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The Effect of a Problem-Solving Approach on Students' Heuristics Knowledge Development

Nurul Akmal Md Nasir & Parmjit Singh
Faculty of Education, Universiti Teknologi MARA, 42300 Bandar Puncak Alam, Selangor, MALAYSIA.
Corresponding Author's Email: nurulakmal@uitm.edu.my

Abstract
Various current studies have shown that high school leavers have a major bridge to gap between the expectations of tertiary level cognitive requirements and their current thinking repertoire in mathematics learning. This study investigates if cognitive strategy or heuristics were the stumbling blocks (other than content knowledge) in inhibiting students' mathematical thinking development. Thus, this three-phase study was undertaken to examine the effect of the problem-solving approach (PSA) on students' heuristic knowledge development in solving non-routine problems. This study employed a quasi-experimental design, comprising 49 first-year college students majoring in mathematics. The first phase findings show that students' lack the repertoire of heuristic knowledge that, to a large extent, inhibited their ability to solve problems. The second phase findings show a positive impact of PSA on students' cognitive heuristic ability in solving problems. The third phase found a significant relationship between heuristic knowledge and the math thinking scores. These heuristics allowed them to generate necessary 'tools' in the absence of the requisite knowledge in seeking solutions to the problems. The findings suggest that college students need to be provided more opportunities to develop their heuristic knowledge and to connect with core math content to bridge the gap with the cognitive requirements of college mathematics.
Keywords: Problem-solving Approach, Heuristics, Non-routine Problems, Cognitive Strategy, Mathematical Thinking.

Introduction

The mathematics curriculum in Malaysia aims to develop students with in-depth mathematical knowledge and abilities so that they can use this knowledge responsibly and effectively to solve more complex problems (MOE, 2018). This has made problem-solving the main element in the framework for the Malaysian Mathematics curriculum, namely KSSM. This new curriculum was implemented due to Malaysian students' low performance in the international studies of TIMSS and PISA (Nasir et al., 2021; Singh et al., 2018; Halim et al., 2014; Yunus, 2015). However, recent findings suggest that not much has changed since the implementation of KSSM. A recent report from the Organization for Economic Cooperation and Development, for instance (OECD), ranked Malaysia in 52nd place out of a total of 76 participating countries in Math and Science (Sonia, 2021). Our southern neighbour, Singapore, emerged as the top-ranking nation, followed closely by Hong Kong, South Korea, and Japan. Singapore has consistently ranked in the top three in the rankings of PISA and TIMSS over the past decade (Educomics, 2021). Why does Singapore excel in math?

Singapore's high scores in every global competitiveness assessment are results of the education system, which is unlike anything in the world (Toh et al., 2019). The heart of their math education framework is based on problem solving, which has been the goal of school mathematics education for the last two decades or so. Similarly, Malaysia's education system adopted a similar philosophical pursuit with the implementation of the Malaysia Education Blueprint 2013–2025, which aims to perform in the top third of all nations in TIMSS and PISA results, which measure performance in international assessments. Yet, we are dismayed with our performance in TIMSS and PISA (Merilyn, 2021; Halim, 2020). Inadvertently, this creates a gap between the expectations of the cognitive requirements of college mathematics and those of the incoming high school leavers.

Studies in both local (Nasir et al., 2021; Hashemi et al., 2019; Singh et al., 2016) and international contexts (Apostol, 2017; Bowyer & Darlington, 2016) have found that the vast majority of students do not possess the knowledge and abilities required to meet the needs and intellectual expectations of college after graduating from high school. According to Hoon et al (2022), many activities which promoting critical thinking skill were organised by the universities. However, many studies found that Malaysia undergraduate students still have low proficiency in solving problem. These studies found that high school leavers faced great difficulty in solving high-order thinking problems due to their inability to apply the learned math content into problem-solving situations. Hashemi et al (2019) found that students could not apply problem-solving steps as the fundamental entry step in deriving a solution. One of the major reasons was the student’s novice cognitive strategy in seeking the process of deriving the solution. Similarly, Nasir et al (2021) also found that undergraduate students struggle to plan their solutions and have limited problem-solving strategies when solving problems. What are these problem-solving strategies?

Applying problem-solving strategies called heuristics is one of the fundamental requirements in the solution process to solve higher-order thinking (HOT) problems. At the macro level, heuristic methods provide some broad suggestions that help in acquiring a better understanding of an issue or successfully addressing it. Heuristics, as a tool, are approaches that provide a pathway in an attempt to derive a solution to a problem. This tool provides understanding strategies, approaches, and techniques for solving problems with the aim of "studying the methods and rules of discovery and invention" (Polya 1973, p. 112). In other words, these heuristics exist because they frequently aid in the discovery of an easy path to the solution of complex problems (Renkl et al., 2008). According to studies, high school graduates who enter college or university have low critical thinking capacity (Singh et al., 2016) and low cognitive thinking ability in problem-solving solutions (Hoon et al., 2018). Parmjit et al (2017) argued that this lack of cognitive ability in students is caused by their teachers' procedural structural orientation and traditional teaching methods, which don't give them enough opportunities to develop their conceptual understanding. Thus, what action should be taken to curb this issue of concern?

One solution is to use a problem-solving approach to enhance students’ mathematical thinking repertoire (Devlin, 2012; Liu & Niess, 2006). This approach provides students with "problem-solving tools" that would enable them to adapt to changing demands (Treffinger et al., 2008). Thus, this study was undertaken to investigate the impact of the Problem-Solving Approach (PSA) on students' heuristic knowledge when solving problems. The specific objectives are:

1. to examine the current level of first-year university students (majoring in mathematics) in heuristic knowledge when solving problems.
2. to examine the effectiveness of the Problem Solving Approach on students' heuristic knowledge development to solve problems.
3. to investigate if heuristic knowledge is a significant predictor of students' mathematical thinking scores (MTS).

Literature Review

The aim of this study was to examine the effect of the problem-solving approach (PSA) on students' heuristic knowledge in solving non-routine problems. The intervention, or manipulating variable of study, namely the problem-solving approach, is planned classroom sessions conducted in the experimental group to solve non-routine problems. The variables of this approach are problem solving, heuristics, and non-routine problems.

Problem-Solving

Polya (1973), a pioneer in problem solving context, elucidated it as a method used to resolve a situation when there isn't a clear solution. According to Lau (2005), it is an attempt to solve a mathematical problem when the solution is unknown. According to both of them, pedagogical approaches based on the problem-solving approach enhances learners' mathematical thinking development. With this approach, learners would actively participate in creating their own mathematical strategies and construct knowledge by solving problems (Lester & Mau, 1993). It helps learners become independent explorers and solves math problems connected to real-world applications. When simplifying the tasks, engages learners in higher order cognitive abilities, including synthesizing, analyzing and reasoning.

Various different problem-solving models have been used in the literature. These models are models developed based on previous experience that provide a recommended approach for solving problems or analysing potential solutions. Polya (1973) proposed a 4-phase model for addressing problems: analysing the problem; formulating a strategy; carrying out the plan; and looking back. In contrast, Schoenfeld (1992) first suggested a 6-stage approach for solving problems, consisting of reading analysis, exploration planning, implementation planning, and verification. Then, he further elaborated the process into six categories: reading or rereading the problem; analysing the problem; investigating parts of the problem; planning all or part of the answer; putting a plan into action; and verifying a solution. In the problem-solving approach, these models help learners face complex problems by guiding them with well-defined guidelines or plans to follow.

Heuristics

Heuristics are tools that provide a basis in deriving a solution to a problem, or at the very least, they provide some basic suggestions that helps understand to solve a problem. (Polya, 1973). Researchers (Wilson, 1993; Bruder, 2016; Tiong, 2006) investigated how general and task-specific heuristics differed. These investigations discovered a beneficial effect on the application of heuristics as a tool for problem-solving. It is crucial to stress once more that heuristic strategies are not a substitute for in-depth knowledge but rather act as a roadmap for specific kinds of challenges. The application of heuristic strategy frequently relies largely on a solid base of domain-specific resources. Expressions like "simplify an algebraic expression by removing parentheses," "make a table," "restate the problem in your own words," or "draw a figure to suggest the line of argument for a proof" are heuristic in nature. These heuristics serve as the baseline tools for success in solving higher-order thinking problems (Polya, 1973). Pattern recognition, writing an equation, organising a list or table, creating drawings or other visual representations, guessing and checking, working backward, solving a smaller problem, and logical thought are among the common forms of heuristics that are frequently mentioned in the literature (Posamentier & Krulik, 2008; Tiong, 2006; Singh, 2012; Watters & Logan, 2006). In the context of this study, the respondents have acquired the necessary subject-matter knowledge through their previous learning in high school. This gives us the opportunity to examine the quality of this learned knowledge when applied to solving non-routine problems that are higher-order thinking in nature.

Non-Routine Problems

One of the fundamental roles of teaching mathematics is to enhance a learner's ability to solve problems. In the literature, there are two different categories of problems: routine problems and non-routine problems. The former is related to 'exercise' problems such as "plug-and-chug" problems (Duch, 2001) and "skill and drill" problems (Polya, 1973), commonly available at the end of a chapter in a traditional textbook. On the other hand, the former significantly contributes to the development of students' cognitive growth (Polya, 1973), where characteristics of problems include "easy yet challenging" and "stumbling block." Examples to exemplify both the problems are as follows:

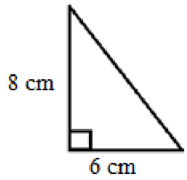

Figure 1

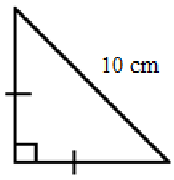

Figure 2

a) Determine the area of the right-angled triangle in Figure 1 (a routine problem).
b) Without using Pythagoras or trigonometry functions, calculate the area of the right-angle isosceles triangle in Figure 2 (non-routine problem).

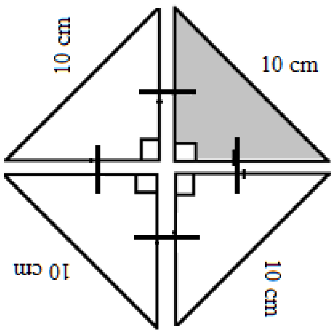

Figure 3

Solution: When the triangles are geometrically constructed to form a square, it yields a quarter of a square with an area of 1/4 x 100 = 25cm2.

School leavers had a 15% success rate (compared to a Figure 1 item that has a 100% success rate) and struggled to understand its solution without using Pythagoras or trigonometric functions. Despite the fact that these students have spent at least 11 years of formal math learning, they lack crucial cognitive strategies and the ability to solve non-routine mathematical problems (Asman and Markowitz, 2001). It is crucial to simulate classroom activities with non-routine math problems in order to equip students with heuristic knowledge to enhance their mathematical thinking development classrooms (Parmjit et al., 2018)

In this problem-solving approach, heuristics were used as a tool to enhance the cognitive thinking process. The students experienced explicit problem-solving stages and various heuristics in engaging with a variety of non-routine exercises and problems in eight parallel sessions. Through a variety of heuristics, the students were actively involved in cognizing their thinking from the problem-solving approach. It focused on using different mathematical contexts to understand the content, identify the strategies, extend the problems and communicate mathematical demonstrations. Building on previous research but non-conclusive findings, can heuristics be taught as a formal activity for classroom practises to enhance students' problem solving prowess?

Methodology

Research Design

This study utilised a quasi-experimental design for the data collection purposes. It aims to investigate the effectiveness of the Problem-Solving Approach (PSA) on students’ heuristic knowledge when solving problems. This design was employed instead of the classic experiment design because of ethical reasons in randomising the subject due to college regulations. The researchers acknowledged the limitation of the quasi-experimental design in terms of its random assignment of subjects for each group.

Subjects of Study

A total of 49 first-year science undergraduates participated in the study using two similar intact groups. The experimental group comprised twenty-four students (n = 24), while the control group comprised twenty-five students (n = 25).

Instrumentation

In this study, the heuristic knowledge of the students was described and compared across indicators. Two sets of paper and pencil tests called mathematical thinking tests (pre-test and post-test) consisting of ten non-routine problems were administered among the students. The content of the tests comprised fundamental mathematics topics, namely numbers, measurement, and estimating; algebra; geometry; statistics & probability; and logical reasoning make up the first four subjects. These five core areas include standards that have received both national and international attention (NCTM, 2000; KPM, 2013) as well as fundamental concepts to be mastered for applications that have been previously learned in high school.

Validity and Reliability

The content validity of both the tests were established by three experts in the field. On the other hand, a test-retest reliability was computed to establish its reliability producing coefficients of 0.859 and 0.894, respectively. Both the validity and reliability measures taken ensure the credibility of the test as a tool for the data collection purpose in answering the research objectives of the study.

Intervention of a Problem-Solving Approach

The PSA integrates two main components in the intervention: (a) problem-solving phases and (b) heuristics. The experimental group was exposed using a series of lessons put together using the framework of (Polya, 1973; Schoenfeld, 1992). The experimental group was also taught specific heuristics and problem-solving strategies to solve the problem. During the intervention, focus was placed on five key heuristics (refer Table 1) as commonly depicted in the literature. The total duration of the intervention was 8 weeks, with 2 hours per week accumulating a total of sixteen hours. The following heuristics were highlighted and explicitly discussed throughout the intervention:

Table 1

List of Heuristics in Problem-Solving Approach Intervention

No	Heuristics	Problem-Solving Strategy
1	Representation heuristics	• Make a table • Draw a diagram (model/picture/graph/visual representation) • Writing Equation
2	Simplification heuristics	• Simplify problem • Pattern recognition • Solving a simpler problem
3	Pathway heuristics	• Work-backward • Before-after concept (combination of working forwards and working backward)
4	Generic heuristics	• Guess and check • Make a systematic list
5	Formula	• Solving using formula

Data Collection and Analysis Techniques

In this study, the heuristic knowledge of the students was described and compared across indicators. Data was collected from the two sets of paper and pencil tests administered before and after the intervention to both the experimental and control groups. Scores of heuristic knowledge obtained in the pre-test and post-test were compared and analysed through split-plot analysis of variance (SPANOVA). The SPANOVA test was used to examine whether the experimental group, which received an eight-week intervention, performed differently over time from the control group.

An adaptation of the Oregon Mathematics Problem Solving Rubrics (OMPSR) as shown in Table 2 was used to evaluate students' heuristic knowledge of each particular item on the Mathematical Thinking Tests (pre-test and post-test). Simple linear regression was used to determine the correlation and the expedient heuristics in predicting students mathematical thinking.

Table 2
The Scoring Rubric

Score	Description
0	No effort was made; this was a failed attempt. No strategy presented.
1	Strategy are random; Strategy used totally wrong.
2	Strategy presented is only partially useful.
3	Strategy is suitable and appropriate but presented with incorrect response.
4	Strategy is suitable and appropriate; The strategy presented along with the correct response.

Results

The term "heuristics knowledge" in this study refers to students' ability to devise useful approaches (including formulae and algorithms) in solving non-routine problems. Any method or strategy for problem-solving that employs a realistic approach or any technique to deliver accurate, ideal, and coherent solutions during the time frame was considered. The first section presents students' current heuristics knowledge based on their pre-test results. This is then followed by examining the impact of the Problem Solving Approach (PSA) on students' heuristic knowledge development to solve problems.

Students' Current Heuristics Knowledge in Solving Problem

Research Question 1: What is the students' current level of heuristic knowledge in solving mathematical thinking test?

This finding was based on the 49 students that participated in the study based on the Pre-Test scores.

Table 3
Heuristics Knowledge Scores in Pre-Test

	N	Mean	Std. Deviation
Heuristic Knowledge (Pre-Test)	49	11.57	2.74

Note: Full mark=40

The data in Table 3 reveal that scores of heuristics knowledge achieved by 49 students engaged in the study are low with a mean of 11.57 (SD=2.74). In other words, these students attained a low score of 28.9% $\left(\frac{11.57}{40}\times 100\right)$ in heuristic knowledge while solving the problem in the Mathematical Thinking Test (pre-test). This finding depicts the notion of students lacks of heuristics knowledge in solving problems

Effect of Problem-Solving Approach on Students' Heuristics Knowledge Development

This section analyses the impact of the PSA intervention on student's heuristic knowledge development.

Research Question 2: Is there a positive effect of the problem-solving approach on students' heuristic knowledge development?

Table 4
Descriptive Statistics of Students' Heuristics Knowledge in Mathematical Thinking Test

	Experimental Group (n=24)		Control Group (n=25)	
	Pre-Test	Post-Test	Pre-Test	Post-Test
Mean	11.33	27.21	11.80	14.80
S.D	2.81	7.24	2.71	3.98
Minimum	5	15	7	7
Maximum	16	39	16	23

Note: Full mark score=40.

Table 4 shows the results of descriptive statistics of students' current heuristics knowledge for solving Mathematical Thinking Tests between pre-test and post-test of both the experimental and control groups. The experimental group's heuristics knowledge score ranged from a low of 5 on the pre-test to a high of 16, based on a maximum score of 40. The heuristics knowledge score range for the control group was 7 to 16, just like it was for the experimental group. The experimental and control groups' respective mean scores for the heuristics knowledge were 11.33 (S.D. = 2.81) and 11.80 (S.D. = 2.71).
In the follow-up analysis, the experimental group's lowest score for heuristics knowledge was 15, while the control group's lowest score was 7, and the highest score was 23. The experimental group's mean post-test scores for the heuristics knowledge were 27.21 (S.D. = 7.24) and the control group's were 14.8 (S.D. = 3.98), respectively. To determine whether there were any statistically significant differences between the scores of the experimental group and the control group, a Split Plot ANOVA Test (SPANOVA) was used (in both pre-test and post-test).
Table 5 shows the result of the variance analysis for the comparison of heuristic knowledge between the experimental and control groups.

Table 5
SPANOVA result for Heuristics Knowledge

Effect		Value	F	Hypothesis df	Error df	Sig.
Heuristics Pre-Heuristics Post	Pillai's Trace	0.73	126.99	1	47	0.00
Heuristics Pre-Heuristics Post * Group	Pillai's Trace	0.56	59.09	1	47	0.00

Table 5 shows that there are significant differences in heuristics knowledge scores between pre-test and post-test [F(1,47) =59.09,p<0.05]. There was a main effect of pre-post tests for the mean score of heuristic knowledge in solving mathematical thinking tests. Pillai Trace value of pre-posttests was 0.73, with a significant value of p<0.05, showing that there was a significant effect in terms of pre-posttests. This indicates that there was a change in heuristics knowledge scores across the two testing times of pre-test and post-test. The effect across the testing time was determined to be significant. Table 6 shows the results of the between-subjects effect for the main effect of group.

Table 6
Between-Subjects Effects (Heuristics Knowledge)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	25980.23	1	25980.23	1074.91	0.00
Group	873.08	1	873.08	36.12	0.00
Error	1135.98	47	24.17		

Data in Table 6 shows a significant difference in heuristics knowledge mean scores [F(1,47) = 36.12, p <0.05] between experimental and control groups at the 0.05 level. This indicates there was a difference in heuristics knowledge score between the experimental group and the control group across the pre-test and post-test. The graph in Figure 1 illustrates the heuristics knowledge scores of experimental and control groups in both tests (pre-test and post-test).

Figure 4: Profile Plot for Achievement of Heuristics Knowledge Scores of the Two Groups

The graph in Figure 1 illustrates an interaction effect of heuristic knowledge scores between the experimental group and the control group across both mathematical thinking tests (pre-test and post-test), in which the mean heuristic knowledge scores for the experimental group (PSA treatment) increased linearly from pre-test to post-test. The mean scores of heuristics knowledge for the control group (conventional method) increased linearly from pre-test to post-test. However, the heuristics knowledge scores of the experimental group were lower compared to the control group at an early stage (pre-test), but these scores of the experimental group surpassed the mean heuristics knowledge scores for the control group across post-test. Findings proved that participants (experimental group) who followed PSA treatment increased their heuristic knowledge in solving mathematical thinking tests more than the control group who received traditional mathematics activity (refer to Figure 1). These findings substantiated that the PSA was able to provide a positive effect on increasing students' heuristic knowledge after the intervention.

Relationship Between Heuristics Knowledge and Mathematical Thinking Scores

This final section examines the extent to which the independent variable Heuristics Knowledge (HK) influences the dependent variables of this research, namely Mathematical

Thinking Scores. Besides, the researchers also examined if heuristic knowledge could be a significant predictor of participants' mathematical thinking achievement. Hence, the scores of heuristics knowledge and overall achievements in the post-test were used to determine how vital heuristics knowledge variables are in predicting mathematical thinking scores after the intervention.

Research Question 3: What is the significant impact of Heuristics Knowledge on learners Mathematical Thinking Scores (MTS)?

H_1: There is a significant impact of Heuristic Knowledge on learners Mathematical Thinking Scores (MTS)?

Table 6
Regression analysis on Mathematical Thinking Scores

Hypotheses	Regression Weights	B	t	p-value	Hypotheses Supported
H_1	HK→ MTS	1.157	35.566	.000	Yes
R^2	.964				
F(1, 47)	1264.97			< .001	Yes

Note: *p< 0.001, HK: Heuristics Knowledge, MTS: Mathematical Thinking Scores

A linear regression was computed to predict students Mathematical Thinking Scores (MTS) based on their Heuristic Knowledge (HK). A significant regression equation was found [F(1,47) = 1264.97, p <.001), with an R2 = .964, which elucidates that the model explains an extreme high 96.4% of the variance in Mathematical Thinking Scores (MTS). This illustrates that Heuristics Knowledge was a significant predictor of students Mathematical Thinking Scores (MTS. The coefficient for heuristic knowledge scores is 1.157. Hence, for every unit increase in heuristic knowledge scores, the researcher expects 1.157 gains in the Mathematical Thinking Test Score. This result is statistically significant. The approximated regression equation for this study is:

$$\textit{Mathematical Thinking Test Score} = -8.768 + 1.157\ (\textit{Heuristics Knowledge})$$

Discussion of Findings

Based on the findings in the pre-test, students in both groups had difficulty attempting the problems that required higher-order thinking abilities. They lacked the cognitive repertoire of heuristic knowledge one expects these high school leavers to have in problem-solving strategy. This result is consistent with previous research from both local and international contexts during the past decade Nasir et al (2021); Parmjit et al (2018); Intan (2016); Aida (2015); Borsuk (2016); Adams (2014); Nasir et al (2021) found that many students struggle to solve these problems, although they have learned the content in their earlier grades. They argued that this was due to an absence of strategies for deriving a solution. Among the reasons why students are not able to use heuristics is that mathematics instruction currently is too focused on procedural paradigm orientation instead of providing the "problem-solving tools" that would allow the students to be adaptative to changing needs towards the development of mathematical thinking (Singh et al., 2017). Kusdinar et al (2017) found that the teacher does introduce various problems to the students but does not explicitly provide the necessary tools to deal with situations when the solution is not obvious. One can conclude that heuristics play a role as a tool for orientation in problem situations; thus, knowledge of heuristics strategies does improve students' problem-solving abilities (Bruder, 2016).

The results of the intervention of the problem-solving approach have greatly influenced the students' heuristic knowledge when solving problems. This intervention, indirectly, had a positive impact on the students' capability to solve the assigned problem correctly. This finding is consistent with findings from Kusdinar et al (2017); Tambunan (2018) where they elucidated that heuristics (problem-solving strategies) should be taught to the undergraduate students. Isa et al (2021) has suggested that there is a needed to apply teaching based problems among undergraduate students in the university to improve the quality of the program. These elements could affect the ability of the students to solve problems. Singh et al (2018) in their seminal work highlighted the need for schools to formally introduce heuristic knowledge in classroom practices as a first step for students to meaningfully solve problems and take ownership of them. This finding is consistent with Harel's (2020) work that some heuristics can be taught as desirable ways of thinking rather than just as rules of thumb or prescriptive procedures.

The findings from the regression analysis elucidated that heuristic knowledge is a significant predictor for mathematical thinking development. The conclusion for this research question is that there is a strong relationship between heuristic knowledge and mathematical thinking test scores. The regression analysis showed that the increment in test score may be explained by the learner's heuristic knowledge acquisition.

We have made an effort to demonstrate that the intervention is a plausible explanation for the observed phenomenon of increasing student scores on mathematical thinking. Even while the predictor of heuristic knowledge in our study accounts for 96.4% of the variance in the post-test scores, we do not assert that the improvements are solely attributable to the intervention. Further research would be required to make such a claim. It ought to involve a more thorough attempt to manage the numerous factors that could have an impact. Such an experiment, which might look at numerous parallel treatments, might make an intriguing subject for further study.

Conclusion

The problem-solving approach significantly influenced the scores of students' heuristic knowledge and types of heuristics used from the pre-test to the post-test. Students' knowledge of heuristics is crucial to developing students' problem-solving skills and mathematical thinking. An essential step in improving students' ability to reason and solve higher-order thinking problems is exposing students to these "tools" (heuristics). Besides, it is crucial to simulate classroom activities with non-routine problems to enhance the mathematical thinking development of students. Since students believed that the university's curriculum and lecturers could help them improve their cognitive domain (Isa et al., 2021), we should support them in achieving this goal.

Mastery of heuristics is the basis for developing mathematical thinking. It assists students in discovering various approaches to solving problems. Schoenfeld (1992) has argued that students should be prepared to "learn to think mathematically". This means that the students should be able to use their knowledge of mathematics with the necessary heuristics tools and apply it to solve problems to make it more meaningful in their learning. The current intervention was to instill students' strategies when attempting the problem with various

heuristics and increase their problem-solving skills. The intervention (PSA) utilised in the study was effective because the learning experiences offered the students engaging problem situations, encouraged the development of core mathematical concepts, and encouraged the use of learnt heuristics to solve the given problems rather than focus on the question and answer only (Salleh et al., 2022).
Singapore has been doing well in international studies such as TIMSS and PISA and constantly features among the top in these education rankings studies. They started with their framework called the "Singapore Mathematics Framework" on problem solving in the late 1990's and have moved on since to tighten their grip on conceptualising it. The framework processes have drawn a lot of interest from around the world. Is there a possibility for a new horizon for Malaysian students in the near future if the introduction of problem solving is combined with an emphasis on heuristic knowledge in a new chapter for school mathematics? More research studies are needed, especially in school mathematics, to further enhance our understanding of its effectiveness.
The small number of participants in this study, which is a common problem when performing an experimental study, was one of its limitations. Future research with bigger sample sizes will improve the external validity of the problem-solving approach treatment, even though the repeated-measures ANOVA utilised in this study should raise the test's power to identify significant changes. The feasibility of implementing this intervention programme in inclusive classroom settings (during and/or after the course hours) and its effects on students' mathematical learning and mathematical thinking need to be further tested by conducting studies with large sample sizes across multiple programmes and universities, and even in schools.

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