a three-win situation.

Apart from a multitude of parameters factors that were investigated such as demand leakage rate, consumer's price-sensitivity and demand variability, which have influence on channel profit , examination of the effects of returns contractual terms was an essence of this study. We summarize what we have obtained from the previous examples below. First, demand leakage rate has a slight impact on the manufacturer with an invariable profit, an adverse impact on segment 1 with a decreasing profit, and a favorable impact on segment 2 with an increasing profit. Secondly, because of the assumed demand patterns, retailers lower prices for sales boom concerns in response to high price-sensitivity, and the chain is advised to sacrifice some sales opportunity in a high price-sensitive market and offset back from a low price-sensitive one; but high price-sensitivity is definitely harmful to each member's profit anyhow. Lastly, wholesale price and buy-back price as well as selling prices are all supposed to be downward in an uncertain demand market, while two corresponding order quantities are upward as a result of the contractual terms. Surprisingly, the manufacturer still makes profit from each unsold return, which equivalently means that two retailers jointly bear the entire risk of demand uncertainty. So, other than a commitment of returns policy, it is more reasonable that a manufacturer needs to guarantee its two retailers a better profit to enhance their willingness to attend the game. Experiments consequently showed this consistent result that returns policy contract is not Pareto improvement to segment 2, thus the manufacturer ought to split some profit; even so, it still receives more profit and at the same time channel profit is also improved. What's more, the contractual terms plays an increasingly important role when demand variability is getting higher.

Two unsolved questions in our study are that: the obtained results from our proposed solution procedure are approximations, not analytical solutions, which still needs deeper discussion if analytical solutions are required; no matter from the managerial or economic point of view, we cannot explain why the ratio of primary demand over price-sensitivity is such a decisive factor to determine who belongs to a high- or a low-priced market. For future research, an extension allowing more than two retailers in this game and a consideration regarding the retailer as a Stackleberg game leader are two directions that deserve more exploration.

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