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Construction of Some New Three Associate Class PBIB Designs with Two Replicates

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Abstract
Some new series of three associate Partially Balanced Incomplete Block Designs with minimal blocks and two replicates have been constructed by introducing another blank diagonal entry to triangular association scheme. The generalization of parameters for any even positive integer n greater than or equal to eight have also been given. Specific design has also been constructed to illustrate the results numerically.
Keywords: Partially Balanced Incomplete Block, Three Associate Classes, Superimpose.

Introduction
By changing the arrangement of treatments or omitting some blocks and or treatments, we obtain designs that may be a class of new designs. Using this technique Bose and Nair (1939) introduced some PBIB designs. Atiqullah (1958) established that considering PBIB designs based on triangular association scheme with $v = n\,(n-2)/2$, $b = (n-1)\,(n-2)/2$, $k = n$, r = n, $\lambda_1 = 1$ and $\lambda_2 = 2$, the necessary condition for existence of these PBIB designs is the existence of symmetrical triangular PBIB designs with $v = b = (n-1)\,(n-2)/2$, $r = k = n-2$, $\lambda_1 = 1$ and $\lambda_2 = 2$. Other methods were given by Shrikande (1960, 1965) and Chang et al (1965) based on the existence of certain BIB designs and considering the dual of a BIB design as omitting certain blocks from the BIB design. John (1966) showed that triangular association scheme can be described by representing the treatments by ordered pairs (x, y) with 1 ≤ x < y ≤ q and then generalized (John, 1966) for the case that $v = q(q-1)\,(q-2)/6$, (q > 3). Arya and Narain (1981) discussed a new association scheme called truncated triangular (TT) with five associate classes when $v = p\,(p-2)/2$ with p an even positive integer ≥ 8, and used to construct partial diallel crosses. Ching-Shui et al (1984) came up with a general and simple method of construction based on the relation of triangular and l_2 type of PBIB design to the line graph theory. A construction of triangular designs with nested rows and columns is given by Agrawal and Prasad (1984). Sinha and Sanpei (2004) gave a series of triangular (tenary) designs with nested row and columns. Recently, Garg

et al (May, 2011) constructed some new series of triangular and four associate PBIB designs with two replications by using dualization technique.

Construction of Designs

The design is constructed from two squared arrays of n rows and n columns (n is even positive integer greater than or equal to eight) with both diagonal entries n_{ij} (i=j and i + j = n+1) in array having no treatments allocated to as illustrated in figure 1.

*	n_{12}	n_{13}	n_{14}	n_{15}	n_{16}	n_{17}	*
n_{21}	*	n_{23}	n_{24}	n_{25}	n_{26}	*	n_{71}
n_{31}	n_{32}	*	n_{34}	n_{35}	*	n_{62}	n_{61}
n_{41}	n_{42}	n_{43}	*	*	n_{53}	n_{52}	n_{51}
n_{51}	n_{52}	n_{53}	*	*	n_{43}	n_{42}	n_{41}
n_{61}	n_{62}	*	n_{35}	n_{34}	*	n_{32}	n_{31}
n_{71}	*	n_{26}	n_{25}	n_{24}	n_{23}	*	n_{21}
*	n_{17}	n_{16}	n_{15}	n_{14}	n_{13}	n_{12}	*

Figure1. Arrangement of treatment entries in a squared array
i and j are integers ($1 \leq$ i, j $\geq$ n) such that;

Each row and column of the square array has $n-2$ treatment entries.

The treatment entries are allocated in the array by following two subsequent steps

1. The initial set of v treatment entries are first filled on one side of the diagonal entries n_{ij} $(3 \leq i+j \geq n,\ i \neq j)$.
2. The second set of v treatment entries n_{ij} $(n+1 < i+j > 2n,\ i \neq j)$ are allocated in such a way that given any two entries n_{ij} and $n_{i'j'}$ are allocated to treatment **x** say, if and only if the subscripts i + i' = j + j'. That is; any two entries allocated to the same treatment are characterized by the sum and equivalence of subscripts i and j of the two entries n_{ij} and $n_{i'j'}$ such that:

- Every treatment in the array appears twice,
- A pair of treatments exist in both the i^th row and j^th column (i, j = 1, 2,. . . n).

The second array is found by transposing the first array and then superimposing it on the first so as to obtain an array containing two treatments in each cell as in Figure 2.

*	n_{12} n_{21}	n_{13} n_{31}	n_{14} n_{41}	n_{15} n_{51}	n_{16} n_{61}	n_{17} n_{71}	*
n_{21} n_{12}	*	n_{23} n_{32}	n_{24} n_{42}	n_{25} n_{52}	n_{26} n_{62}	*	n_{71} n_{17}
n_{31} n_{13}	n_{32} n_{23}	*	n_{34} n_{43}	n_{35} n_{53}	*	n_{62} n_{26}	n_{61} n_{16}
n_{41} n_{14}	n_{42} n_{24}	n_{43} n_{34}	*	*	n_{53} n_{35}	n_{52} n_{25}	n_{51} n_{15}
n_{51} n_{15}	n_{52} n_{25}	n_{53} n_{35}	*	*	n_{43} n_{34}	n_{42} n_{24}	n_{41} n_{14}
n_{61} n_{16}	n_{62} n_{26}	*	n_{35} n_{53}	n_{34} n_{43}	*	n_{32} n_{23}	n_{31} n_{13}
n_{71} n_{17}	*	n_{26} n_{62}	n_{25} n_{52}	n_{24} n_{42}	n_{23} n_{32}	*	n_{21} n_{12}
*	n_{17} n_{71}	n_{16} n_{61}	n_{15} n_{51}	n_{14} n_{41}	n_{13} n_{31}	n_{12} n_{21}	*

Figure2. Arrangement of treatment entries in the superimposed square array

Results

Taking each row or column of figure 2 to constitute a block, we obtain $\frac{n}{2}$ distinct blocks yielding a design with the following parameters

$$v = \frac{n(n-2)}{2} \qquad b = \frac{n}{2} \qquad k = 2(n-2) \qquad r = 2$$

$$\lambda_1 = 2 \qquad \lambda_2 = 1 \qquad \lambda_3 = 0$$

Association Scheme

Two treatments are said to be:

i. First associates if they both occur in the same row and column.
ii. Second associates if they both occur in the same row or the same column but not both.
iii. Third associates if they neither occur in the same row nor in the same column.

Giving rise to the following association parameters

$$n_1 = 3 \qquad n_2 = 4(n-4) \qquad n_3 = \frac{n(n-10)+24}{2}$$

$$p_{jk}^{0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4(n-4) & 0 \\ 0 & 0 & 0 & \frac{n(n-10)+24}{2} \end{bmatrix} \qquad p_{ik}^{1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 4(n-4) & 0 \\ 0 & 0 & 0 & \frac{n(n-10)+24}{2} \end{bmatrix}$$

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$$p_{jk}^{2}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 3 & 2(n-4) & 2(n-6) \\ 0 & 0 & 2(n-6) & \dfrac{n(n-14)+48}{2} \end{bmatrix} \qquad p_{jk}^{3}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 16 & 4(n-8) \\ 1 & 3 & 4(n-8) & \dfrac{n(n-18)+80}{2} \end{bmatrix}$$

Illustration

Taking n=8 we obtain three associate class PBIB design with the blocks

Block 1: (1, 2, 3, 4, 5, 6, 7, 12, 16, 19, 22, 24)
Block 2: (1, 6, 7, 8, 9, 10, 11, 13, 17, 20, 23, 24)
Block 3: (2, 5, 8, 11, 12, 13, 14, 15, 18, 21, 22, 23)
Block 4: (3, 4, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21)

With the parameters

$v = 24$ $\quad b = 4$
$k = 12$ $\quad r = 2$
$\lambda_1 = 2$ $\quad \lambda_2 = 1$ $\quad \lambda_3 = 0$
$n_1 = 3$ $\quad n_2 = 16$ $\quad n_3 = 4$

and the association matrixes p_{jk}^{i} given by:

$$p_{jk}^{0}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad p_{jk}^{1}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$p_{jk}^{2}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 3 & 8 & 4 \\ 0 & 0 & 4 & 0 \end{bmatrix} \qquad p_{jk}^{3}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 16 & 0 \\ 1 & 3 & 0 & 6 \end{bmatrix}$$

Conclusion

In this paper we have constructed some new series of three associate class PBIB designs with minimal possible blocks and two replications from an even squared array greater than or equal to eight by leaving both the diagonal entries blank and superimposing the squared array by its transpose. The restriction of the number of replications to two and the minimal number of blocks constructed in this paper is desirable in large experiments to minimize cost.

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