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The Use of Representations in Supporting Early Mathematics Learning and Problem Solving

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Abstract
Early mathematics or also known as early numeracy is one of the important areas of learning for children’s cognitive development at an early age. The learning of Early Mathematics has the goal of fostering young children’s capabilities for problem-solving and their interest in mathematics. Therefore, a fun and effective learning experience of Early Mathematics very important for prepares children with various important Mathematical concepts in preparation before entering Standard One. One of the ways to enhance the process of learning and to provide a more enjoyable experience of solving Early Mathematics problems is by using representations. Therefore, this study was carried out to identify how representation can help the mastery and support the learning of Early Mathematics to prepare children with a good concept of Mathematics, especially in addition. A total of two 6 years olds children involved as a participant in this study. Descriptive analysis shows that representation can help children make problem solving to the addition operation better and more accurately, as well as easily. The high score in post-test compared to pre-test score also proves that the use of representation in the teaching of additional concepts for Early Mathematics can help children understand Mathematical concepts more effectively. It is hoped this study can serve as a guide for early childhood teachers to provide a more enjoyable experience of solving Early Mathematics problems.

Keywords: Early Mathematics, Problem Solving, Mathematical Concepts, Representations, Addition

Introduction
The learning of Early Mathematics has the goal of fostering young children’s capabilities for problem-solving and their interest in mathematics. One of the ways to enhance the process of learning and to provide a more enjoyable experience of solving Early Mathematics problems is the use of representations. Representations can take the form of physical objects, visual diagrams, spoken language, and numerical symbols, among others. Notably, representations are seen as giving preschool children a relatively more convenient way to solve Early Mathematics problems, since they provide readily accessible and understandable examples.

According to Abu Bakar et al (2020a), representations are seen as giving preschool children a relatively more convenient way to solve Early Mathematics problems, since they provide
readily accessible and understandable examples. Early Mathematics problems related to the addition operation, for example, can be explained to young children in the form of visual objects. When young children are shown objects – such as some pieces of candy, stones, or pieces of paper – they are more likely to understand and come up with the answer quicker. In the same way, when equations related to the addition operation problem are written on a board or paper, preschool children can better understand the problem. This enhances the learning experience by providing relevant examples that are visually comprehensible for the children. Moreover, the use of representations can also help young children connect the mathematical problems with its physical world representation, thus providing a more concrete knowledge towards the concept of addition (Maidin & Abu Bakar, 2021; Sia & Abu Bakar, 2022).

Furthermore, the use of representations can also take the form of numerical symbols. Although numerical symbols may not be easily committed to memory by young children, understanding the symbols can still deepen their understanding of addition by corresponding a numerical symbol with the real-world representation. In addition, representations allow children to connect their own learning process with an image, story, or activity. For example, using addition with pictures, children can physically count the items in each box. It allows them to go beyond analysing problems numerically to analysing them visually (Abramovich et al., 2019). Representations also help children make links to their existing knowledge and offer a more concrete practice and understanding of the concept.

Problem Statement
Mathematics is a fundamental skill needed for success in life and it is important to start learning about mathematics at an early age. One of the important concepts to learn in early mathematics is the addition operations. To help children master this concept, the use of representations is very beneficial. Using representations is an effective tactic to help children solve problems. It provides an opportunity to break down the problem into smaller parts and link them to physical objects. In this way, children can see the problems they are trying to solve as a process which is easier to understand. This is a more engaging way for children to learn because it helps them connect the numbers to the corresponding words (Abu Bakar et al., 2020b). In addition to this, the visual and tactile nature of representations can be useful for helping children practice problem solving. When physical objects are used to make the problem easier to understand, it also strengthens the children’s understanding of the concept. This is because the physical objects help children remember and connect the different parts of the problem.

Other than the use of physical objects, there are also other types of representations that can be implemented to support early mathematics learning and problem solving. For example, as outlined by the Malaysian Ministry of Education (2017), the use of teaching aids such as number lines, fractions pictures, wooden cubes of various colours, story cards, linking trays and pictures of objects can acts as representations for helping them in Mathematics problem solving.

Research Purposes
The addition operation is one of the basic and important concepts that preschool children need to master. By understanding the concept of addition operations, children can solve
problems that exist in their daily routine such as buying and selling activities (Ekowati & Suwandayani, 2020). The use of representation in learning Mathematics helps children understand Math concepts more effectively. Children’s failure to understand the concept of addition operations will cause children to fail to master Early Mathematics skills well, at the same time causing children to dislike learning Mathematics (Perdana & Suswandari, 2021).

According to Maidin and Abu Bakar (2021), the selection of interesting Mathematics teaching pedagogy is very important because it can maximize knowledge for children. Effective learning is learning that can give meaning and create feelings of fun and happiness in children. In fact, effective and meaningful learning also encourages children to accept learning without any pressure, at the same time making children always ready to accept the next learning (Dikilitas & Mumford, 2020). Therefore, this field study was carried out with the aim of introducing the concept of addition to children so that children understand the concept of mathematics better. The use of the representations in introducing the concept of addition to children aims to answer the following 2 questions:
1. How do children demonstrate understanding of numbers and the concept of addition?
2. To what extent do children use various representations in solving problems related to the addition operation?

**Research Objectives**
This study was conducted to:
1. Identify children’s understanding of numbers and the concept of addition.
2. Explore the use of various representations in solving problems involving addition operations.

**Literature Review**

**Early Mathematics**
Early mathematics or also known as early numeracy is one of the areas of learning that is important for the cognitive development of children at an early age. According to the Malaysian Ministry of Education (2017), children’s early experience in gaining basic mathematical knowledge and skills at preschool level is very important as a preparation for children to follow formal education at primary school level later. Harun et al. (2017) also stated that the level of numeracy skills among primary school students depends on the numeracy skills acquired by children when they are in preschool. It is feared that children will be discouraged to learn and will not want to go to school if they do not master the 3M basic skills of reading, writing and counting (Hamid, 2021) as well as the addition of another basic skill that students need to master, which is reasoning. In Malaysia, when four to six-year-old children go to any early childhood education center, they will be provided with basic early mathematical knowledge and skills such as pre-numbers, number concepts (zero to 20; counting numbers from ten to ten 10 to 100), number operations (addition and subtraction operations within 18), value of money, concept of time and time as well as concept of form and space (Malaysian Ministry of Education, 2017).

According to Watts et al (2017), mathematics is a hierarchical subject where mastery of good basic mathematical concepts and skills is essential to understand and solve more difficult mathematical problems. Children’s ability to understand and master basic mathematical concepts at an early age will help them improve their thinking skills to solve problems and
make the right decisions in their daily lives (Dağli et al., 2019). Learning early mathematical concepts will be a positive and meaningful early experience as a result of the development of children’s tendencies such as curiosity, imagination and perseverance if they are given space to interact and manipulate various materials or concrete objects that are easily available around them (Hassan et al., 2017). This is because according to Bakar and Alias (2021), the use of teaching aids that are concrete or existing around children can help improve understanding, facilitate the process of understanding the mathematical concepts taught and meet the needs of children who prefer hands-on activities.

In addition, children need to use their mathematical knowledge to interact with the environment of everyday life spontaneously which involves the use of several mathematical concepts (Çelik, 2021). Such as the concept of money which is for buying at the canteen or school cooperative, the concept of time and time for management of daily routine activities in order to train oneself to be a disciplined and punctual person, the concept of number to express the number or quantity of something and the concept of shape and space to evaluate something based on characteristics that can be classified or matched according to suitability and creativity. In addition, children will also use counting and calculating skills to solve math problems whether it involves the process of addition or subtraction in their various real-life affairs (Harun et al., 2017).

**Add Operational**

According to Harun et al. (2017), children who can master the skills of counting in ascending and descending order and counting numbers by skipping well are more ready to learn the concept of number operations. The implementation of addition operations learning activities in the form of games that involve the use of concrete objects can help improve the skills and efficiency of counting two collections of objects to the level of being able to solve addition operations problems without having to count concrete objects from the collection of objects one by one (Ompok & Emison, 2021). Before performing the addition operation, children must first understand the concept of addition operations, among which is understanding the meaning of using language or early mathematical terminology such as "the sum of all", "the more", "add", "collect", "combine", "mix", “collect all” (Sarudin et al., 2019).

**Use of Representations**

Abu Bakar and Karim (2019) stated that concrete, visual and symbolic representation will be used in learning mathematics especially when it involves the concept of number operations because representation can facilitate mathematical thinking and understanding concepts as well as presenting ideas to solve a problem. Children are also found to be less spontaneous in using various representations when solving problems instead they need to be encouraged or prompted first to use various forms of representation to translate the number operations involved in solving the mathematical problem (Abu Bakar et al., 2020a).

**Problem Solving**

In a study on solving addition operation problems among six-year-old children, Abu Bakar et al. (2020b) found that children face difficulties and it is difficult to translate the addition operation in writing mathematical sentences that use the symbol plus (+) and equal to (=) or in other words is writing arithmetic equations because children are not yet able to express relationships between two objects involved in a problem solving addition operation.
According to the Malaysian Ministry of Education (2017), all early childhood education institutions that implement the KSPK Revision 2017 need to ensure that children in the institution will be taught to complete addition operations around 18. Based on the learning standards enacted in the KSPK curriculum, at the end year of the school session, five-year-old children should be able to express the addition of two sets of objects and be able to express the addition based on the given situation using objects. For six-year-old children, at the end of the school session, they should be able to perform the following math skills well and correctly. The intended skills are to express addition by counting directly from numbers, write and express mathematical sentences using the plus symbol (+) and the equal to (=) symbol, add within the range of basic facts and solve addition operation problems.

**Methodology**

To complete the study related to the use of representation in solving Mathematical problems among children, the researcher has carried out this study in a private kindergarten in Kota Bharu. Since this study aims to help the mastery and support the learning of Early Mathematics to prepare children with a good concept of Mathematics as preparation for Year One, the participants of the study only involve kindergarten children who are 6 years old. By selecting random sampling, the study sample was selected among 6-year-old children without any exclusion criteria to allow each 6-year-old child to have the same chance to be selected as a study sample.

Since this study involves children, the researcher has distributed a letter containing the study information as well as a letter of permission to the parents and guardians of children selected as study participants. The purpose is for parents and guardians of study participants to clearly understand the aims and objectives of the study. In addition, for the researcher to obtain permission from parents and guardians to collect data on the children under their care, which involves Maths-related activities, audio recordings, videos, photos and children’s work results throughout the study.

A total of 5 children were involved as study participants where the researcher carried out a preliminary test (pre-test) to obtain preliminary information about the children’s mastery in Early Mathematics. In this initial test, there are 5 items that cover the aspects of recognizing numbers, counting up, counting down, counting objects and addition operations. From the results of the initial test, the researcher has selected the 2 study participants who recorded the lowest scores for further activities as a real respondent.

Participant of the first study, P1 did not record any score in the initial test for the plus operation item. Having a spoiled personality and loving the attention of teachers and friends, P1 shows a tendency and interest in artistic activities such as singing and painting. Meanwhile, the second study participant, P2 is a quiet child who often loses focus during the teacher’s explanation session but likes to ask the teacher questions. Uniquely, P2 chose to ask the teacher privately by raising her hand and approaching the teacher instead of in class.

To help the mastery of the 2 study participants in Early Mathematics, especially in the addition operation, the researcher introduced the concept of addition using the “Let’s Add” kit in order from enactive, iconic and symbolic to both study participants. The researcher also prepares activities and addition operation questions so that both study participants understand the
concept of addition well. Study participants were also given problem solving questions and they were free to choose to use either counters, draw symbols, write symbols or combine all three in solving the question. The objective is to examine children's understanding of numbers and the concept of addition and to explore the use of various representations in solving problems related to addition operations. This study was conducted for 3 months starting October 2022 which involved data collection, data analysis and report writing.

Research Findings
The findings of the study show that, of the 5 study participants who were selected to answer the initial test (pre-test), 2 of them recorded a low score compared to the other 3 participants (Table 1). All five children have different characters from each other. First participant in initial test, R1 is an active child who has the character of being a good leader and liked by friends. Haziq also showed a high percentage of attendance at kindergarten. In fact, categorized in the group of students who easily master the concepts taught by the teacher and like perfection. The 2nd participant, R2, despite showing a moderate percentage of attendance, he is an active child, friendly and likes to ask teachers and friends. The 3rd participant, R3 is a quiet child, easily understands the teacher’s teaching and likes perfection. Meanwhile, the 4th participant, R4 is a spoiled child who loves the attention of teachers and friends. R4 also still needs guidance from teachers and parents in controlling her emotions. The 5th participant, R5 is a quiet child. However, R5 prefers to ask the teacher but one-to-one, not in class.

Based on the initial test scores, 3 study participants recorded a full score (18/18) for the number recognition item, while 4 of them obtained a full score (3/3) for the addition operation. One participant did not record any score in the plus operation item. Based on the initial test scores, the researcher has selected 2 study participants to be introduced to the concept of addition using the “Let's Add” kit. The two selected participants were the study participants who recorded the lowest initial test scores. Both were introduced to the concept of addition using the “Let's Add” kit. The researcher also prepares problem-solving questions and gives the opportunity to both participants to choose a method of solving the addition
operation. To see the improvement and comparison of scores before using the “Let's Add” kit and after using the “Let's Add” kit, both participants were given a post-test question that had 2 items, namely counting and addition operations. Based on the post test scores there was an increase in P1's plus operation item scores compared to the initial test scores (Table 2).

<table>
<thead>
<tr>
<th>Participant</th>
<th>P1</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Results</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Know the number</strong></td>
<td>Total</td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>13/18</td>
<td>72.2</td>
</tr>
<tr>
<td><strong>Count (Ascending)</strong></td>
<td>8/11</td>
<td>72.7</td>
</tr>
<tr>
<td><strong>Count (Descending)</strong></td>
<td>1/11</td>
<td>9.1</td>
</tr>
<tr>
<td><strong>Count object</strong></td>
<td>2/3</td>
<td>66.7</td>
</tr>
<tr>
<td><strong>Addition</strong></td>
<td>0/3</td>
<td>0</td>
</tr>
<tr>
<td><strong>Score comparison</strong></td>
<td>33.3% improvement in the object counting item and 100% in the addition operation item</td>
<td>Stay</td>
</tr>
</tbody>
</table>

Based on table 2, P1 showed an increase in scores in the item counting objects from 66.7% to 100%. Meanwhile, P2 recorded the same score as the initial test score, which is 100%. As for the plus operation item, P1 managed to score 100% for the post-test compared to 0% in the initial test.

**Discussion**

**How do children demonstrate understanding of numbers and the concept of addition?**

As a result of the researcher's observation during the initial test, it was found that P1 was not able to understand the concept of addition, which is combining 2 groups of objects into one group, which makes the number increase. In line with the findings of the study conducted by Sarudin et al. (2019) who stated that, children must first understand the concept of the addition operation, among which is understanding the meaning of the use of language or early mathematical terminology such as "the sum of all", "the more", "add", "collect", "join", "mix", "collect all" before completing the addition operation. P1's failure to understand the concept of addition caused this child to fail to record any score on the addition operation item in the initial test.
In contrast to P2, where when the pre-test and post-test were carried out, P2 was able to state all the correct answers to the question about the sum of the sum of two sets of objects by calculating all the sum of the objects used based on the addition operation question given. However, P2 was found to begin the calculation of the number of objects starting with the first number which is the number one and followed by the sequence of numbers that follow even though P2 was told about the total number of the first collection is so many numbers. Based on the situation, P2 was found to have understood the basic concept of addition and was able to use the correct mathematical terminology for this concept. However, P2 was found to be less able to remember the number that represents the result of the first collection and has not yet mastered the ascending number sequence, that is saying the name of the number after the number mentioned by the teacher who has encouraged her to always start the calculation of the collection starting with number one. In line with what was stated by Harun et al (2017), children were found to be more ready to learn the concept of number operations and be able to complete number operations quickly if they are skilled in counting in ascending and descending order and counting numbers in skips well and efficiently.

As children are introduced to the concept of representation, children show a good understanding of the concept of addition, which is collecting all objects into a group. P1’s post test score recorded after being introduced to the Let’s Add kit, showed an increase in scores in 2 items, namely an increase of 33.3% for the object counting item and a 100% increase for the addition operation item. The increase in scores recorded by P1 shows the use of representation introduced with the “Let’s Add” kit helps children understand the concept of addition better. Coinciding with the study by Bautista et al. (2019), the use of representation helps children understand the process of determining the difference between 2 different quantities in Mathematical operations.

When the use of representation was introduced to P2 for learning the plus operation, P2 was found to be able to use concrete and visual representations well. However, P2 is not yet able to use representation in the form of symbols where she seems to hesitate and there are times when she forgets to put the plus sign (+) and the equal sign (=) when she writes mathematical sentences. This shows that P2 is still not able to master the skill of making connections between objects and symbols. This finding is supported by Abu Bakar et al. (2020a) who found that the difficulty faced by children in translating the addition operation to writing mathematical sentences or arithmetic equations is because they are not yet able to express the relationship between two objects involved in solving a number operation problem.

To what extent do children use various representations in solving problems related to the addition operation?

When given a high-level problem-solving question, P1 chose to use counters to solve the question. As can be seen in Figure 1, P1 showed the act of counting that she had done as the researcher had shown when introducing the Let’s Add kit, showing her reliance on the representation she had chosen to solve the problem.
Therefore, counters are fundamental to P1 as a problem-solving strategy. This reinforces that the use of representation in learning mathematics, especially when it involves the concept of number operations, can facilitate mathematical thinking and conceptual understanding as well as the presentation of ideas to solve a problem (Abu Bakar et al. 2020a). In line with the findings of a study by Bakar and Alias (2021), who stated that the use of concrete teaching aids can help improve understanding and facilitate the process of children understanding the mathematical concepts being taught. In contrast to P2, who chose to use symbols and visual drawings to solve problem-solving questions (Figure 2).

However, representation in the form of symbols cannot be used well and correctly by her. After the teacher told P2 to change to the use of representations other than symbols, only then was she able to solve the problem-solving questions. When P2 uses visual representation, she tends to draw the objects involved in the problem-solving question. P2's tendency to draw the object caused her to take quite a long time to solve the problem-solving questions. In addition, when given a plus operation question, P2 was found to continue to write answers confidently without revising the answers using any form of representation.

After the teacher was told to check the answer of the addition operation by using any representation, then he started to use representation in the form of visual drawings for the review activity. After revisions were made using the representation form, P2 was able to repair the original answers which were mostly wrong answers. This is in line with the findings of Abu Bakar et al. (2020a) who stated that children were found to be less likely to use various representations spontaneously in problem solving but instead need to be encouraged or
prompted first to use any form of representation to translate the number operations involved in solving the mathematical problem.

**Research Implications**

Nowadays, the use of representation to solve Math problems has become common among educational professionals. By using representation, early childhood teacher can help children to represent numbers in meaningful ways. Representing numbers is a crucial skill in Math problem solving, as it helps to make sense of the problem and provides an anchor for comparison. Representing numbers can be done in various ways such as using words, symbols, or visuals. This helps children see a real-life situation and application of the problem which increases their interest in solving it.

Therefore, these findings act as evident that representation plays an important role in problem solving. Representation not only helps learners represent the problem in a more meaningful and visually appealing manner, but more importantly, it helps to develop problem-solving and reasoning skills. Representation aids learners to comprehend the underlying concepts of Math problems, as well as to use strategies and different techniques to solve the problem. It is imperative that learners understand the importance of representation in proper problem solving. This will contribute to the development of a more solid foundation for Math skills, and ultimately better Math problem solving skills.

**Conclusion**

In conclusion, the use of the Let's Add kit with the method of using representation in the order of i) enactive, ii) iconic and iii) symbolic can help children understand the concept of addition more quickly and effectively. In addition, through the representation method as well, can help children make problem solving to the addition operation better and more accurately, as well as easily. The increase in post test scores also proves that the use of representation in the teaching of additional concepts for Early Mathematics can help children understand Mathematical concepts more effectively.

Therefore, early education teacher is advised to use representation because it is an essential skill in learning Mathematics as it helps to make sense of Math problems and to develop better problem-solving skills. This is because, representation serves as a way of seeing, understanding, communicating, and interpreting the problem. It helps learners to develop better strategies to solve Math problems which helps them to understand the underlying concepts better.

Clearly, the use of representation in solving Mathematical problems is one of the most effective and efficient ways to stimulate children's thinking processes. This allows the researchers to conclude that there is progress shown by the children in understanding mathematics, as shown by the paths taken by the children while engaging in problem solving.
References


