The Teaching Strategy of Teaching Quadratic Functions Using Geogebra Following The 5e Instructional Model

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Abstract

The research presents the teaching strategy of teaching quadratic functions using GeoGebra following the 5E instructional model. The 5E instructional model is a kind of exploration teaching model based on constructivism, emphasizing the exploration of students to build knowledge independently (Chen et al., 2022). As the GeoGebra is the one of the knowledge technology tools, which are widely used in education, especially widely used in the teaching of middle school mathematics. This research aims to present how to make teaching design of teaching quadratic functions using GeoGebra following the 5E instructional model, which aims to improve students' understanding of quadratic function graphs and properties. This research provides an initial framework for how students learn quadratic functions, and continued research in this area will support curriculum development and instructional decisions. In addition, this research also provides evidence and reference for educational researchers, and provides a reference for improving the teaching strategy of quadratic functions.

Keywords: Teaching Strategy, Quadratic Functions, Geogebra, The 5e Instructional Model

Introduction

In China, the conventional teaching method is an “examination-oriented method”, which make our classroom, school, and personality alienated (Zhong, 2006). With the promulgation and implementation of the Compulsory Education Curriculum Plan and Curriculum Standards (2022 Version), the conventional teaching method has been unable to meet the requirements of the new compulsory education mathematics curriculum. So, it is necessary to improve the teaching method in China.

The quadratic function is one of the key points of mathematics learning in junior high school and also one of the difficult topics. It has always been the focus of the examination of the high school entrance examination. However, many students have a poor learning effect on concepts, graphics and properties, analytical formulas, and comprehensive use of this part of knowledge. Based on the interviews with junior high school students and teachers, Li (2021) found that there are subjective reasons such as poor learning emotions, uncorrected attitudes, and lack of motivation in learning the quadratic function for students and teachers still use conventional methods, although they think that the use of multimedia-assisted
teaching is often more advantageous in the teaching of quadratic functions, but many teachers are not good at using this technology and it is more time-consuming.

GeoGebra is an open-source, dynamic mathematics software program which is created by Markus Hohenwarter in 2001 (Hohenwarter et al., 2009). As a currently widely used auxiliary software in mathematics teaching worldwide, GeoGebra can more accurately and intuitively display the corresponding graphics and changing laws, and can better help teachers realize the elaboration and explanation of corresponding teaching priorities, and receive ideal results in teaching practice. There are past studies show that GeoGebra has a positive impact on students’ achievement and learning attitude (Arbain & Shukor, 2015; Dogan & Içel, 2011). Through quasi-experimental research design, Arbain and Shukor (2015) indicated that the using of GeoGebra software has positive impact on students’ achievement in Mathematics and it can increase students’ interest, confidence and their motivation in learning Mathematics. Through a pre test, a post test and a recall test, Dogan and Içel (2011) found that GeoGebra has positive effects on students’ learning and achievements and it can improve students’ motivation with positive impact. It also indicated that teaching using GeoGebra would enhance students’ retention of achievement and attitude.

According to Chen et al (2022), the 5E instructional model is a kind of exploration teaching mode based on constructivism, emphasizing students’ independent exploration, communication and collaboration and it requires students’ self-construction as a way to improve their cognitive level. The 5E instructional model can cultivate students’ explorative ability, and help them build a scientific mathematical concept, which is good for the students’ lifelong development.

Through designing the process of teaching quadratic functions using GeoGebra based on 5E instructional model, this research analyzes the teaching strategies of applying the 5E instructional model, including the stages of engagement, exploration, explanation, elaboration, and evaluation. Taking the graph and properties of quadratic functions as examples, using GeoGebra for teaching and designing the 5E instructional model based teaching case, it is hoped to provide a reference for Chinese mathematics teachers and contribute to studies on teaching using GeoGebra and the 5E instructional model in China.

This research is dedicated to the use of GeoGebra to carry out junior high school mathematics teaching. The results of the research help teachers understand the process of students learning mathematics, especially those related to the use of GeoGebra and mathematics, and reveal the factors that influence the use of GeoGebra for mathematics teaching. At the same time, the results of this research will be beneficial to improve students’ ability to use the combination of number and shape, and to improve their mathematical thinking.

The 5E Instructional Model

The 5E Instructional Model is based on cognitive psychology, constructivist-learning theory, and best practices in science teaching, which can be used to design a science lesson and it consists of cognitive stages of learning that comprise engage, explore, explain, elaborate, and evaluate (Bybee & Landes, 1990). Using the instructional model, students can redefine, reorganize, elaborate, and change their original concepts through self-reflection and interaction with their peers and the environment and the learners explain objects and phenomena, and internalize these explanations according to their current concept understanding (Bybee, 1997). According to Bybee (1997), this model can be the organizing
pattern of a sequence of daily courses, various units, or annual plans. As it has been modified from Bybee, each stage of the 5E Instructional Learning Cycle is now described as follows.

**Engagement:** In the first stage of the cycle, the teacher’s goal is to assess student prior knowledge or identify possible misunderstandings and this student-centered phase should be a period of motivation, which can create a desire to understand more about the topic of the upcoming (Durán & Durán, 2004). Through associated with the students’ past experience and current learning experience and concepts, instructors allow students to participate in a topic (Lam et al., 2022).

**Exploration:** The students use their known concepts to generate new knowledge through self-research and observation during the exploration stage (Lam et al., 2022). In this stage, the students have time to think, plan, investigate and organize information collected. During the exploration stage, it now provides students with a common and specific learning experience and this stage are also student-centered and include positive exploration. Students are encouraged to apply process skills, such as observation, questions, investigations, test predictions, assumptions, and communicate with other peers (Duran & Duran, 2004).

**Explanation:** During the explanation stage, students are now participating in the analysis of their explorations, and their understanding is clarified and modified due to reflective activities (Cardak et al., 2008). Students show the results of their research and observations, and get further explanations from the instructors in order to get a deeper understanding (Lam et al., 2022). The explanation stage enables students to describe their understanding and asks questions about the concepts they have been exploring, which is likely to cause new questions. Before the teacher tries to provide an explanation, students must first have the opportunity to express their own explanations and ideas. Therefore, the initial part of the explanation phase was the teacher as a promoter, and asked students to describe and discuss their exploratory learning experience (Durán & Durán, 2004).

**Elaboration:** In the elaboration stage, students apply new concepts and may further explore the topic to improve their ability to understand and apply new knowledge (Lam et al., 2022). This section gives students the opportunity to expand and consolidate their understanding of this concept, or apply them to the real-world situation (Cardak et al., 2008). At this stage of the learning cycle, the teacher should encourage students to apply their new understanding of concepts and strengthen new skills and the goal of this stage is to help development a more deeper and extensive understanding of the concepts (Durán & Durán, 2004).

**Evaluation:** During the evaluation stage, the teacher encourages students to evaluate their learning ability and achievements. Evaluation should run through the entire lesson and the teacher should observe students’ knowledge and skills, the application and the change of thinking of the new concepts (Cardak et al., 2008). Evaluation should be regarded as a continuous process and the teacher observes students when they apply new concepts and skills, and find evidence that the students have changed or modified their thinking and the students can also have the opportunity to conduct self-assessment or peer-assessment (Durán & Durán, 2004). Figure 1 shows the 5E Instructional Model.
Figure 1: The 5E Instructional Model

Stages to Designing the Lesson Plans based on the 5E Instructional Model

Engagement

The first stage of designing the Lesson Plans based on the 5E Instructional Model is Engagement. The activities of this section will seize the attention of students, stimulate their thinking, and help them obtain prior knowledge (Cardak et al., 2008). According to Duran and Duran (2004), in the first stage of the cycle, the teacher’s goal is to assess student prior knowledge or identify possible misunderstandings. At the same time, teachers have associated students’ past experience and current learning experience and concepts (Lam et al., 2022). Therefore, for the teaching of quadratic functions, this stage should allow students to review the knowledge they have learned before and connect new and old knowledge. At the same time, the teacher can grasp students’ attention by using GeoGebra software to generate graph and stimulate students’ interest in learning. The time of this stage is about 3 minutes.

For the teaching topic of Graph and Properties of Quadratic Function $y=ax^2$, the teacher asks the questions:
1. How are the properties of a linear function studied?
2. Can we study the properties of quadratic functions by analogy with studying the properties of linear functions? If so, what should we study first?

Teacher asks the questions and uses GeoGebra to demonstrate the graph of common linear function $y=x$ and guide students to review the graphs and properties of linear functions they have learned before. By observing the graph of the linear function performed by GeoGebra, students analyze, and induce to obtain the properties of linear functions. Teacher reviews the prior knowledge that students have learned, which may lead to cognitive conflicts among students and it is concluded that the properties of quadratic functions can be studied by the method of studying the properties of linear functions, and the graph of quadratic functions should be studied first.

For the teaching topic of Graph and Properties of Quadratic Function $y=ax^2+k$, the teacher asks the questions
1. How are the properties of quadratic function $y=2x^2$?
2. Are the opening direction, symmetry axis and vertex coordinates of the graph of the quadratic function \( y=2x^2+1 \) the same as the graph of the quadratic function \( y=2x^2 \)?

The teacher asks the questions and uses GeoGebra to demonstrate the graph of quadratic function \( y=2x^2 \) and guides students to review the graphs and properties of quadratic function \( y=2x^2 \) they have learned before. By observing the graph of the quadratic function performed by GeoGebra, students analyze, and induce to obtain the properties of quadratic function \( y=2x^2 \). Teacher guides students to review prior knowledge, which may lead to cognitive conflicts among students.

For the teaching topic of Graph and Properties of Quadratic Function \( y=a(x-h)^2 \), the teacher asks the questions:

1. How are the properties of quadratic function \( y=-\frac{1}{2}x^2 \)?

2. Are the opening direction, symmetry axis and vertex coordinates of the graph of the quadratic function \( y=-\frac{1}{2}(x+1)^2 \) the same as the graph of the quadratic function \( y=-\frac{1}{2}x^2 \)?

The teacher asks the questions and uses GeoGebra to demonstrate the graph of quadratic function \( y=-\frac{1}{2}x^2 \) and guides students to review the graphs and properties of quadratic function \( y=-\frac{1}{2}x^2 \) they have learned before. By observing the graph of the quadratic function performed by GeoGebra, students analyze, and induce to obtain the properties of quadratic function \( y=-\frac{1}{2}x^2 \). Teacher guides students to review prior knowledge, which may lead to cognitive conflicts among students.

For the teaching topic of Graph and Properties of Quadratic Function \( y=a(x-h)^2+k \), the teacher asks the questions:

1. How are the properties of quadratic function \( y=-\frac{1}{2}(x+1)^2 \)?

2. Are the opening direction, symmetry axis and vertex coordinates of the graph of the quadratic function \( y=-\frac{1}{2}(x+1)^2-1 \) the same as the graph of the quadratic function \( y=-\frac{1}{2}(x+1)^2 \)?

The teacher asks the questions and uses GeoGebra to demonstrate the graph of quadratic function \( y=-\frac{1}{2}(x+1)^2 \) and guides students to review the graphs and properties of quadratic function \( y=-\frac{1}{2}(x+1)^2 \) they have learned before. By observing the graph of the quadratic function performed by GeoGebra, students analyze, and induce to obtain the properties of quadratic function \( y=-\frac{1}{2}(x+1)^2 \). Teacher guides students to review prior knowledge, which may lead to cognitive conflicts among students.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2+bx+c \), the teacher uses GeoGebra software to show the graph of the following three quadratic functions one by one and asks the question: What is the relationship between the graph and properties of these functions?
Teacher asks the question and guides students to review the graphs and properties of simple quadratic functions, that is, quadratic functions can be transformed through the translation of images. Under the guidance of teacher, students review the knowledge and research methods previously learned. Teacher reviews the prior knowledge that students have learned, which may lead to cognitive conflicts among students and pave the way for the study of general quadratic function graph later.

For the teaching topic of Quadratic Function and Quadratic Equation in one variable, the teacher asks the questions:

1. How are the properties of quadratic function and quadratic equation in one variable studied?
2. What is the relationship between quadratic function and quadratic equation in one variable?

Teacher asks the questions and guides students to review the properties of quadratic function and quadratic equation in one variable they have learned before. Under the guidance of teacher, students recall the concepts and properties of quadratic functions and quadratic equations in one variable. Teacher reviews the prior knowledge that students have learned, which may lead to cognitive conflicts.

Exploration

The second stage of designing the Lesson Plans based on the 5E Instructional Model is Exploration. In this stage, students have time to think, plan, investigate and organize information collected (Cardak et al., 2008). During the exploration stage, according to Lam et al. (2022), the students use their own concepts to generate new knowledge through observation and self-research. In this stage, it contains positive exploration and is student-centered. Therefore, for the teaching of quadratic functions, this stage should allow students to explore new knowledge by themselves, such as drawing the graph of the quadratic functions and explore the nature and laws of the graph of the quadratic functions. At the same time, students generate quadratic functions graph by using GeoGebra software and the students observe whether the quadratic function graph drawn by themselves is consistent with the quadratic function graph demonstrated by GeoGebra. The time of this stage is about 10 minutes.

For the teaching topic of Graph and Properties of Quadratic Function $y=ax^2$, the teacher guides the students to draw the graph of the quadratic function $y=x^2$ through lists, points, and lines. At the same time, the teacher uses GeoGebra to demonstrate the graph of the quadratic function $y=x^2$.

Students observe whether the quadratic function graph drawn by themselves is consistent with the quadratic function graph demonstrated by GeoGebra.

Through observation, thinking, discussion, and communication, students can conclude that the quadratic function $y=x^2$ has an axis of symmetry, and the axis of symmetry and the image have a point of intersection, and derive the following concepts:

Parabola Concept: Curves like this are often called parabolas.
Vertex concept: The intersection of a parabola with its axis of symmetry is called the vertex of the parabola.

For the teaching topic of Graph and Properties of Quadratic Function \(y=ax^2+k\), the teacher guides the students to draw the graph of the quadratic functions \(y=2x^2+1\) and \(y=2x^2-1\) through lists, points, and lines. At the same time, the teacher uses GeoGebra to demonstrate the graph of the quadratic functions \(y=2x^2+1\) and \(y=2x^2-1\). Students observe whether the quadratic function graph drawn by themselves is consistent with the quadratic function graph demonstrated by GeoGebra. Through observation, thinking, discussion, and communication, students can conclude that the quadratic functions’ opening directions, axes of symmetry and vertex coordinates.

For the teaching topic of Graph and Properties of Quadratic Function \(y=a(x-h)^2\), the teacher guides the students to draw the graph of the quadratic functions \(y=-\frac{1}{2}(x+1)^2\) and \(y=-\frac{1}{2}(x-1)^2\) through lists, points, and lines. At the same time, the teacher uses GeoGebra to demonstrate the graph of the quadratic functions \(y=-\frac{1}{2}(x+1)^2\) and \(y=-\frac{1}{2}(x-1)^2\). Students observe whether the quadratic function graph drawn by themselves is consistent with the quadratic function graph demonstrated by GeoGebra. Through observation, thinking, discussion, and communication, students can conclude that the quadratic functions’ opening directions, axes of symmetry and vertex coordinates.

For the teaching topic of Graph and Properties of Quadratic Function \(y=a(x-h)^2+k\), the teacher guides the students to draw the graph of the quadratic function \(y=-\frac{1}{2}(x+1)^2-1\) through lists, points, and lines. At the same time, the teacher uses GeoGebra to demonstrate the graph of the quadratic function \(y=-\frac{1}{2}(x+1)^2-1\). Students observe whether the quadratic function graph drawn by themselves is consistent with the quadratic function graph demonstrated by GeoGebra. Through observation, thinking, discussion, and communication, students can conclude that the quadratic function’ opening directions, axes of symmetry and vertex coordinates.

For the teaching topic of Graph and Properties of Quadratic Function \(y=ax^2+bx+c\), The teacher guides the students to transform the quadratic function \(y=\frac{1}{2}x^2-6x+21\) into \(y=\frac{1}{2}(x-6)^2+3\) through the method of completing the square. On the basis of the existing quadratic function graph \(y=\frac{1}{2}(x-6)^2+3\), the teacher uses GeoGebra to make a quadratic function graph \(y=\frac{1}{2}x^2-6x+21\). The students observe the quadratic function graphs and state the properties. Students observe that the graphs of the quadratic functions overlap, which leads students to know that the analytical expressions of these two functions can be transformed, so that students can try to transform the unknown into the known for research, and use the method of completing the square to transform the two quadratic functions.

For the teaching topic of Quadratic Function and Quadratic Equation in one variable, the teacher guides the students to use the known conditions to list the quadratic equation in one variable. At the same time, the teacher uses GeoGebra to demonstrate the graph of the
quadratic function \( h=20t-5t^2 \). Students observe whether the quadratic function graph drawn by themselves is consistent with the quadratic function graph demonstrated by GeoGebra. Under the guidance of the teacher, students list quadratic equations in one variable and obtain the answers to the questions.

**Explanation**

The third stage of designing the Lesson Plans based on the 5E Instructional Model is Explanation. In this stage, the students show their research and observations results, and get further explanations from the instructors to get a deeper understanding (Lam et al., 2022). The students are participating in the analysis of their explorations, and by reflecting on activities, their understanding is clarified and modified (Cardak et al., 2008). Therefore, for the teaching of quadratic functions, this stage should allow students to show their research and observations results of quadratic functions, and get further explanations from the teacher. By analyzing their explorations, the students finally get the properties of the quadratic functions. The time of this stage is about 10 minutes.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2 \), the teacher uses GeoGebra to demonstrate the graphs of two quadratic functions \( y=\frac{1}{2}x^2 \) and \( y=2x^2 \) and guides the students to observe the similarities and differences between these two graphs compared with the graph of \( y=x^2 \) and get the characteristics of the graph of the quadratic function \( y=ax^2 \) by using GeoGebra to demonstrate different quadratic functions \( (a>0) \). Through observation and comparison, students come to the conclusion: When \( a>0 \), the opening of the parabola \( y=ax^2 \) is upward, the axis of symmetry is the \( y \)-axis, the vertex is the origin, and the vertex is the lowest point of the parabola. The larger the \( a \), the smaller the opening of the parabola. When \( x<0 \), \( y \) decreases as \( x \) increases, and when \( x>0 \), \( y \) increases as \( x \) increases.

The teacher uses GeoGebra to demonstrate the graphs of two quadratic functions \( y=-\frac{1}{2}x^2 \) and \( y=-2x^2 \) and guides the students to observe the similarities and differences between these three graphs and get the characteristics of the graph of the quadratic function \( y=ax^2 \) by using GeoGebra to demonstrate different quadratic functions \( (a<0) \). Through observation and comparison, students come to the conclusion: when \( a<0 \), the opening of the parabola \( y=ax^2 \) is downward, the axis of symmetry is the \( y \)-axis, the vertex is the origin, and the vertex is the highest point of the parabola. The smaller the \( a \), the smaller the opening of the parabola. When \( x<0 \), \( y \) increases as \( x \) increases, and when \( x>0 \), \( y \) decreases as \( x \) increases.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2+k \), the teacher uses GeoGebra to demonstrate the graphs of two quadratic functions \( y=2x^2+1 \), \( y=2x^2-1 \) and guides the students to observe the similarities and differences between these two graphs compared with the graph of \( y=2x^2 \). Through observation and comparison, students come to the conclusion: Translate the parabola \( y=2x^2 \) up by 1 unit length to get the parabola \( y=2x^2+1 \); translate the parabola \( y=2x^2-1 \) down by 1 unit length to get the parabola \( y=2x^2-2 \).

The teacher uses GeoGebra to demonstrate different graphs of two quadratic functions \( y=ax^2+k \), \( y=ax^2 \) and guide students to compare and summarize, get the relationship of the graphs of the quadratic functions \( y=ax^2+k \) and \( y=ax^2 \). Through observation and comparison, students come to the conclusion: When \( k>0 \), translate the parabola \( y=ax^2 \) up by \( k \) unit length to get the parabola \( y=ax^2+k \); When \( k<0 \), translate the parabola \( y=ax^2 \) down by \( |k| \) unit length to get the parabola \( y=ax^2+k \).
For the teaching topic of Graph and Properties of Quadratic Function \( y = a(x-h)^2 \), the teacher uses GeoGebra to demonstrate the graphs of two quadratic functions \( y = -\frac{1}{2}(x+1)^2 \), \( y = -\frac{1}{2}(x-1)^2 \) and guides the students to observe the similarities and differences between these two graphs compared with the graph of \( y = -\frac{1}{2}x^2 \). Through observation and comparison, students come to the conclusion: Translate the parabola \( y = -\frac{1}{2}x^2 \) to the left by 1 unit length to get the parabola \( y = -\frac{1}{2}(x+1)^2 \); translate the parabola \( y = -\frac{1}{2}x^2 \) to the right by 1 unit length to get the parabola \( y = -\frac{1}{2}(x-1)^2 \).

The teacher uses GeoGebra to demonstrate different graphs of two quadratic functions \( y = a(x-h)^2 \), \( y = ax^2 \) and guides students to compare and summarize, get the relationship of the graphs of the quadratic functions \( y = a(x-h)^2 \) and \( y = ax^2 \). Through observation and comparison, students come to the conclusion: When \( h > 0 \), translate the parabola \( y = ax^2 \) to the right by \( h \) unit length to get the parabola \( y = a(x-h)^2 \); When \( h < 0 \), translate the parabola \( y = ax^2 \) to the left by \(|h|\) unit length to get the parabola \( y = a(x-h)^2 \).

For the teaching topic of Graph and Properties of Quadratic Function \( y = a(x-h)^2 + k \), the teacher uses GeoGebra to demonstrate the graph of quadratic function \( y = -\frac{1}{2}(x+1)^2 - 1 \) and guides the students to observe the similarities and differences between the graph of quadratic function \( y = -\frac{1}{2}(x+1)^2 - 1 \) compared with the graph of \( y = -\frac{1}{2}x^2 \). Through observation and comparison, students come to the conclusion: Translate the parabola \( y = -\frac{1}{2}x^2 \) down by 1 unit length and then to the left by 1 unit length to get the parabola \( y = -\frac{1}{2}(x+1)^2 - 1 \).

The teacher uses GeoGebra to demonstrate different graphs of two quadratic functions \( y = a(x-h)^2 + k \), \( y = ax^2 \) and guides students to compare and summarize, get the relationship of the graphs of the quadratic functions \( y = a(x-h)^2 + k \) and \( y = ax^2 \). Through observation and comparison, students come to the conclusion: In general, the parabola \( y = a(x-h)^2 + k \) has the same shape as \( y = ax^2 \), but in a different position. By translating the parabola \( y = ax^2 \) up (down) to the left (right), the parabola \( y = a(x-h)^2 + k \) can be obtained. The direction and distance of the translation are determined according to the values of \( h \) and \( k \).

The parabola \( y = a(x-h)^2 + k \) has the following characteristics:
1. When \( a > 0 \), the opening is upward; when \( a < 0 \), the opening is downward.
2. The axis of symmetry is \( x = h \).
3. The vertex is \((h, k)\).

For the teaching topic of Graph and Properties of Quadratic Function \( y = ax^2 + bx + c \), the teacher proposes conjecture and guides students to verify conjecture. Students work together to explore the process of transformation through group discussions. By analogy with the two special quadratic functions transformation process, students get the process of...
transforming the general form Quadratic Function \( y=ax^2+bx+c \) into \( y=a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a} \), so as to find out symmetry axis \( x=-\frac{b}{2a} \) and the vertex coordinates \((-\frac{b}{2a}, \frac{4ac-b^2}{4a})\).

According to sliding the \( a, b, c \) sliders in the GeoGebra software, the teacher prompts the students to discuss and write the opening direction of the quadratic function, the axis of symmetry, the intersection with the \( y \)-axis, the intersection with the \( x \)-axis, and the change trend of the function. Students themselves write the relevant properties corresponding to the relevant quadratic functions. Through practical operation, the teacher lets students write the relevant properties according to the images and feel the connection between knowledge.

For the teaching topic of Quadratic Function and Quadratic Equation in one variable, the teacher uses GeoGebra to demonstrate the graph of quadratic function \( y=-x^2+4x \) and guides the students to convert the quadratic function into a quadratic equation of one variable for solving. Under the guidance of teachers, students can convert quadratic functions into quadratic equation of one variable for solving, or convert the constructed quadratic equation of one variable into quadratic functions for solving. Students can use GeoGebra to demonstrate the graphs of the quadratic functions.

The teacher uses GeoGebra to demonstrate the three graphs of quadratic functions and guides the students to find the roots of the corresponding quadratic equation of one variable. Through observation and comparison, students come to the conclusion:

1. The parabola \( y=x^2+x-2 \) and the \( x \)-axis have two common points, and their abscissas are -2, 1. When \( x \) takes the abscissa of the common point, the function value is 0, and the equation is obtained. The root of \( x^2+x-2=0 \) is -2, 1.

2. The parabola \( y=x^2-6x+9 \) has a common point with the \( x \)-axis, and the abscissa of this point is 3. When \( x=3 \), the function value is 0. From this, the equation \( x^2-6x+9=0 \) is obtained. There are two equal real roots of 3.

3. The parabola \( y=x^2-x+1 \) has no common point with the \( x \)-axis. It can be seen that the equation \( x^2-x+1=0 \) has no real root.

**Elaboration**

The fourth stage of designing the Lesson Plans based on the 5E Instructional Model is Elaboration. This stage allows students to have the opportunity to expand and consolidate their understanding of concepts and applies it to the real world situation (Cardak et al., 2008). In this stage, students apply new concepts and may further explore the topic to improve their ability to understand and apply new knowledge (Lam et al., 2022). Therefore, for the teaching of quadratic functions, this stage should allows students to expand and consolidate their understanding of quadratic functions and apply the new concepts and properties of quadratic functions to solve the problems. The time of this stage is about 10 minutes.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2 \), the teacher applies GeoGebra to the aspects of the teaching process, and set content and time for each teaching link. The teacher uses GeoGebra to demonstrate the graphs of four quadratic functions and guides students to write the opening direction, symmetry axis and vertex of the quadratic functions, describe the changing trend of the graphs. Students complete the exercises based on the methods they have learned today. The teacher consolidates students’ new knowledge through four exercises.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2+k \), the teacher applies GeoGebra to the aspects of the teaching process, and sets content and time
for each teaching link. The teacher uses GeoGebra to demonstrate the graphs of three quadratic functions and guides students to write the opening direction, symmetry axis and vertex of the quadratic functions, and the relationship between the parabola $y = \frac{1}{2} x^2 + k$ and the parabola $y = \frac{1}{2} x^2$. Students complete the exercises based on the methods they have learned today. The teacher consolidates students’ new knowledge through exercises.

For the teaching topic of Graph and Properties of Quadratic Function $y = a(x-h)^2$, the teacher applies GeoGebra to the aspects of the teaching process, and set content and time for each teaching link. The teacher uses GeoGebra to demonstrate the graphs of three quadratic functions and guides students to write the opening direction, symmetry axis and vertex of the quadratic functions, and the relationship between the parabola $y = \frac{1}{2} (x-h)^2$ and the parabola $y = \frac{1}{2} x^2$. Students complete the exercises based on the methods they have learned today. The teacher consolidates students’ new knowledge through exercises.

For the teaching topic of Graph and Properties of Quadratic Function $y = a(x-h)^2 + k$, the teacher applies GeoGebra to the aspects of the teaching process, and sets content and time for each teaching link. The teacher uses GeoGebra to demonstrate the graphs of four quadratic functions and guides students to write the opening direction, symmetry axis and vertex of the quadratic functions. Students complete the exercises based on the methods they have learned today. The teacher consolidates students’ new knowledge through exercises.

For the teaching topic of Graph and Properties of Quadratic Function $y = ax^2 + bx + c$, the teacher applies GeoGebra to the aspects of the teaching process, and sets content and time for each teaching link. The teacher guides students to write the opening direction, symmetry axis and vertex of the quadratic functions, describe the changing trend of the image, and use GeoGebra to demonstrate. Students complete the exercises based on the methods they have learned today. The teacher consolidates students’ new knowledge through exercises, and uses GeoGebra software to demonstrate to deepen students’ impression.

For the teaching topic of Quadratic Function and Quadratic Equation in one variable, The teacher applies GeoGebra to the aspects of the teaching process, and sets content and time for each teaching link. The teacher uses GeoGebra to demonstrate different graphs of quadratic functions $y = ax^2 + bx + c$ and guides students to draw conclusions through quadratic function graph. Students draw conclusions under the guidance of teachers:

(1) If the parabola $y = ax^2 + bx + c$ has a common point with the x-axis, and the abscissa of the common point is $x_0$, then when $x = x_0$, the function value is 0, so $x = x_0$ is the equation $ax^2 + bx + c = 0$ a root.

(2) There are three positional relationships between the graph of the quadratic function $y = ax^2 + bx + c$ and the x-axis: no common point, one common point, and two common points. This corresponds to the three cases of the roots of the quadratic equation $ax^2 + bx + c = 0$: no real roots, two equal real roots, and two unequal real roots.

Evaluation
The fifth stage of designing the Lesson Plans based on the 5E Instructional Model is Evaluation. During this stage, teachers encouraged students to evaluate their learning ability and achievements and the evaluation should run through the entire lesson (Cardak, Dikmenli, & Saritas, 2008). Evaluation should be regarded as a continuous process and when the students apply new concepts and skills, the teacher observes them and tries to find evidence that the students have changed or modified their thinking and the students also have the opportunity to conduct self-assessment or companion assessment (Duran & Duran, 2004). Therefore, for the teaching of quadratic functions, this stage should allow students to review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize. The time of this stage is about 2 minutes.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2 \), the teacher asks questions: What knowledge did you learn in this class, and what did you gain? The teacher can analyze, evaluate and reflect on all aspects of the classroom according to the students’ performance in the classroom by reviewing the teaching process. Students review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2+k \), the teacher asks questions: What knowledge did you learn in this class, and what did you gain? The teacher can analyze, evaluate and reflect on all aspects of the classroom according to the students’ performance in the classroom by reviewing the teaching process. Students review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize.

For the teaching topic of Graph and Properties of Quadratic Function \( y=a(x-h)^2 \), the teacher asks questions: What knowledge did you learn in this class, and what did you gain? The teacher can analyze, evaluate and reflect on all aspects of the classroom according to the students’ performance in the classroom by reviewing the teaching process. Students review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize.

For the teaching topic of Graph and Properties of Quadratic Function \( y=a(x-h)^2+k \), the teacher asks questions: What knowledge did you learn in this class, and what did you gain? The teacher can analyze, evaluate and reflect on all aspects of the classroom according to the students’ performance in the classroom by reviewing the teaching process. Students review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize.

For the teaching topic of Graph and Properties of Quadratic Function \( y=ax^2+bx+c \), the teacher asks questions: What knowledge did you learn in this class, and what did you gain? The teacher can analyze, evaluate and reflect on all aspects of the classroom according to the students’ performance in the classroom by reviewing the teaching process. Students review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize.

For the teaching topic of Quadratic Function and Quadratic Equation in one variable, the teacher asks questions: What knowledge did you learn in this class, and what did you gain? The teacher can analyze, evaluate and reflect on all aspects of the classroom according to the students’ performance in the classroom by reviewing the teaching process. Students review and summarize what they have learned in this lesson. The teacher clarifies ideas for students by summarizing and guides students to learn to summarize.
Conclusion

As one of the key points and the difficult topics of mathematics learning, the teaching of quadratic functions is very difficult for junior high school mathematics teachers and the learning of quadratic functions is also very difficult for junior high school students. This research uses GeoGebra for teaching and the 5E instructional model as the basic structure, including the five basic stages of engagement, exploration, explanation, elaboration, and evaluation. This research conducted specific teaching design and descriptions of quadratic functions during each teaching stage, which aims to improve students’ understanding of quadratic function graphs and properties and then to improve teacher’s teaching effect. This research provides an initial framework for how students learn quadratic functions, and continues research in this area will support curriculum development and instructional decisions. In addition, this research also provides evidence and reference for educational researchers, and provides a reference for improving the teaching strategy of quadratic functions.

References