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Teaching Geometry Using Van Hiele's Phase-Based Instructional Strategy

Nasir Ahmad Saleh Aldiabat
Ministry of Education, The United Arab Emirates
Email: Nasurdiabat@yahoo.com, nasrdiabat@student.usm.my

Wun Thiam Yew
School of Educational Studies, University Sains Malaysia, Malaysia
Email: tywun@usm.my

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Abstract

The purpose of this concept paper is to 1. emphasize the use of Van Hiele's Phase-Based Instructional Strategy in teaching geometry, 2. evaluate the advantages of using Van Hiele's Phase-Based Instructional Strategy in teaching geometry, and 3. highlight significant previous studies undertaken in this specific field. The researchers elucidate the significance of learning geometry, its intimate correlation with other fields of mathematics, and the challenges students face when delving into the subject. The researchers offer valuable pedagogical perspectives, comparing the conventional approach to learning geometry with Van Hiele's Phase-Based Instructional Strategy. As the literature indicated, the writers partitioned the crucial paper into three sections. Challenges faced by students in learning geometry in school, factors contributing to teaching difficulties in school geometry, and instructional strategy based on Van Hiele's phases. The literature presents several advantages of Van Hiele's Phase-Based Instructional Strategy. Teachers must be cognizant of the advantages of employing Van Hiele's Phase-Based Instructional Strategy when teaching geometry. Further investigation is necessary to broaden the scope of research in this domain, intending to identify efficacious educational approaches that promote the utilization of Van Hiele's Phase-Based Instructional Strategy in diverse mathematical areas, as well as in other subjects, including physics, science, computer science, and the STEM projects.

Keywords: Geometry, Van Hiele's Phase-Based Instructional Strategy.

Introduction

Geometry is a fundamental discipline within the field of mathematics. The National Council of Teachers of Mathematics (NCTM) emphasizes the significance of geometry in education. This subject involves the study of geometric patterns and designs and the analysis of properties and characteristics of geometric shapes. It also includes understanding spatial arrangement, utilizing transformations, and applying symmetry in mathematical analysis (NCTM, 2000). As described by Bora and Ahmed (2018), Geometry is a branch of mathematics that specifically examines the properties of particular things, the spatial relationships between various objects, and the qualities these items contain.

The norms and regulations of mathematics instruction confirm that geometry is crucial in the early stages of education, as demonstrated by the inclusion of geometry in the mathematics curriculum. Many students find exploring problems, concepts, and geometric expressions stimulating and beneficial (Abdullah & Zakaria, 2013b). According to Bozkurt (2018), the study of geometry offers numerous advantages and positive outcomes. It plays a crucial role in enhancing students' cognitive abilities and facilitates the creation of new environmental structures by integrating two- and three-dimensional shapes. Additionally, geometry is essential for establishing connections between lines and shapes and for visualizing and understanding the significance of images.

Chew and Lim (2013) also provide support for geometry education. Geometry is important for comprehending other branches of mathematics, such as fractions, decimals, percentages, polynomial functions, derivatives, and integral calculus. Acquiring knowledge in geometry allows us to understand the universe in which we reside comprehensively. Geometric patterns occur naturally in several phenomena, including geological formations, the solar system's structure, plants and flowers, stones and crystals, and even living organisms like animals (Ajai & Ogungbile, 2023). Kuzle (2023) stated that Geometry fosters understanding and abilities that are applicable and interconnected with several academic subjects, extending beyond mathematics.

Geometry holds significant value both within and beyond the confines of educational institutions. Students acquire a wide range of knowledge and skills pertaining to many applications of geometry. They possess knowledge of geometric construction and engage in studying geometric shapes. Geometry facilitates the exploration and understanding of the features and attributes of various shapes. Geometry is crucial in maps as it identifies positions, explains spatial connections, and comprehends transformations. Geometry subjects facilitate a more profound comprehension of various mathematical, artistic, scientific, and social studies issues for students (Usman, 2019).

Sherard (1981) presents six justifications for the inclusion of geometry as a core mathematical discipline in high school mathematics education:

1. Geometry is a crucial aptitude as it is important for effective communication. Basic spoken and written languages have several geometric terms, such as point, line, angle, parallel, perpendicular, plane, circle, square, triangle, and rectangle. This geometric expression enables us to articulate our perspectives to others with exactitude.
2. Geometry is applicable in diverse practical scenarios. Geometrical applications are necessary for measuring various aspects of our homes and other aspects of our daily existence.
3. Geometry plays a crucial role in various disciplines within the realm of basic mathematics. Geometric explanations can provide valuable insights into various arithmetical, algebraic, and statistical concepts.
4. Geometry provides a robust mathematical basis for subsequent academic exploration. Euclidean geometry was a mandatory prerequisite for university entry in the United Kingdom.
5. Geometry is an essential skill as it is one of the cultural inheritances of humanity.
6. Geometry, like mathematics, offers a setting for students to enhance their logical thinking abilities.

The aforementioned six elements contribute to the logical inference that geometry is a prominent mathematical aptitude, as it serves as both a unifying subject within the overall mathematics curriculum and a means of enhancing learners' logical and deductive reasoning skills.

Learners' Challenges in Learning School Geometry

School geometry is a branch of mathematics that focuses on studying shapes, spatial relationships, and their properties. The primary objective of school geometry is to facilitate students' comprehension of spatial reasoning, logical cognition, and problem-solving abilities about the characteristics and interconnections of geometric objects. The fundamental concepts covered in school geometry include points, lines, planes, contours, angles, polygons, congruence and similarity, circles, area and perimeter, dimensions, 2-dimensional figures, symmetry, three-dimensional figures, coordinate geometry, and transformations (Russell, 2020). The study of geometry in school plays a crucial role in developing mathematical concepts. It serves as a link that connects diverse ideas across many branches of mathematics (Mammana & Villiani, 1998; Muschla & Muschla, 2000).

Geometry learning challenges manifest in several forms. Misunderstandings, ambiguous terminology, recognition of basic geometric forms, characteristics of shapes, parallel and perpendicular lines, the concept of angles, the sum of angles in a triangle, and the skill of constructing mathematical proofs are among the topics extensively discussed in the literature (Burger & Shaughnessy,1986; Clements & Battista, 1992; Fuys et al., 1988). The results of the research conducted by Chiphambo & Feza (2020) indicate that a majority of students encounter difficulties in comprehending geometry due to the following factors: Insufficient comprehension of mathematical symbols, Obstacles in understanding geometric language that hinder comprehension of query requirements, and insufficient comprehension of many classifications of triangles and their inherent characteristics.

Misconceptions

According to Feza and Webb (2005) and French (2004), students have various and distinct misconceptions about geometric concepts. However, these misconceptions are intriguing. Teachers must deeply understand learner misconceptions to effectively organize and execute accurate geometry instruction. Clements and Battista (1992) recognized that many elementary and secondary school students possess erroneous beliefs about the relationships between geometric shapes and their characteristics. Many high school students believe a square ceases to be a square when its base is not horizontal. Put simply, many secondary school students can only perceive shapes in a consistent orientation.

Cho and Win (2020) stated that students' misconceptions about geometry could result in disorientation, annoyance, and mistakes, hindering their ability to develop confidence, an optimistic educational attitude, and an appreciation for the elegance of mathematics. Hence, it is imperative to mitigate pupils' misconceptions. To mitigate misunderstandings in geometry, it is imperative to comprehend the underlying causes and enduring nature of these fallacies.

Unclear Terms

Feza and Webb (2005) argue that a lack of linguistic fluency hinders the development of geometric knowledge. Mastery of geometry necessitates familiarity with technical terminology. Children have a limited vocabulary that prevents them from describing the distinguishing features of a figure or making accurate comparisons across figures (Feza & Webb, 2005). Oberdorf and Taylor-Cox (1999) argue that pupils' geometric misconceptions are caused by a deficiency in exposure to appropriate language.

Bombio and Del Rosario (2022) discovered a notable correlation between reading comprehension and the accuracy of proof and rational thinking. The researchers recommended that teachers provide pupils with a diverse range of reading resources to improve their proficiency in geometric proof.

Attributes of Shapes

According to Mayberry (1983), high school students often disregard shape attributes. As a result, a significant number of individuals require assistance in accurately describing shapes using the specific language of their characteristics (Burger & Shaughnessy, 1986; Feza & Webb, 2005). In their research, Clements and Battista (1992) found that less than 25% of 11th-grade students in the United States could accurately identify whether pictures had lines of symmetry.

Proof Writing

According to Senk (1985), a crucial objective of geometry curricula in numerous nations is to instruct students in constructing proofs. Several studies have found that many secondary students struggle with writing proofs in geometry, considering it one of the most difficult tasks (Hoffer, 1981; Senk, 1985; Siyepu, 2005). There is a growing issue with the caliber of mathematical proofs in geometry, posing a significant obstacle for pupils. The students encounter challenges in analyzing geometric properties and understanding the theorems that form the foundation of the proofs, which is the cause of the difficulties they face. The issue of proofs has emerged as a crucial factor in developing geometry curricula in many nations (Noto et al., 2019).

Visualizing spatial relationships, understanding abstract concepts, applying theorems and postulates, differentiating between properties and definitions, translating between two-dimensional and three-dimensional representations, remembering and applying formulas, and integrating algebraic concepts are only a few examples of students' conceptual challenges when studying geometry in class (NCTM 2020a, 2020b).

Hassan (2015) identifies several challenges that students encounter in geometry, including their struggle with comprehending geometric shapes and their properties, understanding geometric proof, grasping trigonometric ratios and their interconnections, using imprecise terminology, arranging shapes accurately, and correctly identifying them.

To provide students with the necessary assistance and direction to overcome these challenges, teachers must be cognizant of these obstacles. By employing methodologies such as hands-on exercises, visual aids, practical applications, and personalized instruction, these challenges can be efficiently resolved, leading to a more profound understanding of geometry.

Reasons for Teaching Difficulties in School Geometry

Students in schools encounter challenges and barriers when it comes to acquiring mathematical knowledge, particularly in geometry. Although significant efforts have been made in the instruction of geometry, a considerable number of pupils still struggle to grasp geometric concepts and lack the motivation to acquire them (Crowley, 1987). Much research deals with interpreting and explaining the factors of difficulty in geometry (Hassan, 2015; Ogundele et al., 2014).

The difficulty Reasons were categorized as factors about students, curricula and their delivery, government policies, the learning environment, and other educational factors (Atebe, 2008; Mahwah, 2007; Odili, 2006;).Several factors contribute to the complexity of the school geometry topic for students. Some factors are associated with motivation and students' disinterest, as well as teachers' inadequacy due to their lack of qualifications for teaching. Additional variables may contribute to the density of pupils in the classroom, while another element is linked to the learners' beliefs that mathematics is a challenging topic that is incomprehensible, leading to emotional distress (Amazigo, 2000)._While other scholars define the difficulty factors in geometry into three factors: curriculum factors, textual factors and related educational factors (Atebe, 2008; Clements & Battista, 1992).

The data analysis performed by Sulistiowati, Herman, and Jupri (2019, February) found that students struggle to translate the problem into a mathematical model. Additional challenges encountered by students at the visualization level were a limited comprehension of the topic, insufficient knowledge of effective strategies, and an inability to apply accurate mathematical techniques. Conversely, pupils encountered the most significant challenge in solving problems at the analysis and informal deduction levels. The challenges faced by students at the analysis level include not being able to translate problems into a mathematical model and not applying the appropriate mathematical techniques. On the other hand, learners at the informal deduction level encounter challenges in using the correct mathematics and often make computational mistakes.

In any case, this study will focus on the difficulty factors associated with educational strategy. The reason is that the teaching method is crucial in transforming the educational content into a stimulating and efficient academic curriculum.

The Factors of Curriculum

There is evidence to suggest that a lack of a comprehensive and well-structured geometry curriculum during primary education is a significant contributing factor to students' poor performance in geometry throughout high school. Secondary school students have had minimal or nonexistent exposure to geometry (Clements & Battista, 1992; Pegg, 1995). Usiskin (1982) asserts that there needs to be more curricula specifically designed for primary school education. Consequently, many arrive in high school with inadequate geometry proficiency to excel.

Baah-Duodu et al (2020) suggest that those involved in curriculum design should assess the arrangement of mathematics qualifications to enhance the prospects for learners to pursue geometry. Offering complex and captivating geometric concepts can enhance the appeal of mathematics as a subject for a wider range of students. Consequently, this would help address the existing scarcity of students proficient in mathematics.

The designated curriculum, which refers to the teaching strategies established by an authorized governing body, has a crucial impact since it affects all aspects of the operational curriculum, including the teacher's planned curriculum, the curriculum that is actually implemented, and the curriculum that the students achieve. Similarly, this could also impact countries with highly centralized educational systems when authorities give minimal consideration to geometry in the prescribed curriculum (Kuzle, 2023).

Textual Factors

Van Hiele (1986) asserts that a top-notch geometry textbook is characterized by the repetition of teaching content, starting from the beginning of the subject. Van Hiele states that a textbook structured in this manner satisfies the concept of "telescoped reteaching." This requires the implementation of a spiral curriculum, which involves organizing content information in textbooks.

Textual factors can indeed complicate the comprehension of classroom geometry. Textual issues can pose challenges for geometry students for several reasons: Abstract nature, ambiguity, lack of visual cues, symbolic notation, and language barrier. To overcome the difficulties students face in learning geometry, educators can employ a range of strategies, including visual aids, verbal explanations, hands-on activities, language support, and technology. By doing so, educators can effectively enhance students' understanding of geometry and optimize their learning experience (Antunovic-Piton & Baranovic, 2022).

Teaching Strategy Factor

In the view of Szymkowiak et al (2021), Traditional teaching is a long-standing instructional approach that relies solely on the teacher as the primary source of information. In this method, students are expected to listen and take written notes without actively participating in learning and teaching.

The Traditional geometry teaching method lacks innovation, lucidity, and profound thinking (Keiser, 1997; NCTM, 2000). Schoenfeld (1983) concurred that students are deprived of the autonomy to engage in creative thinking within the confines of a school environment. Furthermore, some students and educators encountered difficulties with this approach and reached a consensus that geometry was an intimidating discipline. Unsurprisingly, numerous children lack interest in geometry due to this reason.

Traditional geometry education has become ambiguous and vexing (Awad, 2014). According to Malloy and Friel (1999), geometry is the branch of mathematics that poses the most significant challenge for most learners. Several independent studies have determined that the poor academic performance of children can be attributed to inadequate methods of teaching and learning geometry in schools (Ali et al., 2014; Sunzuma et al., 2013).

In their study, Jaber, Southerland, and Dake (2018) delineated the specific domains of knowledge that teachers must possess to facilitate effective instruction. These encompass three key areas: content knowledge (CK), pedagogical knowledge (Teaching Strategy), and pedagogical content knowledge (PCK). Content knowledge refers to the specific knowledge about a particular subject that is intended to be studied. Pedagogical knowledge, on the other hand, pertains to the understanding of teaching strategies and methods. It encompasses knowledge in classroom management, assessment, lesson plan construction, and student learning. Pedagogical content knowledge, on the other hand, serves as the connection between different components of knowledge (Chew & Lim, 2013).

According to Tomlinson (2000), differentiated education consistently aims to discover alternate methods for pupils to grasp the subject effectively. Owolabi and Adedayo (2012) assert that students assimilate the concepts they are taught through the teaching methods and styles employed by teachers, who are regarded as facilitators with a significant impact on pupils.

The instructor serves as the fundamental and focal point of the educational process (Findell, 2001). The efficacy and dynamism of learning are entirely contingent upon the teacher. The teacher knows how to efficiently and distinctly convey information to the pupil (Akinsolu, 2010). According to Moore (2004), for teachers to teach their students effectively, they must have the chance to improve and expand their teaching knowledge. This includes demonstrating their understanding of mathematics and using appropriate teaching methods for the content being taught.

Acquiring knowledge and skills is essential for personal growth and development throughout one's lifetime. Knowledge acquisition is achieved proficiently and effectively when teachers play a significant role. Their contributions are not only essential but also irreplaceable, as no existing or future technology will be able to replace them. The primary responsibility of instructors in the teaching and learning process has elevated the significance of teaching ability, particularly at the foundational level of education, in acquiring knowledge (Ndagiwenimana & Maniraho, 2022).

Many of our instructors possess inadequate Pedagogical Content Knowledge (PCK), which significantly hinders students' ability to focus during class, thus resulting in below-average academic performance (Usman et al., 2019). The observations indicated a limited degree of pedagogical content knowledge (PCK) integration in classroom practices, emphasizing the difficulties encountered by instructors in effectively employing PCK. Hence, it is crucial to offer supplementary in-service training to augment teachers' Pedagogical Content Knowledge (PCK) levels, ultimately resulting in improved mathematics instruction and learning (Moh'd et al., 2021).

Numerous scholars have attributed the difficulties that students encounter in geometry specific and mathematics overall to the teacher's ignorance of the proper subject matter (instructional strategy). (Awofala, & Lawani, 2020; Chai, & Tsai, 2010; Guerriero, 2012; Shulman, 1987; Stoker, 2003; van Hiele, 1986). Accordingly, it is necessary to find effective and valuable teaching methods to make geometry subjects accessible to all.

According to Usman et al (2019), using van Hiele's phase-based teaching strategy can help students overcome most of their geometry learning challenges. In particular, van Hiele's model is the most effective and well-characterized approach to geometry learning (Abdullah & Zakaria, 2013b). Many previous studies have asserted that the van Hiele model has a significant impact on the development of geometric thinking and the achievement in geometry (Abu et al., 2012; Altakhayneh, 2021; Armah et al., 2018; Hamzeh, 2017; Naufal et al., 2021; Skrbec & Cadez, 2015).

Van Hiele's Phase-Based Instructional Strategy

The van Hiele model suggests five learning phases to help students progress from one level to the next. The learning phases include inquiry (information), direct orientation, explication, free orientation, and integration. If students receive geometry instruction based on these phases, they will be able to progress from one level of van Hiele's geometric thinking to the next. The levels in order from the first level to the fifth are Visual level, descriptive level, logical level, inference level, and abstract level (van Hiele, 1986; Usiskin, 1982). It is one of the most important theoretical frameworks for understanding the instructional processes of students because it puts pupils at the center of the learning process and provides treatments that can effectively increase students' levels of geometric thinking (Al-ebous, 2016; Howse & Howse, 2014; Mostafa et al., 2017).

The instructional process consists of the following five phases:1- Information or Inquiry: This instruction phase includes two-way interaction between teacher and student, observation, asking more questions, and studying the terminology for a specific geometrical shape. 2- Guided orientation or Directed Orientation: During this phase, the instructor leads the students to explore the topic by having them complete a series of straightforward, accurate, structured activities, such as folding, measuring or constructing. 3- Explicitation: During this phase, students express their thoughts on the relationships they discovered through practical learning activities. They use their own vocabulary to convey what they have learned about the subject. Conversely, the instructor guarantees that the correct and appropriate terms are advanced and used (Armah et al., 2018). 4- Free orientation: An instructor assigns activities that may be finished in various ways, allowing students to become more skillful and proficient compared with their previous knowledge. 5- Integration: Learners summarize what they have understood from the lectures (FengSer et al., 2022).

The impact of the van Hiele Theory on current K–12 geometry education has been extensively studied. Multiple studies have demonstrated that the van Hiele Geometric Thinking Levels provide a framework for understanding how students' geometric thinking evolves from elementary school to university and enhances academic performance in geometry (Armah & Kissi, 2019; Burger & Shaughnessy, 1986; Mohd et al., 2019; Usiskin, 1982).

The research conducted by Usman et al (2020) demonstrated that van Hiele's phase-based teaching method is more successful than the traditional teaching methods in enhancing the geometry performance of pre-service teachers. Hence, the implementation of the van Hiele phase-based teaching method can be seen as an effective approach to improve the performance of pre-service mathematics instructors in Niger state and Nigeria as a whole.

Armah et al (2018) found that implementing the van Hiele Phase-based Instruction can enhance students' geometric reasoning and deepen their comprehension of geometry ideas. This approach also prepares them to effectively teach geometry at the primary level after completing their undergraduate studies. Using van Hiele Phase-based Instruction is crucial for teachers aiming to improve global and national tests such as TIMSS, BECE, WASSCE, and DBE examinations.

Altakhayneh (2021) suggested employing the Van Hiele model to foster geometric reasoning in students, mainly when teaching geometry. This model is known for its ability to systematically and comprehensively impart knowledge and skills in geometry. This model is efficacious in fostering geometric reasoning for individuals of both genders. The researcher emphasized the necessity of training mathematics instructors on contemporary tactics and models for teaching geometry according to the Van Hiele model, specifically focusing on geometry proofs and the effective presentation and validation of geometry concepts.

According to Naufal et al (2021), The Van Hiele model was commonly utilized and demonstrated a beneficial influence in fostering geometric cognition as a pedagogical approach, whether as a standalone intervention or in conjunction with other instructional methods or aids. This was observed in most reviewed studies that implemented the Van Hiele model and incorporated metacognition in developing geometric thinking. Combining the Van Hiele model and implementing metacognition can enhance the advancement of geometric thinking.

Mohammed and Zakariyya (2023) concluded that the Van Hiele Teaching Model improves the academic achievement of high school learners in geometry, and it confirmed that implementing Van Hiele's teaching style when instructing Mathematics has effectively alleviated students' anxiety and discomfort.

Multiple studies have demonstrated that Van Hiele's Phase-Based Instructional Strategy is superior to traditional instructional strategies in enhancing students' disposition toward geometry. Therefore, using Van Hiele's phase-based instructional technique can be seen as one of the effective strategies for improving students' attitudes toward geometry (Awofala et al., 2013; Karjanto, 2017; Ma & Kishor, 1997; Peker & Mirasyediolu, 2003; Recber et al., 2018; Salifu et al., 2021; Usman et al., 2019)

Van Hiele's Phase-Based Instructional Strategy possesses strengths from both the students' and teachers' perspectives. The teaching technique employed a sequential approach, carefully organized in phases, inspiring students to learn geometry effectively. Furthermore, the strategy took into account the individual requirements of the students (Chua et al., 2017; Fajet et al., 2005; Usman et al., 2019).

Based on the above, the researcher determined that Van Hiele's Phase-Based Instructional Strategy is superior to the traditional teaching strategy in enhancing students' academic performance, geometric thinking, and attitude towards geometry. Therefore, using Van Hiele's Phase-Based Instructional Strategy can be seen as a genuine approach to improving students' academic performance, geometric thinking, and attitude towards geometry. Therefore, the researcher suggests that teachers employ Van Hiele's Phase-Based Instructional Strategy when teaching geometry and other branches of mathematics.

Based on the above, the researcher determined that Van Hiele's Phase-Based Instructional Strategy is an effective strategy in enhancing students' academic performance, geometric thinking, and attitude toward geometry and is superior to the traditional teaching strategies in reducing the challenges that teachers and students face during teaching and learning geometry. Therefore, using Van Hiele's Phase-Based Instructional Strategy can be seen as a genuine approach to improving students' academic performance, geometric thinking, and attitude towards geometry. Therefore, the researcher suggests the necessity of training mathematics instructors on contemporary tactics and models for teaching geometry according to the Van Hiele model teachers and employing Van Hiele's Phase-Based Instructional Strategy when teaching geometry and other branches of mathematics.

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