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Mathematical Modeling as A Tool for Sustainable Development in Nigeria

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Abstract
The Nigerian economy, being a developing economy, requires mathematics that can effectively put science and technology in the forefront of national development. Many developmental challenges currently facing the Nation could be solved if it is possible to get mathematical models that could describe them. A mathematical model is a description of a system using mathematical concepts and language. In this paper we review the role mathematical modeling can play in launching Nigeria onto the path of sustainable development in this 21st century. Different forms of mathematical models were highlighted and classified. Some applications and challenges of mathematical modeling were examined. It was found out that mathematical modeling is one of the tools needed to transform the Nigerian economy from a developing to a developed economy. From the review, it is recommended that the Nigerian Government should invest more in mathematics education at all levels – primary, secondary and tertiary – in order to train more mathematical modelers and provide the enabling environment for the teaching and learning of mathematics. Again, students should be motivated or encouraged to study mathematics in general and mathematical modeling in particular through grants, loans and scholarships.
Keywords: Mathematical Modeling, Sustainable Development, Mathematical Model, Developing Economy.

Introduction

Mathematical modeling is the process of developing a mathematical model. A mathematical model, also known as symbolic model, is a description of a system using mathematical concepts and language. It is the application of mathematics to solve real life problems. It replaces components or variables in the real-life system with symbols and the systems are generally related mathematically. For example Newton's second law of motion modeled the relationship between force, mass and acceleration as $F = Ma$. Apart from mathematical models there are other types of models such as: (1) ICONIC MODELS and (2) ANALOGUE MODELS. Iconic models are models that represent the real –system by something that "look like" what is being represented. These models scale up or down the real system but retain some of the physical features of the real system they represent. Examples of iconic models are photographs, globes, world aerodynamicist's and three-dimensional models of physical facilities often used in architecture and factory planning (Inyama, 2007). An Analogue model is a model that establishes a relationship between a variable in the system and an analogous variable in the model. Examples of analogue model include graphs, electrician's schematic diagrams, flow charts, maps and building plans. Unlike iconic models, analogue models are useful for the study of dynamic situations. Usually changes can be made easily in analogue model than in iconic model. However, analogue models may over simplify the real-life system. A major advantage of mathematical models over iconic or analogue models is that interrelationship between variables can be revealed through manipulation of the model. For example, the model $F = ma$ can be expressed as $a = F/m$ or $m = F/a$. Furthermore, mathematical models are usually the most general in application and yield the most useful information, though they are often the most difficult and expensive to construct.

Mathematical models are used not only in the natural sciences (such as physics, chemistry, Biology, earth sciences, meteorology) and engineering disciplines (e.g computer science, construction, manufacturing, artificial intelligence) but also in the social sciences (such as economics, psychology, sociology and political sciences). Physicists, engineers, statisticians, operations research analysts and economists use mathematical models most extensively (Dangelmayr, 2005).

Mathematical models can take many forms, including but not limited to dynamical systems, statistical models, differential equations, or game theoretic models. These and other types of models can overlap, with a given model involving a variety of abstract structures. In general, mathematical models may include logical models, as far as logic is taken to be part of mathematics. In many cases, the quality of a scientific field depends on how well the mathematical models developed on the theoretical side agree with results of repeatable experiments. Lack of agreement between theoretical mathematical models and experimental measurements often leads to important advances as better theories are developed (Benda, 2000).

Classification of Mathematical Models

Mathematical models can be classified in some of the following ways:

1. **Linear Vs. Nonlinear**

   Mathematical model are usually composed of variables, which are abstractions of quantities of interest in the described systems, and operators that act on these variables, which can be algebraic operators, functions, differential operators etc. A mathematical model is regarded as linear if all the operators in the model exhibit linearity, (that is, if they satisfy the two properties of additivity, i.e. $f(x+y) = f(x)+f(y)$ and Homogeneity of degree one, i.e. $f(\alpha x) = \alpha f(x)$. A model is considered nonlinear if it does not exhibit any or

Vol. 1 , No. 2, 2012, E-ISSN: 2226-6348 © 2012 HRMARS

both of the above properties. A linear model may have nonlinear expressions in them. For example, in a statistical linear model, it is assumed that a relationship is linear in the parameters, but may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operator, but it can still have nonlinear expressions in it. In a mathematical programming model if the objectives function and constraints are represented entirely by linear equations, then the model is regarded as a linear model. If one or more of the objective functions or constrains are represented with a nonlinear equation, then the model is said to be a nonlinear model. Nonlinearity is often associated with phenomena such as chaos and irreversibility. Nonlinear systems and models tend to be more difficult to study than linear ones. A common approach to nonlinear problems is to linearize them, but this can be problematic if one is trying to study aspects such as irreversibility which are strongly tied to nonlinearity.

2. Deterministic Vs Stochastic (Probabilistic) Models

A deterministic model is a model in which every set of variable state is uniquely determined by parameters in the model and by sets of previous states of these variables. Therefore, deterministic models perform the same way for a given set of initial conditions. On the other hand, stochastic models are models that incorporate randomness and so variable states are not described by unique values but rather by probability distributions (Lin, 1988).

3. Static Vs Dynamic Models

A static mathematical model is a model that does not account for the element of time, while a dynamic model does. Dynamic models are typically represented with differential equations.

4. Discrete Vs Continuous Models

Discrete models do not take into account the function of time and usually uses time – advance methods. On the other hand, continuous models do take into account the function of time. They are typically represented with F(t) and the changes are reflected over continuous time interval.

5. Deductive, Inductive or Floating Model

A Deductive model is a logical structure based on a theory. An inductive model arise from empirical findings and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure (Andreski, 1972).

Examples of Mathematical Modeling

Many examples of mathematical modeling abound but we shall consider them under the following headings:

Vol. 1 , No. 2, 2012, E-ISSN: 2226-6348 © 2012 HRMARS

1. Modeling with difference Equations,
2. Modeling with Ordinary differential Equations,
3. Modeling with partial differential Equations,
4. Optimization Modeling,
5. Modeling with simulation,
6. Function fitting: Data modeling.

Modeling with Difference Equations

A difference equation is a formula for computing an output sample at time t based on past and present input samples and past output samples in the time domain. The general, causal, difference equation may be written as follows.

$$y(t) = b_o x(t) + b_1 x(t-1) + \ldots t\, bm\, x(t-m)$$
$$- a_1 y(t-1) - \ldots - aNy(t-N)$$
$$= \sum_{i=0}^{M} bi\, x(t-i) - \sum_{j=1}^{N} ajy(t-j)$$

Where x is the input signal, y is the output signal, and the constants b_i, i=0,1,2,…, M a_i, i=1,2,…, N are called the coefficients.

Consider the situation in which a variable changes in discrete time steps. If the current value of the variable is a_n then the predicted value of the variable will be a_{n+i}. A mathematical model for the evolution of the (still unspecified) quantity a_n could take the form.

$$a_{n+1} = \alpha a_n + \beta$$

In words, the new value is a scalar multiple of the old value offset by some constant β. This model is common, eg, it is used for modeling bank loans (Dangelmayr *et al.,* 2005). One might amend the model to make the dependence depend on more terms and to include the possibility that in every iteration the offset can change, thus,

$$a_{n+1} = \alpha_1 a_n + \alpha_2 a^2{}_n + \beta_n$$

This could correspond to, for example, a population model where the migration levels change every time step.

Modeling with Ordinary Differential Equations

A differential equation is a mathematical equation for an unknown function of one or several variables that relates the values of the function itself and its derivatives of various order. Dynamics models are often represented by differential equations. Although modeling with ordinary differential equations share many of the ideas of modeling with the different equations discussed above, there are many fundamental differences. At the center of these difference is the assumption that time is a continuous variable. One of the simplest differential equations is also an extremely important model, i.e.

$$\frac{dx}{dt} = \alpha x$$

In words, the rate of change of the quantity x depends on the amount of the quantity. If $\alpha > 0$ then we have exponential growth. If $\alpha < 0$, the situation is exponential decay. Of course, additional terms can be added that fundamentally alter the evolution of x (t). For example;

$$\frac{dx}{dt} = \alpha_1 x + \alpha_2 x^2$$

The model formulation again requires the development of the appropriate right-hand side.
In the above model, the value x on the right hand side is implicitly assumed to be evaluated at the time t. It may be that there is evidence that the instantaneous rate of change at time t is actually a function of a previous time, i.e.

$$\frac{dx}{dt} = f[x(t) + g(x(t-i)]$$

This is referred to as a delay differential equation (Dangelmayr et al., 2005).

Modeling with Partial Differential Equations

In modeling with ordinary differential equations it is assumed that there was only one independent variable. Many situations arise in practice where there are more than one independent variables. In such a case the mathematical modeling will be done using partial differential equations. For spatio-temporal models we have space and time as the independent variables. So we have

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial x^2}$$

or $$\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Optimization

In many modeling problems the goal is to compute the optimum of a given objective function subject to some constraints, restrictions or limitations. This many correspond to maximizing profit, revenue, productivity, production etc in a company, utility of a consumer or to minimize losses, space, time etc in a company; given some constraints such as funds availability, space, time etc. This type of optimization problem is referred to as a constrained optimization problem. If the optimization is to be carried out without considering any constraints, it is known as unconstrained optimization.

If the objective function as well as the equations that define the constraint set are linear, then the optimization problem is called linear programming problem. Otherwise the problem is referred to as nonlinear programming problem. The solution method for both types of problem are quite different.

Modeling with Simulation

Many problems may afford a mathematical formulation yet be analytically in tractable. In these situations, a computer can implement the mathematics literally and repetitively often times to extreme advantage. Computer simulations can be employed to model evolution equations. Applications in the realm of fluid dynamics and weather prediction are well established. A striking new example of such simulation modeling is attempting to model electrical activity in the brain.

Function Fitting: Data Modeling

Often data is available from a process to assist in the modeling. To compute functions that reflect the relationships between variables in the data we produce a model.

$$Y = f(x; w)$$

And using the set of input-output pairs computer the parameter w. in some cases the form of f may be guessed. In other cases a model free approach can be used (Aris, 1994).

The Values of Mathematical Modeling

Some of the values of mathematical modeling includes;

1. One is forced to choose what to focus on. You must prioritize factors.
2. The modeling process helps make thoughts more precise.
3. A model helps one go beyond the surface of a phenomenon to understanding of mechanisms and relationships
4. One can play out different scenarios, modifying assumptions, initial values of parameters, to see the resulting effects.
5. Unlike the iconic and analogue models, mathematical models make it easy for the interrelationships between variables to be revealed through manipulation of the model (Gershenfield, 1998).

Problems Associated with Mathematical Modeling

The following are some of the problems associated with mathematical modeling:

1. The model does not address what you want to accomplish.
2. The model is very sensitive to initial conditions or to the values of parameters.
3. The model creates a mathematical solution to a problem that does not lend itself to a mathematical solution.
4. The model is too simple to mirror adequately.
5. The model is too complex to aid understanding.
6. The results are too technical to communicate.
7. The results are not in a form that can be implemented.
8. Resources are not adequate to implement a suggested solution.

Vol. 1 , No. 2, 2012, E-ISSN: 2226-6348 © 2012 HRMARS

Conclusion

This paper reviewed the role mathematical modeling can play for a sustainable development in Nigeria. It looked at the advantage of mathematical modeling over other types of modeling and classified mathematical models. It also looked at some examples and difficulties of mathematical modeling and submitted that it has a vital role to play for a sustainable development of Nigeria.

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