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An Economic Order Quantity Model for Defective Items under Permissible Delay in Payments and Shortage

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Abstract

Economic Order Quantity models have many assumptions that are not satisfied completely with recent economic conditions. One of these assumptions is that all items in an ordered lot are perfect quality. But a portion of ordered lot may be defective. The other unrealistic assumption is that the payments are made as soon as the items received. However, in today's business transactions it is more common that the supplier will allow certain fixed period known as permissible delay in payment to the retailer for settling the total amount of received goods. In this study, by loosening these two unrealistic assumptions, a new model is proposed in the case of defective items, permissible delay in payments and shortage. For two case of permissible delay, the optimal values are determined. Furthermore, numerical examples are given for the developed model and changes in the optimal values are analyzed with sensitivity

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analysis. Finally some previously published results are deduced as special cases of proposed model.

Key words: *EOQ, Defective Items, Permissible Delay in Payments, Shortage*

JEL Codes: *C6, M11*

Introduction

The traditional inventory models consider many unrealistic assumptions that are not valid in real life situations. Therefore, many researchers developed new EOQ models by loosening these unrealistic assumptions and a vast literature has occurred.

A common unrealistic assumption of the EOQ model is that all units obtained by purchasing are of perfect quality. Salameh and Jaber (2000) developed an EOQ model for situation where a random proportion of the ordered lot is of defective item. They assume that received orders are subject to 100% inspection process and defective items are kept in the stock until the end of the screening period than they are sold as a single lot at a discounted price. They concluded that as the percentage of the defective items increases, economic order quantity tends to increases. Papachristos and Konstantaras (2006) examined and corrected Salameh and Jaber's (2000) model such that many of its assumptions are not accurately met and clearly set especially for avoiding shortages. Wee et al. (2007) and Eroğlu and Özdemir (2007) extended Salameh and Jaber's (2000) model by allowing for shortages. The difference between these two models is in explaining the elimination of backorders. Maddah and Jaber (2008) revisited the Salameh and Jaber's (2000) study, corrected some flaws it has and extended it in many ways. Main correction for flaws of Salameh and Jaber's (2000) model was using renewal theory to obtain exact expression for the expected profit. Hsu and Hsu (2013) also analyzed an EOQ model for imperfect quality items with shortage backordering.

Another unrealistic assumption of EOQ models is that the retailer must pay for the items as soon as the items received. However, in today's business transactions it is more common that the supplier will allow certain fixed period known as permissible delay in payment to the retailer for settling the total amount of received goods. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. Goyal (1985) developed an EOQ model under conditions of permissible delay in payments. In his model it is assumed that unit purchase cost and unit selling price are the same and it is concluded that the cycle time and order quantity generally increases under the permissible delay in payments. Other notable works in this direction are those of Huang and Chung (2003), Teng et al. (2005), Chung and Liao (2006), Sana and Chaudhuri (2008), Liao (2008), Chung and Huang (2009) and their references.

In this paper, we relaxed two unrealistic assumptions of the classical EOQ model and developed a new model that reflects the practical business situations. Defective items and permissible delay in payments together is studied by Chung and Huang's (2006) and Chung (2013) works but they did not allow backlogging. However in many real-life conditions, stock out is unavoidable because of various uncertainties in the related system. So, the occurrence of shortage in inventory control models could be considered as a natural phenomenon. Jaggi et al. (2013) developed a joined model for defective items under credit financing with allowable shortages but they analyzed five cases of credit financing time. In this study we incorporated both Salameh and Jaber (2000) and Goyal (1985) to propose new model for imperfect quality items under the permissible delay in payments and also allowing shortages. Our model is also an extension of Eroğlu and Özdemir's (2007) model for the case of permissible delay in payment and Chung and Huang's (2006) model for allowing shortages. Also this study is simplification of Jaggi et al. (2013) model with two case and a simple algorithm. For two case of permissible delay in payments, the relationships between permissible delay time, rate of defective item and optimal values are determined by solving objective function that is modeled as an expected total annual profit maximization problem. Furthermore, numerical examples are given and analyzed for the developed model. Changes in the optimal values with respect to the rate of defective item and permissible delay in payments are analyzed with sensitivity analysis. Finally we deduce some previously published studies of other researchers as special cases.

Notations and Assumptions
The following notation is used:

Q order quantity

B maximum backorder level permitted

D demand rate in units per unit time

p percentage of defective items in Q

$f(p)$ probability density function of p

K fixed cost of placing an order

c unit variable cost

s unit selling price of good-quality items

v unit selling price of imperfect-quality items, ($v < s$)

h unit holding cost per item per unit time excluding interest charges

π backorder cost per unit per unit time

x screening rate in units per unit time

d unit screening cost

T cycle length (year)

M permissible delay in payment (year)

t time to screen Q units ordered per cycle

t_1 time to eliminate the backorder level of 'B' units

t_2 after eliminating shortages, time elapsed until screening process ended, $t - t_1$

t_3 after deducting imperfect quality items from inventory, time elapsed until on hand inventory comes down to zero

t_4 time to build up a backorder level of 'B' units

F time when on hand inventory finishes

I_e annual interest rate earned per \$

I_o annual interest rate charged per \$ in stocks by the supplier, $I_o \geq I_e$

Following assumptions are made for the proposed model:

1. Demand rate is known and constant.
2. The inventory system involves only one type of inventory.
3. The lead time is zero.
4. Replenishment is instantaneous.
5. Each lot received contains percentage defectives. Defective rate, p, is a random variable with a known probability density function, $f(p)$.
6. A lot of size Q is conducted 100% screening process at a rate of x units per unit time. At the end of the screening process, defective items are sold as a single batch at a discounted price.
7. Shortages are allowed. Allowable shortages are completely backordered with perfect-quality items while imperfect-quality items are left from inventory after screening process.

8. The screening rate is sufficiently large such that perfect-quality items that are determined in screening time are adequate for demand occurred in that period. Moreover, since the screening rate is sufficiently large, screening time, t, is always smaller than permissible delay time in payments, M.

9. Supplier allows certain fixed period known as permissible delay in payment to the retailer for settling the total amount of received goods. During the time the account is not settled, generated sales revenue is deposited in an interest bearing account at a rate of I_e. At the end of this period, the account is settled and interest is charged for unpaid amount at a higher interest rate, I_o, $I_o \geq I_e$.

Mathematical model

The behavior of the inventory level is illustrated in Fig. 1. Since each lot contains defective items at a rate p, the rate of perfect-quality items which are screened during t_1 is $(1-p)$ in Fig. 1. A part of these perfect-quality items met the demand with a rate of D and the remaining is used to eliminate backorders with a rate of $(1-p)x - D = (1 - p - D/x)$. In Fig. 1. this rate is depicted as A. The screening process finishes up at the end of the time t and defective items of pQ are subtracted from inventory.
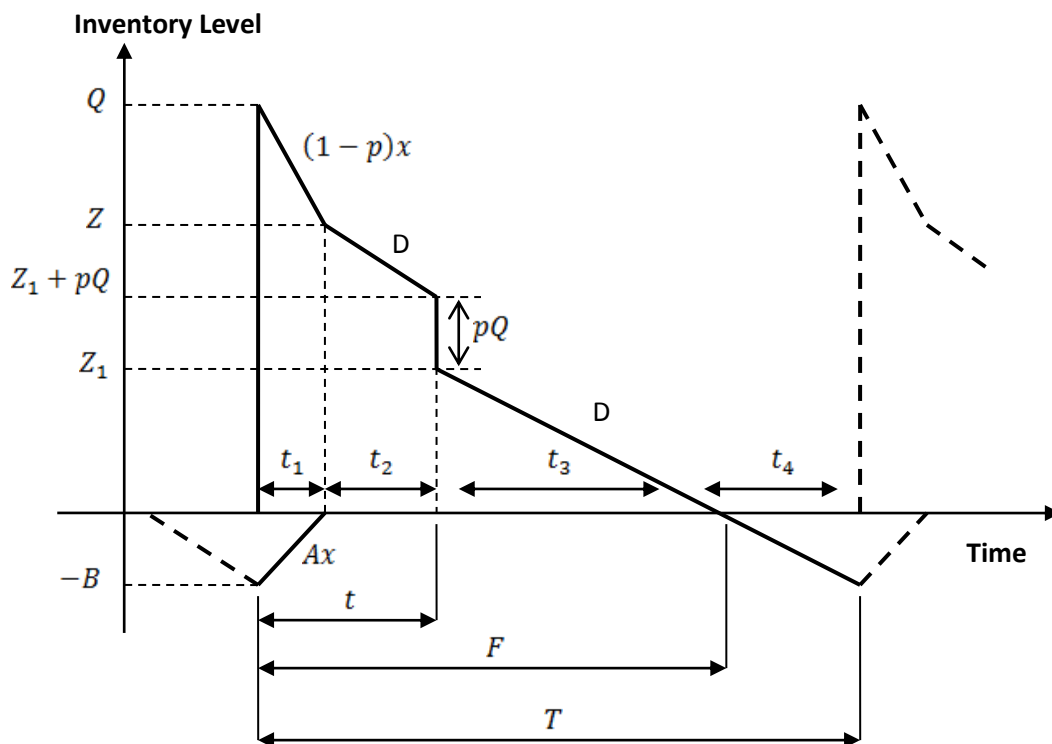

Figure 1. Behavior of the inventory level over time

According to the above notations and assumptions, under the case of permissible delay in payments there will be two cases, namely (I) $t < M \leq F$ and (II)$M > F$.

Case I: $t < M \leq F$

The behavior of the inventory level for Case I can be depicted as Figure 2 .

When $t < M \leq F,$ we let $TR_1(Q,B)$, $TC_1(Q,B)$ and $TPU_1(Q,B)$denote the total revenue per cycle, the total cost per cycle and total profit per unit time, respectively.
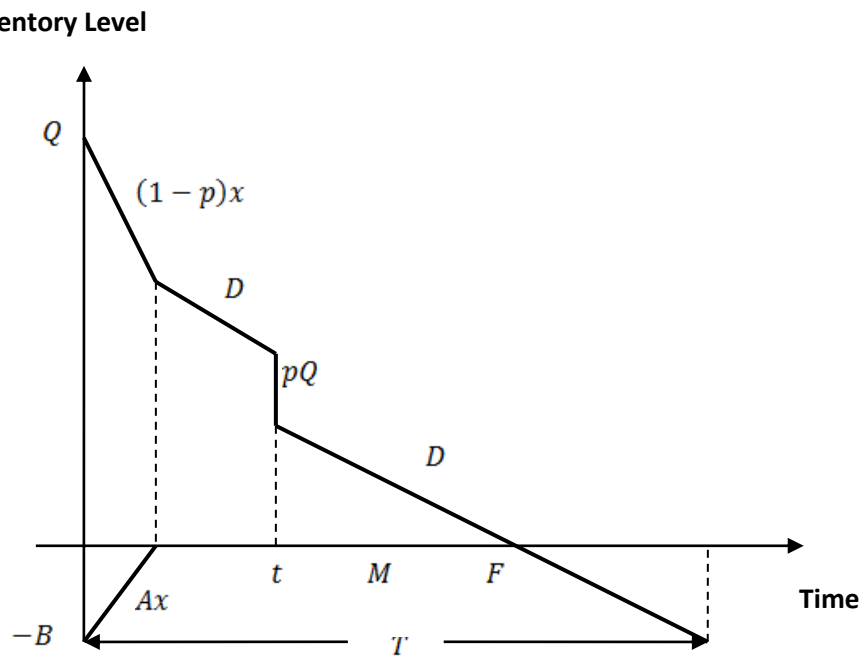

Figure 2. Case I. $t < M \leq F$

The components of total revenue per cycle, $TR_1(Q,B)$, are; sales revenue of perfect and imperfect quality items and interest earned from sales revenue during the permissible period. The components of total cost, $TC_1(Q,B)$, are; order cost per cycle, procurement per cycle, screening cost per cycle, holding cost per cycle, shortage cost per cycle and interest payable for cycle for the inventory not being paid after the due date M.

Since defective rate is a random variable, the values of cycle length and total profit are also random. So by using the renewal reward theorem, expected total profit per unit time for case I is given as:

$$E(TPU_1) = \frac{E(TR_1) - E(TC_1)}{E(T)}$$

$E(TPU_1)$ is conditionally concave under some circumstances. Partial derivatives of $E(TPU_1)$, with respect to Q and B are set equal to zero separately to obtain optimal order size Q^* and allowable maximum shortage level B^*. Then: for case I, Q_1^* and B_1^* are found as follows:

$$Q_1^* = \sqrt{\frac{[(h+\pi)E_2+cI_0]B^2+2cMI_0DB+(cI_0-sI_e)D^2M^2+2KD}{\frac{2vE(p)I_eD}{x}+cE_5I_0+hE_4}}$$

(1)

$$B_1^* = \frac{(h+cI_0)E_1Q_1^*-cMI_0D}{(h+\pi)E_2+cI_0}$$

(2)

By substituting B_1^* in eq. (1), the value of Q_1^* is found independent from shortage level as follows:

$$Q_1^* = \sqrt{\frac{2KD+(cI_0-sI_e)D^2M^2-\frac{(cMI_0D)^2}{(h+\pi)E_2+cI_0}}{\frac{2vE(p)I_eD}{x}+cE_5I_0+hE_4-\frac{(h+cI_0)^2E_1^2}{(h+\pi)E_2+cI_0}}}$$

(3)

where;

$$E_1 = E(1-p) = 1 - E(p)$$

$$E_2 = E\left(\frac{(1-p)}{1-p-D/x}\right)$$

$$E_3 = E[(1-p-D/x)^2]$$

$$E_4 = \frac{D(2-D/x)}{x} + E_3$$

$$E_5 = E[(1-p)^2]$$

Case I: *M > F*
The behavior of the inventory level for Case II can be depicted as Figure 3 .

When $M > F$, we let $TR_2(Q,B)$, $TC_2(Q,B)$ and $TPU_2(Q,B)$ denote the total revenue per cycle, the total cost per cycle and total profit per unit time, respectively.

The components of total revenue per cycle, $TR_2(Q,B)$, are; sales revenue of perfect and imperfect quality items and interest earned from sales revenue during the permissible period.

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The components of total cost, $TC_2(Q,B)$, are; order cost per cycle, procurement per cycle, screening cost per cycle, holding cost per cycle and shortage cost per cycle. Since defective rate is a random variable, as in Case I, by using the renewal reward theorem, expected total profit per unit time for case II is given as:
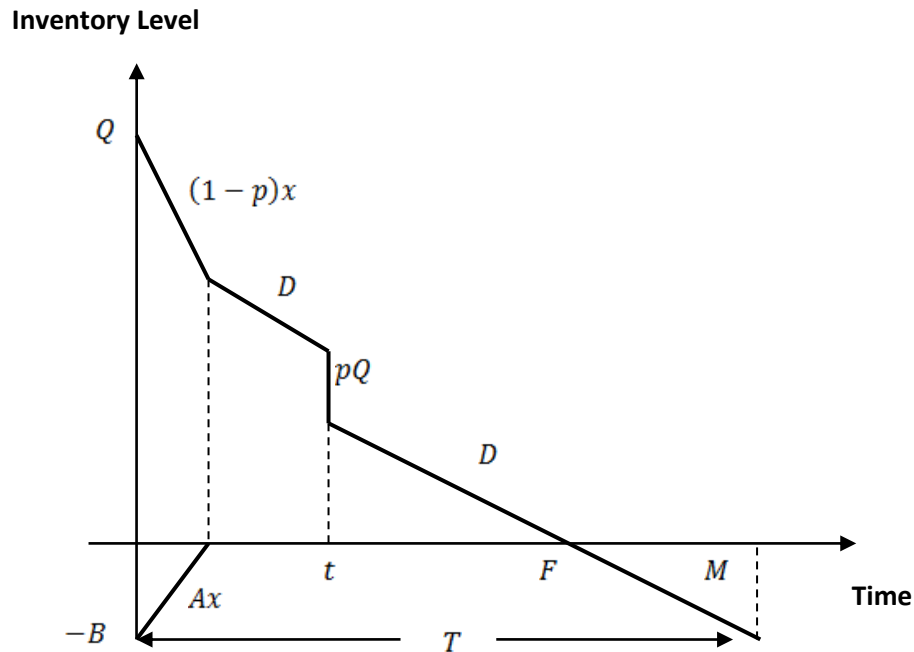

Figure 3. Case II. $M > F$

$$E(TPU_2) = \frac{E(TR_2) - E(TC_2)}{E(T)}$$

Since $E(TPU_2)$ is strictly concave, partial derivatives of $E(TPU_2)$ with respect to Q and B are set equal to zero separately to obtain optimal order size Q^* and allowable maximum shortage level B^*. Then: for case I, Q_2^* and B_2^* are found as follows:

$$Q_2^* = \sqrt{\frac{[(h+\pi)E_2 + sI_\varrho]B^2 + 2sMI_\varrho DB + 2KD}{\frac{2vE(p)I_\varrho D}{x} + sE_5 I_\varrho + hE_4}} \tag{4}$$

$$B_2^* = \frac{(h+sI_\varrho)E_1 Q_2^* - sMI_\varrho D}{(h+\pi)E_2 + sI_\varrho} \tag{5}$$

By substituting B_2^* in eq. (4), the value of Q_2^* is found independent from shortage level as follows:

$$Q_2^* = \sqrt{\frac{2KD - \frac{(sMI_\varrho D)^2}{(h+\pi)E_2 + sI_\varrho}}{\frac{2vE(p)I_\varrho D}{x} + sE_5 I_\varrho + hE_4 - \frac{(h+sI_\varrho)^2 E_1^2}{(h+\pi)E_2 + sI_\varrho}}}$$

(6)

Where;

$$E_1 = E(1-p) = 1 - E(p)$$

$$E_2 = E\left(\frac{(1-p)}{1-p-D/x}\right)$$

$$E_3 = E[(1-p-D/x)^2]$$

$$E_4 = \frac{D(2-D/x)}{x} + E_3$$

$$E_5 = E[(1-p)^2]$$

The following conditions must be held that the developed model is valid:

- To prevent shortages at any time during cycle length, perfect quality items in each lot must be greater or equal to the demand.
  $Q - pQ \geq Dt$ and $t = Q/x$ then; $p \leq 1 - D/x$ Since p is a random variable; $E(p) \leq 1 - D/x$

- Moreover, to eliminate and prevent backorders in screening period following conditions must be met:
  - Screening rate must be sufficiently greater than demand rate; $x > D$
  - Eliminating rate of backorders from perfect quality items must be positive. It means $Ax \geq 0$.
  Thus $xE(1-p-D/x) \geq 0$ and $E(1-p-D/x) \geq 0$or $E(p) \leq 1 - D/x$. (This is the same condition to prevent shortage as stated above)
- Screening time, t must be at least equal or greater than the expected value of the time to eliminate backorder,$E(t_1)$. Otherwise a portion of the backorder would not be eliminated at the end of the cycle.
  $t \geq E(t_1)$

- Screening rate x, is sufficiently large such that less $t \leq M$ and screening time t must be less than cycle length, T in order to avoid shortages.

Special Cases

Eroğlu and Özdemir (2007) Model
If permissible delay in payments is not allowed then the model for defective items with shortages is attained. Thus, the following reduced forms of Equations (2), (5), (3) and (6) are achieved:

$$B_1^* = B_2^* = B^* = \frac{hE_1Q^*}{(h+\pi)E_2}$$

$$Q_1^* = Q_2^* = Q^* = \sqrt{\frac{2KD}{h\left[E_4 - \frac{hE_1^2}{(h+\pi)E_2}\right]}}$$

These results are the same as equation (13) and (14) in Eroğlu and Özdemir (2007) model. But Eroğlu and Özdemir used w instead of B for shortage size and y instead of Q for order size. So, Eroğlu and Özdemir (2007) model is a special case of this paper.

Maddah and Jaber (2008) Model
Salameh and Jaber (2000) model is revisited by Maddah and Jaber (2008). The revisited form of their model is also a special case of this paper.

In this paper if shortage and permissible delay in payments are not allowed, (in this case $M = I_o = I_e = B = 0$ and $\pi = \infty$) Equations (2) and (5) are achieved as follows:

$$Q_1^* = Q_2^* = Q^* = \sqrt{\frac{2KD}{hE_5 + \frac{2E(p)D}{x}}}$$

These results are similar to Equations (6) and (7) obtained by Maddah and Jaber (2008). So Maddah and Jaber (2008) model with revisited form of Salameh and Jaber (2000) model is special case of developed model.

Traditional Economic Order Quantity Model with Shortages
If permissible delay is not allowed and there is no defective items, in this case $p = M = 0$ so $I_o = I_e = 0$, $E_1 = E_2 = E_3 = E_4 = E_5 = 0$ and $x = \infty$. Thus, the equations (2), (5), (3) and (6) are reduced to following equations:

$$B_1^* = B_2^* = B^* = \frac{hQ^*}{(h+\pi)}$$

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$$Q_1^* = Q_2^* = Q^* = \sqrt{\frac{2KD}{h}\frac{(h+\pi)}{\pi}}$$

These results are same for traditional economic order quantity model with shortages.

Traditional Economic Order Quantity Model
If assumptions of permissible delay, defective items and shortage are not accepted then the model is reduced to classical economic order quantity model. Since permissible delay, shortage and defective items is not allowed then $p = B = M = I_o = I_e = 0$ and $x = \pi = \infty$, so Equations (2), (5), (3) and (6) are calculated as follows:

$$B_1^* = B_2^* = B^* = 0$$

$$Q_1^* = Q_2^* = Q^* = \sqrt{\frac{2KD}{h}}$$

As a result, these four cases mentioned above are depicted as a special case of this paper.

A simple algorithm
A simple algorithm is developed to determine the case for which the value of permissible delay is suited and what are the optimal values of the model.

Step 1. Calculate the optimal values for case I with given values. Obtain value of F and compare with given value of permissible delay, M. If $M \leq F$, the results are optimal values for case I. Optimal values of model are: $Q_1^*, B_1^*, T_1^*, E[TPU_1(Q_1^*, B_1^*)]$. Other wise go to step 2.

Step 2. Calculate the optimal values for case II with given values. Obtain value of F and compare with given value of permissible delay, M. If $> F$, the results are optimal values for case II. So the optimal values of model are: $Q_2^*, B_2^*, T_2^*, E[TPU_2(Q_2^*, B_2^*)]$.

Numerical Examples
For two cases of permissible delay in payment two numerical examples are given.

Numerical example for case I
A company orders a product as lots to meet outside demand. The defective rate in each lot has a uniform distribution with the following probability density function.

$$f(p) = \begin{cases} 10, & 0 \leq p \leq 0.1 \\ 0, & \text{otherwise} \end{cases}$$

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The demand rate is 5000 units while the screening rate is 60000 units annually. Order cost is 400 TL per order unit holding and shortage costs per year are 4 and 6 TL respectively. Unit purchase and screening costs are 35 and 1 TL, respectively. Selling price of good and imperfect quality items are 60 and 25 TL, respectively. Permissible delay in payment is 30 days. The interest rate earned and charged are 12% and 15%, respectively. Thus, the model parameters are given as follows:

D = 5,000 unit, x = 60,000 unit, K = 400 TL, h = 4 TL, π = 6 TL, c = 35 TL, d = 1 TL, s = 60 TL, v = 25 TL, M =30 day=30/360 year= 0.083 year, I_e =0.12, I_o = 0.15.

Since defective rate is a random variable, expected values are as follows: $E(p)$=0.05, E_1=0.95, E_2= 1.096261, E_3= 0.751944, E_4= 0.911667, E_5= 0.903333.

The optimal values of solution are calculated as:

$Q_1^* \cong$ 960 units

$B_1^* \cong$ 386 units

T_1^* = 0.182 year $\cong$ 66 day

$E[TPU_1(Q_1^*, B_1^*)] \cong$ 114,420 TL

For these values F = 0.105 years $\cong$ 38 day. Since for case I; permissible delay in payment, M(=30) <F (=38), the results are optimal values for case I. The results are satisfied by the necessary conditions of the model.

Numerical example for case II
While parameters for case I is valid for case II, permissible delay in payment is 60 day instead of 30 day. So for case II, M=60/360=0.166 year. Then, optimal values are obtained as:

$Q_2^* \cong$ 715 units

$B_2^* \cong$ 89 units

T_2^* = 0.136 year $\cong$ 49 day

$E[TPU_2(Q_2^*, B_2^*)] \cong$ 116,941 TL

For these values F = 0.118 years $\cong$ 43 day. Since for case II; permissible delay in payment, M(=60) >F (=43), the results are optimal values for case II. The results are also satisfied by the necessary conditions of the model.

Sensitivity analysis

It is important that how much effect the permissible delay in payments and defective rate has on the order size, shortage size and the retailer's profit. For this, a sensitivity analysis is made for different values of permissible delay in payments and defective rates. Four different values of permissible delay are adopted, M=15, 30, 45, 60. for each value of M, four different values of defective rate are tested. The results are shown in Table 1, and the following conclusions can be made which are consistent with our expectations:

Table 1. Sensitivity analysis with various values of M and $E(p)$

M		$E(p)$ 0.10	0.20	0.30	0.40
15	Q^*	1,006	1,142	1,204	1,236
	B^*	478	447	401	331
	$E(TPU)^*$	110,426	102,679	92,623	79,074
30	Q^*	998	1,069	1,128	1,159
	B^*	376	348	306	244
	$E(TPU)^*$	111,223	103,58	93,667	80,323
45	Q^*	890	954	1,006	1,034
	B^*	245	220	181	127
	$E(TPU)^*$	113,085	106,279	97,099	84,418
60	Q^*	745	801	848	880
	B^*	83	64	35	0
	$E(TPU)^*$	114,18	107,375	98,271	85,774

(D = 5,000 unit, x = 60,000 unit, K = 400 TL, h = 4 TL, π = 6 TL, c = 35 TL, d = 1 TL, s = 60 TL, v = 25 TL, I_e =0,12, I_o = 0.15)

i.   With fixed defective rate, $E(p)$, as permissible delay in payments, M increases, the expected value of the total unit profit, $E(TPU)^*$ increases while optimal order size, Q^* decreases.

ii. With permissible delay in payments, M fixed, as defective rate, $E(p)$ increases, the expected value of the total unit profit, $E(TPU)^*$ decreases while optimal order size, Q^* increases.

iii. As either M or $E(p)$ increases, the maximum allowable shortage size, $B*$ decreases.

Conclusion

Classical EOQ model have some unreal assumptions such that all ordered quantity is good quality and the payments are made when the order quantity is received. However in real world, the ordered lots have some defective items and the retailer is allowed a permissible delay in payments. Therefore, new models are developed for more realistic solutions in real life

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problems. Such an EOQ model is developed in this paper for defective items with shortages under the condition of permissible delay in payments. It is assumed that defective rate is a random variable with uniformly distributed and retailer can earn an interest revenue during the permissible delay in payments by selling the items. For two cases of permissible delay in payment, two analyses were made and the optimal values of order size, maximum allowable shortage size and expected value of total unit profit. Furthermore some previously published results of other researchers were deduced as special cases of this paper. Finally, numerical examples were given for two case of the developed model and the effects of variations of permissible delay and defective rates on optimal values were examined with sensitivity analysis. The analysis showed that, with increasing of permissible delay in payment, total profit increases while order size decreases; but if defective rate increases, total profit decreases while order size increases.

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