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Understanding the Challenges and Strategies in Teaching Fractions: Insights from Experienced Mathematics Teachers

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Abstract

Teaching fractions effectively at the primary school level is a persistent challenge faced by mathematics educators worldwide. This study explores the specific difficulties encountered in teaching fractions, the pedagogical strategies employed to overcome these challenges and the impact of these strategies on students' understanding. Semi-structured interviews were conducted with three experienced Malaysian primary school mathematics teachers and the data were analyzed using thematic analysis. The analysis identified key challenges related to both conceptual and procedural understanding. Additionally, the study highlighted effective teaching strategies, including active and collaborative learning, as well as the use of diverse instructional materials. The findings underscore the importance of tailored teaching approaches and resources in enhancing students' comprehension of fractions, offering valuable insights for educators and curriculum developers aiming to improve mathematics instruction. These insights provide practical recommendations for educators and curriculum developers to improve mathematics instruction, particularly through integrating hands-on and technology-based learning methods.

Keywords: Teaching fractions, Primary mathematics, Conceptual and procedural understanding, Instructional strategies.

Introduction

Understanding fractions is a crucial part of mathematics education because it is an essential foundation to understanding ratios, proportions and algebra (Siegler et al., 2013; Xu et al., 2023). Achieving proficiency in fractions is not only fundamental for educational advancement in mathematics but also plays a significant role in day-to-day problem-solving and decision-making skills (Lamon, 2020). However, fractions often known for being a difficult area for primary students, often causing an obstacle that affects their mathematical progress in the future (Behr et al., 1992; Lee et al., 2023).

The difficulty in understanding fractions comes from their abstract concept and their role as both numbers and operators, which can be a tough concept for young students (Braithwaite & Siegler, 2024). It is common for students to find it hard to differentiate between whole numbers and fractions as well as understanding the concept of part-whole relationships and

carry out operations with fractions (Ren & Gunderson, 2021). This misapplication can cause numerous comprehension and calculation errors. For example, a student might incorrectly judge $\frac{1}{4}$ as larger than $\frac{1}{2}$ by just looking at the numerators, ignoring the importance of the relationship between numerator and denominator (Siegler et al., 2011). Without correcting these misconceptions, students may find it difficult to progress in other mathematical areas such as ratios, proportions and algebra (Xu et al., 2023). Thus, it's essential to clear these conceptual blocks for students' mathematical advancement.

The key to helping students navigate the complexities of fractions lies in effective teaching strategies. Understanding fractions requires grasping its fundamental conceptual and procedural elements. The conceptual side involves getting the essentials and relationships within mathematical concepts, such as seeing fractions as parts of a whole or understanding the equivalence between two fractions. Conversely, procedural knowledge consists of the skills needed to perform operations such as addition, subtraction or simplification of fractions (Rittle-Johnson et al., 2015). Merging these two aspects can be problematic for students, often resulting in a fragmented understanding of the topic (Lopez, 2020). The importance of clearing these conceptual hurdles cannot be overstated, as difficulties with fractions hinder long-term mathematical progress (Barbieri et al., 2020).

Several strategies have been identified to aid in the teaching of fractions. Hands-on activities and the use of manipulatives has been recognized as beneficial in enriching students' understanding of fractions. Materials such as fraction bars or pie charts offer visual and physical ways for students to explore and intuitively grasp fractions (Alshehri, 2017; Amo-Asante & Bonyah, 2023). Engaging with these resources can help students recognize fractions as parts of a whole. According to constructivist learning theories, such hands-on experiences are vital for gaining a solid understanding of mathematical concepts.

Furthermore, collaborative learning methods, including peer instruction and teamwork, enhance student engagement and understanding. Through these interactions, students can share and refine their ideas, gaining different perspectives and strengthening their knowledge through social engagement and debate (Aqilah et al., 2022; Fleury & Garrison, 2014). Collaborative learning not only deepens mathematical understanding but also boosts communication and critical thinking skills. It's also found that explaining concepts to peers can significantly enhance learning, making collaboration an effective technique in fraction education.

Scaffolding is another effective instructional strategy that guides students through incrementally challenging tasks, with appropriate support, to foster confidence and skill in managing complex fraction operations (Prihantini et al., 2021). This approach provides the necessary support at the start of the learning process, gradually reducing assistance as students' proficiency grows. Scaffolding is especially beneficial for teaching intricate operations, like adding and subtracting fractions with different denominators, helping students navigate the steps involved.

Lastly, the use of visual aids and interactive technology is crucial in reinforcing learning and catering to various learning preferences. Tools like number lines or area models support students in visualizing and understanding fraction operations more (Alkhateeb, 2020; Mavrikis

et al., 2022). Moreover, advancements in educational technology and digital resources have broadened the horizons for engaging and interactive fraction education. Virtual manipulatives, for example, allow for a dynamic exploration of fractions, providing instant feedback and opportunities for experimentation in a manner unachievable with traditional methods (Reinhold et al., 2020). Such technologies have proven to be effective in enhancing engagement and understanding by offering diverse and interactive learning experiences.

The aim of this study is to investigate the firsthand experiences of primary school mathematics teachers in the instruction of fractions, looking into the obstacles they encounter, the methods they apply to overcome these challenges and how various educational tools prove effective in promoting students' understanding. Through a qualitative research approach, this study aims to enrich the existing literature on mathematics education and offer insights into successful fraction teaching practices.

Research Objectives

1. To identify the challenges primary school teachers encounter in fraction education.
2. To explore the teaching strategies used by teachers to aid students' understanding of fractions.
3. To examine the role and impact of various teaching aids in improving students' conceptual and procedural knowledge of fractions.

Methodology

This research adopted a qualitative method, using in-depth semi-structured interviews to delve into the experiences and methodologies of mathematics teachers in imparting knowledge about fractions. By choosing this qualitative route, we were able to deeply explore the issues, methodologies and instruments that play a critical role in teaching fractions, as viewed through the lens of experienced teachers. The participants comprised three primary school mathematics teachers from Malaysia as in Table 1.

Table 1
Overview of participant profiles

Participants	Gender	Years of Experience
Teacher A	Male	11
Teacher B	Female	20
Teacher C	Female	12

Having over five years of teaching experience is regarded as having professional level expertise in the educational field (Muhonen et al., 2021). All participants currently teach Year 4 students and have substantial experience in delivering fraction content within the Malaysian education context. An interview protocol was developed that consisted of open-ended questions designed to elicit information about the challenges, strategies and tools used in teaching fractions. To ensure the quality of the protocol, two experts in mathematics education were consulted to review the questions for clarity, relevance and alignment with the research objectives. Expert validation of interview protocols helps ensure content validity and alignment with the study's goals (Knott et al., 2022). Feedback from these experts was incorporated to refine the questions and improve the interview process (May et al., 2018).

Data were collected through comprehensive, semi-structured one-on-one interviews with each teacher. These interviews ranged from 60-90 minutes in duration and were audio-recorded with the participants' permission to ensure precise transcription and analysis. To analyze the interview transcripts, we utilized thematic analysis as suggested by Braun and Clarke (2006). This involved steps like getting familiar with the data, generating initial codes, searching for and reviewing themes and finally, defining, naming and reporting on the themes. This systematic approach ensured a rigorous and comprehensive understanding of the data, highlighting commonalities and differences in teachers' experiences and practices. To enhance the validity of the findings, data were cross-checked with the participants for accuracy through member checking. This process ensured that the participants' perspectives were accurately represented (Motulsky, 2021).

Results

The analysis yielded three primary themes corresponding to the research objectives:

Challenges in Teaching Fractions

All participants collectively recognized that fractions pose a significant challenge for primary students due to both conceptual and operational complexities. Understanding fractions as parts of a whole is a fundamental concept that students often find difficult, hindering their advancement in more complex operations. Teacher A observed:

"If they don't understand the basic concept, they cannot perform the correct procedures. They often get confused about what a fraction represents."

Teacher B added:
"Students fail to visualize fractions properly, which makes it hard for them to comprehend more complex problems."

Furthermore, operations like adding and subtracting fractions with different denominators are notably challenging. According to Teacher C:
"Subtraction involving mixed numbers and different denominators is particularly difficult. Students get confused with the multiple steps involved and often make errors."

Students tend to rely on memorizing procedures without understanding underlying principles, leading to errors and misconceptions. Teacher B commented:
"They often memorize mechanical steps but don't understand why they're doing them, so when faced with slightly different problems, they struggle."

A surprising but significant challenge identified was students' difficulty in recalling multiplication tables, which is essential for simplifying fractions and finding common denominators. The difficulty with fractions is further deepened by students' reliance on prior knowledge of whole numbers, which can interfere with their understanding of fractions as distinct from whole numbers. Teacher A pointed out:

"Students often apply their understanding of whole numbers to fractions, leading to confusion, especially when they are asked to perform operations like addition or subtraction with fractions."

Teaching Strategies

To address the challenges associated with teaching fractions, the teachers employed a variety of strategies aimed to improve both conceptual and procedural understanding. All three participants emphasized the importance of using hands-on activities to help students grasp the abstract nature of fractions. Teacher A shared:

"I use tangible materials like fraction strips and pie charts to make fractions more concrete for the students. These materials allow them to manipulate fractions and see how different fractions relate to each other."

Collaborative learning was another key strategy highlighted by the teachers. Teacher C noted:
"Group work and peer teaching are crucial. When students explain concepts to each other, they often find ways to simplify complex ideas, making the material more accessible."

The teachers also used scaffolded instruction to gradually build students' understanding. Teacher B described her approach:
"I start with simple, concrete examples and slowly move to more abstract problems as the students become more comfortable with the material. It's important not to overwhelm them too quickly."

Teacher B, in particular, integrated technology into her lessons to enhance engagement and understanding. She explained:
"I use educational software that allows students to explore fractions in an interactive way. These tools provide immediate feedback, which is essential for learning."

Teacher C emphasized the use of visual aids and repetition to reinforce learning:
"Visuals like fraction walls and number lines are essential. Repetition through varied practice helps reinforce the concepts and ensure that students remember the procedures."

Teaching Aids

The use of teaching aids was identified as a critical component in supporting the strategies discussed above. The teachers employed a range of materials to make learning fractions more accessible and engaging. Teacher A discussed the use of concrete materials:

"I use physical objects like cut-out paper fractions and blocks. These help students physically manipulate the fractions and see the part-whole relationships."

Teacher B added that she uses semi-concrete materials:
"Things like drawings and diagrams, which are not quite as tangible as physical objects but still provide a visual representation that students can grasp."

In addition to physical materials, digital tools played a significant role. Teacher B mentioned:
"There are several apps that simulate fraction problems and allow students to interact with the fractions in real-time. This is particularly useful for students who might struggle with traditional methods."

Teacher C highlighted the importance of using a variety of instructional materials to cater to different learning styles. She stated:

"Some students learn better through visuals, others through hands-on activities. I try to use a mix of videos, diagrams and physical materials to reach all students."

Teachers also noted the value of custom-made teaching aids tailored to their students' specific needs. Teacher C shared:
"I often create my own materials, like fraction kits, which I can modify based on the difficulties my students are experiencing. This personalized approach helps address the specific challenges my students face."

Discussion

This study confirms the significant challenges primary school students face in grasping fraction concepts, as evidenced by observations from three experienced teachers. Students frequently display difficulties in perceiving fractions as parts of a whole and have difficulty transitioning from whole-number reasoning to fractional reasoning (Siegler et al., 2011). It is observed that children often misunderstand a fraction's representation, making it difficult for them to apply the correct procedures. This aligns with prior studies indicating that reliance on whole-number logic can lead to misunderstandings in fraction (Ren & Gunderson, 2021).

The struggle students face in visualizing fractions further impedes their ability to comprehend more complex fraction problems. Additionally, executing multi-step operations, such as fraction subtraction with different denominators, proves the difficulties for many learners. The habit of memorizing procedures without understanding the underlying concepts exacerbates these challenges (Wijaya et al., 2021). These issues underscore the need for instructional methods that equally emphasize conceptual and procedural understanding since failings in basic understanding prevent long-term mathematical progress (Barbieri et al., 2020).

The teachers employed a variety of strategies to address the challenges associated with teaching fractions. Engaging students in hands-on activities, using manipulatives like fraction bars and pie charts, significantly aids in visualizing and manipulating fractions more effectively. Such tools not only make fractions more relatable but also align with findings that highlight the effectiveness of physical manipulatives in deepening conceptual understanding (Cramer et al., 2002).

Collaborative learning such as group work and peer instruction also play a vital role in learning fractions (Charalambous et al., 2021). It will encourage students to explain concepts to each other, simplifying complex ideas and strengthening their understanding through social interaction (Klang et al., 2021). This collaborative approach aligns with Vygotsky's theory of social learning, emphasizing the importance of peer interaction in cognitive development. Additionally, scaffolded instruction was a common approach. Step-by-step instruction facilitates learners' gradual familiarization with increasingly complex ideas, promoting confidence and solid understanding. Scaffolding is especially important for complex operations like adding fractions with different denominators, as it allows students to learn incrementally (Prihantini et al., 2021).

Moreover, the strategic deployment of various teaching aids significantly bolsters fraction learning. The incorporation of both concrete materials like fraction cutouts and semi-concrete materials such as diagrammatic representations, proves invaluable in supporting conceptual understanding (Morano & Riccomini, 2020). Customizing fraction tools to cater to specific student needs further expands understanding, reflecting the criticality of customizing learning aids (Issa et al., 2013).

The integration of technology into fraction learning also emerges as a critical strategy. Digital tools including educational software, offer prompt feedback, helping students correct their mistakes in real time. The use of digital tools, such as fraction apps, provides interactive, engaging experiences that make abstract concepts more accessible (Alkhateeb, 2020). By incorporating both traditional and digital resources, teachers can offer multi-modal instruction that caters to diverse learning styles (Alshehri, 2017; Amo-Asante & Bonyah, 2023).

Implications

Theoretical Implications

This study contributes to the existing literature by expanding our understanding of how constructivist approaches and Vygotsky's social learning theory (Vygotsky, 1978) can be applied in teaching fractions. While previous studies emphasize peer learning, this research highlights the importance of structured, scaffolded support in the early stages of fraction learning, before collaborative activities. These findings suggest that scaffolding should be tailored to individual students' needs before moving to group-based activities, supporting the idea of differentiated instruction within a constructivist framework (Rahmawati & Purwaningrum, 2022). Furthermore, the study demonstrates how technology-enhanced learning can align with 21st-century learning frameworks, where digital tools not only serve as supplementary resources but also as central components of conceptual understanding. Future research could explore how different types of digital tools (e.g., gamified learning apps vs. traditional virtual manipulatives) influence student engagement and long-term retention (Reinhold et al., 2020).

Practical Applications

Based on these findings, teachers are encouraged to integrate a multi-modal approach to fraction instruction. Specifically, the use of manipulatives like fraction strips and pie charts should be a standard practice, combined with visual aids and digital tools (Alshehri, 2017; Amo-Asante & Bonyah, 2023). Teachers should leverage technology to provide real-time feedback and interactive learning experiences, especially when introducing abstract concepts like fractions (Reinhold et al., 2020). Incorporating scaffolded instruction should also become more deliberate. Teachers can start by providing significant guidance for complex operations like fraction subtraction with different denominators and gradually reduce support as students become more confident (Lamon, 2020). Training programs for teachers should emphasize the importance of gradual, incremental learning supported by a balance of direct instruction and peer collaboration.

Impact on Student Understanding

The use of manipulatives, visual aids and digital tools significantly enhances students' ability to internalize fraction concepts and apply them in varied contexts. These strategies not only

address immediate conceptual challenges but also prepare students for more complex topics such as ratios and algebra (Barbieri et al., 2020). By adopting a multi-modal approach, teachers can better cater to diverse learning needs, ensuring that students who struggle with traditional methods of instruction can benefit from alternative forms of engagement. These findings suggest that a well-rounded approach combining physical, visual and technological tools is key to fostering deep, lasting understanding of fractions (Issa et al., 2013).

Conclusion

This study provides valuable insights on the ongoing difficulties faced when teaching fractions in primary schools, alongside effective tactics that experienced teachers employ to navigate these issues. The findings from this study highlight the necessity of employing diverse teaching tools and techniques to meet the varied educational needs of students. While the study offers meaningful contributions, there are certain limitations to consider. The small sample of three teachers, all from the same educational context in Malaysia, means the findings are context specific. Although not broadly generalizable, these insights may be transferable to other similar educational settings, where teachers face comparable challenges in teaching fractions. Additionally, relying on teacher-reported strategies rather than direct classroom observations may have limited the scope of the findings. Future research could expand the sample size, explore different educational contexts and incorporate classroom observations to enrich the understanding of effective fraction instruction. Further research should also investigate the long-term impact of these strategies on students' mathematical achievements and explore how these methods can be adapted to other challenging topics within the mathematics curriculum.

The study also contributes to existing educational theories by reinforcing the importance of constructivist and social learning approaches in mathematics education. By illustrating how hands-on manipulatives, scaffolded instruction and collaborative learning foster deeper conceptual understanding, the study provides empirical support for Vygotsky's social learning theory and Bruner's scaffolding theory. These findings highlight how step-by-step guidance, paired with peer interaction, enables students to master complex fraction operations. In the Malaysian educational context, this research offers practical insights into how teachers navigate the challenges of teaching fractions, despite resource limitations and diverse learning needs. Teachers can adapt these strategies, from traditional manipulatives to digital tools, to improve fraction instruction across a variety of educational settings. Overall, this research underscores the critical role of diverse, tailored teaching strategies in improving fraction instruction and highlights how both theoretical frameworks and practical classroom approaches can work together to enhance students' mathematical understanding and success.

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