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The Effects of Interest Rate on the Optimal Consumption Path in a Bewley Model with the Co-existence of Currency and Credit[1]

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Abstract
In the Bewley models, the endowment is faced to idiosyncratic risks. But contingent claims markets is restricted or completely excluded by assumption and so households couldn't insure themselves against these risks. Consequently, households will have strong motive to precautionary saving for self-insurance. Households' only option is to "self-insure" by managing a stock of a single asset to buffer their consumption against adverse shocks. The bewley models differ mainly with respect to the particular asset that is the instrument for self-insurance: fiat currency, credit (such as IOU's, bank deposits, government bonds and so on) or capital. In these models if the interest rate would be equal to the rate of the time preference then asset and consumption diverge to infinity and so monetary equilibrium doesn't exist. Therefore these

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models conclude that the use of Friedman rule can be misleading in an incomplete market setup. Therefore these models reduce the interest rate so that asset and consumption converge and consequently the monetary equilibrium exists.
In this paper we extend the bewley models and construct a heterogeneous model with idiosyncratic risks and borrowing constraint where agents hold money and bearing interest assets as government bonds for precautionary motives and self-insurance. We show that the consequences of bewley models in this condition are still true: There should be the interest rate lower than time preference ($r < \rho$) to insure the existence of monetary equilibrium. With sufficient uncertainty in the income and interest rate sequences, consumption will grow without bound even if the rate of interest is equal to or greater than the discount rate.

Introduction

This paper describes a version of what is sometimes called a "savings problem" (Chamberlain and Wilson, 2000). A consumer wants to maximize the expected discounted sum of a concave function of one-period consumption rates. However, the consumer is cut off from all insurance markets and almost all asset markets. The consumer can only purchase nonnegative amounts of a single risk-free asset. The absence of insurance opportunities induces the consumer to adjust his asset holdings to acquire “self-insurance.” Self-insurance occurs when the agent uses savings to insure himself against income fluctuations. On the one hand, in response to low income realizations, an agent can draw down his savings and avoid temporary large drops in consumption. On the other hand, the agent can partly save high income realizations in anticipation of poor outcomes in the future (Sargent and ljungqvist, 2004).

Bewley models

The bewley models are the class of models were invented by Bewley (1977, 1980, 1983, 1986), to study a set of classic issues in monetary theory such as inside and outside money, a free banking regime, the criticism of Friedman’s optimal quantity of money (Sargent and ljungqvist, 2004). Bewley (1983) was the first to derive the properties of a model were money was used as a self-insurance device to cope with some idiosyncratic risk. He notably shows that Friedman's rule cannot hold in such a framework. If the interest rate is close to the discount rate, the money demand explodes because of precautionary motives (Jelloul, 2007). The equilibrium thus necessitates a low rate of interest such that $r < \rho$ thereby violating the Friedman rule (Huggett, 1993). Imrohoroglu (1992) used numerical simulation to compute the welfare cost related to inflation by using the steady state average utilities.
In this paper we extend the bewley models and construct a heterogeneous model with idiosyncratic risks and borrowing constraint where agents hold money and bearing interest assets as government bonds for precautionary motives and self-insurance. We implement along the lines of Aiyagari (1994) and Imrohoroglu (1992) a model with two assets that can be used as store of value.

The Model

The present model describes a particular type of incomplete markets model. The models have a large number of ex-ante identical but ex-post heterogeneous agents who trade a single security. We use a general equilibrium framework and infinite horizon savings problem.

We assume that the government augments the nominal supply of currency over time to finance a fixed aggregate flow of real transfer *T*.The government budget constraint at *t*≥0 is:

(1) $T_t = T_{t-1}(1+g_t)$

Where g is the rate of money supply growth.

We consider a continuum of agents with distinct money and bonds holdings. Every agent occupies a state *s* at time *t* with a probability $\pi(s'|s)$ of transiting to a state *s*' at *t + 1*. The sequence of household's endowment evolves according to an m-state Markov chain. Each period *t*, every agent receives a wage depending on its state. If the realization of this process in time *t* is equal to s_i , then the income of household is equal to ωs_i . The households for self-insuring themselves can some cash money or to adjust its bond holdings by paying a transaction cost (δ). Money and bonds holdings are nonnegative so no borrowing is allowed. The agent's sequence of consumption and money holding and bond holding is $\{c_t, m_t, b_t\}_{t=0}^{\infty}$. Bond has a net rate of interest *i* and fiat currency has an implicit rate of return $r = \frac{p_{t-1}}{p_t} - 1$. Agents hold $m_t \geq 0$ money and $b_t \geq 0$ bonds at the beginning of period t. households decide to consume c_t to maximize their intertemporal discounted utility:

(2) $$U = E_t \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Where $\beta \in (0,1)$ is discount factor and we assume that $\beta(1+r) < 1$. $u(c_t)$ is a strictly increasing, strictly concave, twice continuously differentiable function of the consumption of a single good c and $\lim_{c \to 0} u'(c) = \infty$. The agent faces the sequence of budget constraints:

(3) $c_t + m_t + b_t \leq \omega s_t + (1+r_t)m_{t-1} + (1+r_t)(1+i_{t-1})b_{t-1} - \delta_t b_{t-1} + g_t T_t$

The Bellman equation for an agent is:

(4) $V(m,b,s) = \max\left\{u(c) + \beta EV(\tilde{m}, \tilde{b}, s') \,|\, s\right\}$

subject to budget constraint (3). $\tilde{m}, \tilde{b}$ are next period money holding and bond holding respectively. s' is the amount of stochastic shock in next period. The value function $V(m,b,s)$ inherits the basic properties of u(c); that is,V is increasing, strictly concave, and differentiable.

Now we summarize the model's axioms:

Axioms

Axiom 1: The economy is pure exchange economy.

Axiom 2: $u(c_t)$ is a strictly increasing, strictly concave, twice continuously differentiable and marginal utility function is convex ($u''' > 0$).

Axiom 3: Households encounter to uninsurable idiosyncratic risk (unemployment in this case) but we have no aggregate risk.

Axiom 4: Endowment of households almost surely is positive.
Axiom 5: There are two type assets for self-insuring: fiat currency and bonds.
Axiom 6: There is no borrowing and lending ($m_t \geq 0$ and $b_t \geq 0$).
Axiom 7: Only money buys goods and its value depends on inflation. Therefore the implicit rate of interest on money is equal $r = \frac{p_{t-1}}{p_t} - 1$.
Axiom 8: Only financial assets provide explicit return (nominal rate of interest rate *i*). There are transaction costs to access financial markets (δ). Therefore the explicit rate of return on bonds is equal .
Axiom 9: economy is in stationary situation (without population growth).
Axiom 10: Agents are infinitely lived.
In the next section we proof that in this environment, the bewley model properties are still true: If interest rate is equal to the rate of time preference (such as Friedman rule (1969)) then consumption diverge to infinity. So to converging the consumption there should be reducing interest rate.

The impact of interest rate on optimal consumption path

In this section we prove that if implicit interest on money is lower than the rate of time preference ($r < \rho$) or equivalently $\beta(1+r) < 1$, and if the gross explicit interest rate on bonds is $\beta\left[(1+r)(1+i) - \delta\right] < 1$, then we show that almost surely consumption converges.

Theorem 1: *Assume that the axioms 1-10 are true and* $\beta(1+r) < 1$ *or* $\beta\left[(1+r)(1+i) - \delta\right] < 1$ *, then if t* $\to \infty$ *so with probability one we have* $\lim_{t\to\infty} c_t^* = \bar{c}$.

Proof: The Lagrangian function is:

(5) $$\begin{aligned}\ell = u(c_t) - \lambda_t(c_t + m_t + b_t - \omega s_t - (1+r_t)m_{t-1} - (1+r_t)(1+i_{t-1})b_{t-1} + \delta_t b_{t-1} - g_t T_t) \\ - \beta E_t \lambda_{t+1}(c_{t+1} + m_{t+1} + b_{t+1} - \omega s_{t+1} - (1+r_{t+1})m_t - (1+r_{t+1})(1+i_t)b_t + \delta_{t+1} b_t - g_{t+1} T_{t+1})\end{aligned}$$

So the first order condition is:

(6) $$\frac{\partial \ell}{\partial c_t} = u'(c_t) - \lambda_t = 0$$

(7) $$\frac{\partial \ell}{\partial m_t} = -\lambda_t + \beta E_t \lambda_{t+1}(1 + r_{t+1}) = 0$$

(8) $$\frac{\partial \ell}{\partial b_t} = -\lambda_t + \beta E_t \lambda_{t+1}\left[(1+r_{t+1})(1+i_t) - \delta_{t+1}\right]$$

By composing the condition (6) and (7) we have:

(9) $$u'(c_t) = \beta E_t\left[(1+r_{t+1})u'(c_{t+1})\right]$$

We prove this theorem in an specification of utility function, of course we easily can extend this proof to the general form. Assume that the utility function has a CRRA form:

(10) $$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \;,(\gamma > 1)$$

Thus the Euler condition (9) changes to this form:

(11) $c_t^{-\gamma} = \beta E_t\left[(1+r_{t+1})c_{t+1}^{-\gamma}\right]$

Now we increase two sides to the power of $-1/\gamma$, we have:

(12) $c_t = \left(\beta(1+r_{t+1})\right)^{-1/\gamma} E_t\left[c_{t+1}\right]$

Due to assuming that $\beta(1+r)<1$, so $\left(\beta(1+r)\right)^{-1/\gamma}>1$. Thus $c_t \geq E_t\left[c_{t+1}\right]$. Consequently c_t is a nonnegative supermartingle and thus we can use "the supermartingle convergence theorem" that implies that c_t with probability 1 converge to a finite limit. By composing condition (6) and (8) and following a similar condition we see that if $\beta\left[(1+r)(1+i)-\delta\right]<1$ then t c_t with probability 1 converge to a finite limit $\blacksquare$

Now we prove that if prove that if implicit interest on money is higher than the rate of time preference ($r>\rho$) or equivalently $\beta(1+r)>1$, or if the gross explicit interest rate on bonds is $\beta\left[(1+r)(1+i)-\delta\right]>1$, then we show that almost surely consumption diverge to infinity:

Theorem 2: *Assume that the axioms 1-10 are true and* $\beta(1+r)>1$ *or* $\beta\left[(1+r)(1+i)-\delta\right]>1$. *If* $t\to\infty$ *then with probability one we have* $\lim_{t\to\infty} c_t^* = \infty$.

Proof: The value function (Bellman Function) for dynamic programming problem of consumer is:

(13) $V(m_{t-1},b_{t-1}) = \max_{m_t,b_t,c_t}\left[u(c)+\beta EV(m_t,b_t)\right]$

Subject to budget constraint:

$c_t + m_t + b_t \leq \omega s_t + (1+r_t)m_{t-1} + (1+r_t)(1+i_{t-1})b_{t-1} - \delta_t b_{t-1} + g_t T_t$

By manipulating this constraint we have:

(14) $b_t = \omega s_t - c_t - m_t + (1+r_t)m_{t-1} + (1+r_t)(1+i_{t-1})b_{t-1} - \delta_t b_{t-1} + g_t T_t$

(15) $m_t = \omega s_t - c_t - b_t + (1+r_t)m_{t-1} + (1+r_t)(1+i_{t-1})b_{t-1} - \delta_t b_{t-1} + g_t T_t$

We have substitute (14) and (15) in value function to get:

$$V(m_{t-1},b_{t-1}) = \max_{m_t,b_t,c_t}\Big\{u(c)+\beta EV\Big[\left(\omega s_t - c_t - m_t + (1+r_t)m_{t-1} + (1+r_t)(1+i_{t-1})b_{t-1} - \delta_t b_{t-1} + g_t T_t\right);$$
$$\left(\omega s_t - c_t - b_t + (1+r_t)m_{t-1} + (1+r_t)(1+i_{t-1})b_{t-1} - \delta_t b_{t-1} + g_t T_t\right)\Big]\Big\}$$

By using the Benveniste-Scheinkman formula, the first-order condition can be written as:

(16) $V_m'(m_{t-1}^*,b_{t-1}^*) = \beta E(1+r_t)V_m'(m_t^*,b_t^*)$

(17) $V_b'(m_{t-1}^*,b_{t-1}^*) = \beta E\left[(1+r_t)(1+i_{t-1})-\delta_t\right]V_b'(m_t^*,b_t^*)$

Also Benveniste-Scheinkman formula implies $V_c'(m_{t-1}^*,b_{t-1}^*) = u'(c_t^*)$. As we said, the value function inherits the basic properties of u(c) and so is strictly concave and bounded. If endowment is positive, we prove this lemma:

Lemma 1: *There is a real-value random variable z such that almost surely*:

(18) $\lim_{t\to\infty}\beta E\left[(1+r_t)(1+i_{t-1})-\delta_t\right]V_b'(m_t^*,b_t^*) = \bar{z}_b$

(19) $\lim_{t\to\infty}\beta E(1+r_t)V_m'(m_t^*,b_t^*) = \bar{z}_m$

Proof: This is sufficient that we show that $\beta E(1+r_t)V'_m(m_t^*,b_t^*)$ and $\beta E\left[(1+r_t)(1+i_{t-1})-\delta_t\right]V'_b(m_t^*,b_t^*)$ are nonnegative supermartingle and consequently according to supermartingle convergence theorem, these expression converge to a finite limit. First, we show that these expression couldn't diverge to infinity. Assume that the discounted value of total income stream is always positive. Then there is an "stopping time" *T* such that $m_T^*>0, b_T^*>0$. So we can use the supermartingle convergence theorem.

Since by axiom 4 endowment is positive, there is an stopping time *T* that $\omega s_T>0$. So $m_T^*+b_T^*\geq \omega s_T>0$. Axiom 6 implies that $m_T^*\geq 0, b_T^*\geq 0$. The concavity of value function implies that $V'(b_t^*,m_t^*)$ is finite. Let be d_t^b and d_t^m defined so:

(20) $$d_t^b=\frac{\beta E\left[(1+r_{t+T})(1+i_{t+T-1})-\delta_{t+T}\right]V'_{bt+T}(m_{t+T}^*,b_{t+T}^*)}{\beta E\left[(1+r_T)(1+i_{T-1})-\delta_T\right]V'_{bT}(m_T^*,b_T^*)}$$

(21) $$d_t^m=\frac{\beta E(1+r_{t+T})V'_{mt+T}(m_{t+T}^*,b_{t+T}^*)}{\beta E(1+r_T)V'_{mT}(m_T^*,b_T^*)}$$

Benveniste-Scheinkman formula ($V'_c(m_{t-1}^*,b_{t-1}^*)=u'(c_t^*)$) together with the concavity of utility function imply that V' is decreasing and so $V'_{m;T+t+s}(m_{t+T}^*,b_{t+T}^*)\leq V'_{m;T+t}(m_{t+T}^*,b_{t+T}^*)$, thus $d_t^b\geq E(d_{t+s}^b)$, $d_t^m\geq E(d_{t+s}^m)$. Since *T* is stopping time so the sequences $\{d_0^m,d_1^m,\ldots\}$ and $\{d_0^b,d_1^b,\ldots\}$ are nonnegative supermartingle. So we can use the supermartingle convergence theorem to show that d_t^m and d_t^b converge to a finite limit. Then almost surely we have: $\lim_{t\to\infty}d_t^m=\bar{d}_m$ and $\lim_{t\to\infty}d_t^b=\bar{d}_b$. It is trivial that $d_0^m=1$ and $d_0^b=1$. Therefore since the sequence is decreasing, $E(\bar{d}_b)\leq 1$ and $E(\bar{d}_m)\leq 1$.

Since $\lim_{t\to\infty}d_t^m=\bar{d}_m$, so according to (20) we have:

$$\lim_{t\to\infty}d_t^b=\lim_{t\to\infty}\left(\frac{\beta E\left[(1+r_{t+T})(1+i_{t+T-1})-\delta_{t+T}\right]V'_{bt+T}(m_{t+T}^*,b_{t+T}^*)}{\beta E\left[(1+r_T)(1+i_{T-1})-\delta_T\right]V'_{bT}(m_T^*,b_T^*)}\right)\Rightarrow$$

$$\beta E\left[(1+r_T)(1+i_{T-1})-\delta_T\right]V'_{bT}(m_T^*,b_T^*).\lim_{t\to\infty}d_t^b=\lim_{t\to\infty}\left(\beta E\left[(1+r_{t+T})(1+i_{t+T-1})-\delta_{t+T}\right]V'_{bt+T}(m_{t+T}^*,b_{t+T}^*)\right)$$

$$\Rightarrow \beta E\left[(1+r_T)(1+i_{T-1})-\delta_T\right]V'_{bT}(m_T^*,b_T^*).\bar{d}_b=\lim_{t\to\infty}\left(\beta E\left[(1+r_{t+T})(1+i_{t+T-1})-\delta_{t+T}\right]V'_{bt+T}(m_{t+T}^*,b_{t+T}^*)\right)$$

If we define $\bar{z}_b$ is equal to $\beta E\left[(1+r_T)(1+i_{T-1})-\delta_T\right]V'_{bT}(m_T^*,b_T^*).\bar{d}_b=\bar{z}_b$, so:

(22) $$\lim_{t\to\infty}\left(\beta E\left[(1+r_t)(1+i_{t-1})-\delta_t\right]V'_{bt}(m_t^*,b_t^*)\right)=\bar{z}_b$$

By following similar process we have:

(23) $$\lim_{t\to\infty}\left(\beta E(1+r_t)V'_{m;t}(m_t^*,b_t^*)\right)=\bar{z}_m$$

So the proof of lemma 1 is completed ■

Lemma 1 and these assumptions that $\beta(1+r)>1$ and endowment is positive and stochastic led that consumption diverge to infinity. If in limit with probability 1: $\lim_{t\to\infty}V'(b_t^*,m_t^*)=0$ so Benveniste-Scheinkman formula implies that with probability 1: $\lim_{t\to\infty}c_t^*=\infty$ ■

Conclusion

Idiosyncratic risk provides a strong stimulus for precautionary savings. Asset detention acts as insurance against future adverse shocks. However, the role of fiat money is not clear when another dominating asset, like bonds, is present. Following the tradition of heterogeneous agents model, we adopted a framework where agents are subject to some uninsurable idiosyncratic risk. We prove that in a bewley model with coexistence of credit and currency, if the rate of interest is greater than or equal to the discount rate, then consumption grows without bound. But if the rate of interest is lower than the discount factor, then consumption generally converges to a finite limit.

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