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Hands-on Activity with Manipulatives Framework for Facilitating Early Algebraic Thinking

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Abstract

This module was developed to address students's weaknesses in algebraic topics. Particularly in understanding the fundamentals of algebraic concepts. Research indicated that many students encounter difficulties with key algebraic terms and find it challenging to connect abstract algebraic concepts to real-life situations. To overcome these issues, this module is designed based on the 5E learning model (engage, explore, explain, elaborate, and evaluate) and incorporates the Engineering Design Process (EDP) application. The module incorporates hands-on activities to enhance students' visualisation of algebraic concepts. The module's design aligns with the Mathematics Curriculum and Assessment Standard Document (DSKP) by the Malaysian Ministry of Education, ensuring its relevance to the national curriculum. By enhancing the students fundamental understanding of algebra, this module also encourages active participation in the learning process, ultimately aiming to strengthen the algebraic thinking skills as well as engagement during learning. This paper presents the conceptual framework of the Hands-on 5E (HO5E) module along with a concise example of a lesson for teaching and learning early algebra topics using this methodology. Utilising a context analysis methodology, we examined and evaluated material and documents pertaining to algebraic thinking, hands-on activities, the 5E model, and engineering design processes (EDP) to formulate the HO5E framework, which connects algebra with practical experiences. This framework offers essential resources for mathematics educators and researchers, compromising conventional lesson materials, assessment instruments, and new pedagogical strategies for core algebra topics. By employing this framework, teachers and researchers can establish a shared foundation that outlines the methodology, methods, process, and activities for visualising algebraic concepts through hands-on activities with tangible manipulatives. This approach not only helps students grasp algebra more clearly but also enables them to appreciate its beauty and relevance to real-world context.

Keywords: Algebraic Thinking, 5E Instructional Model, Engineering-Design Process, Hands-on Activity-based Strategy, Manipulatives.

Introduction

In elementary classrooms around the world, children learn about numbers and the four basic operations through real-world contexts, making these concepts clear and concrete.However, secondary school expects students to grasp more abstract ideas. Algebra is frequently regarded as more difficult than arithmetic due to its focus on sets of numbers and variables rather than solely on individual numbers.

Algebra is an essential topic that demands students employ logical reasoning, testing their abilities by highlighting arithmetic operations while utilising symbols to represent equations and define connections within mathematical procedures. (Thomas, M. O. J. & Tall, D., 2013). Furthermore, algebra is an essential element of mathematics that provides several advantages in daily life. It functions as an entrance to more sophisticated mathematical concepts. (Kriegler S, 2008).

Problem Statement

Algebraic thinking commences with concrete experiences related to numbers and progressively evolves into generalisation and abstract reasoning by many activities Chimoni et al. (2023). However, algebra failure rates continue to be high, particularly among kids with lower academic capabilities. (McCoy, 2005). Young children often encounter difficulties articulating their ideas in algebraic thinking due to their limited grasp of foundational concepts. Studies show that while children possess significant abilities to learn mathematical concepts at an early age, their understanding is still developing, which can hinder accurate expression of algebraic ideas. (Li, 2023). Also, research has shown that kids can get wrong ideas about math early on if they don't understand the basics of algebra, which makes it harder for them to explain and use these ideas correctly. (Hansen et al., 2014). Addressing these misconceptions through appropriate instructional strategies is crucial for fostering a solid foundation in algebraic thinking. Besides, students also have trouble understanding the different meanings of symbols and the logic behind them. They also have trouble moving from basic math to algebra throughout their algebraic thinking (Knuth et al., 2005).

Scenario Algebra In Malaysia

The proficiency of students of lower secondary schools in Malaysia in mastering algebraic concepts remains substandard. According to the Sijil Pelajaran Malaysia (SPM) result of the national secondary school examination, most students struggle with mathematics, particularly in algebra. Lower secondary school students perform below in mastering algebraic concepts (Mullis, 2019). Furthermore, research by Stephens (2008) mentioned that the low proficiency in algebra is due to the limited grasp of fundamental algebraic concepts. Therefore, lacking in this foundational understanding of the concepts may cause more problems to the skill of problem solving, which involves equations, expressions, and functions (Zakaria & Maat, 2010).

This result of inefficiency in algebra was also shown in the Trends in International Mathematics and Science Study (Mullis, 2019). The Malaysian average is below the OECD average. Only 5% of the students utilised algebraic problem-solving skills involving application, reasoning, and generalisation. According to the findings of the studies by Stephens (2008), one of the factors contributing to this challenge is insufficiency comprehension of algebraic properties.

Algebraic Thinking

Algebraic thinking is a part of mathematical thinking that combines mathematical thinking skills and areas of algebraic content. Algebraic thinking is using mathematical thinking skills such as reasoning, representation, and problem solving to understand the idea of algebraic content. Numerous studies propose dividing the algebraic content in secondary school into three distinct sections The first section is to understand the concept of variables, expressions, functions, and equations (NCTM and Mullis 2010). The second part is the area of pattern and generalising the situations, quantities, and patterns. The last part of algebra is the use of multiple representations, where students can create and analyse graphs, tables, and algebraic symbols. Although the content areas of algebra in the secondary schools incorporate pattern, variable, and representation, some researchers define algebra topics in slightly different ways. Herbert and Brown (1997) described algebraic thinking as the application of mathematical symbols and methods to analyse different situations.

Use of Manipulative with Collaborative Learning

To reduce students's difficulties in understanding algebra concepts, manipulatives can be employed to facilitate the meaningful and successful learning process. Students can establish links between algebraic concepts through manipulatives. (Chappell & Strutchens, 2001).Studies by Hizman 1997 also suggest that using hands-on manipulatives in groups is very effective in significantly enhancing student performance in algebraic concepts. Furthermore, students expressed their appreciation for the incorporation of manipulatives in activities designed to facilitate their understanding of algebraic concepts. Student algebraic skills and self-efficacy are enhanced through collaborative activities (Fletcher, 2008). Windsor (2010) also suggests that algebraic thinking is fostered when students are given the chance to articulate their mathematical concepts in a classroom setting that appreciates and fosters collaborative learning. Additionally, collaborative work can foster students's positive attitudes towards mathematics and enhance students' procedural fluency for mathematical reasoning (Jansen 2012).

Hands-on Activity-Based Strategy

Learning by doing is the core principle of hands-on activity-based strategy. Manipulatives are a frequent practice in hands-on mathematics education. Students can understand the mathematical concept easily when they can visualise.(Clements & McMillen, 1996; NCTM, 2000). The teacher may employ a variety of resources as long as they are concrete and can be manipulated to support reflection and discussion as well as to foster a hands-on approach. Students may utilise structured everyday materials to solve tasks.

According to Haury and Rillero (2015), the hands-on learning approach engages students in comprehensive learning experiences that improve their critical thinking skills.As a result, a hands-on approach has been suggested as a method to enhance academic performance and comprehension of mathematics concepts by allowing students to visualise so they can see the abstract concepts more clearly. Through this hands-on approach, students are able to engage with the real-world illustration and observe the effect of change in the various variables. This method is learner-centred, enabling the learner to observe, manipulate, and handle objects while learning.

Furthermore, research by Nesin (2012) states that hands-on activities are also more effective in enhancing the attention engagement level of students, particularly younger students, by deviating from the normal practice of remaining seated in the classroom and listening to the teacher's instructions.

On the contrary Fischer et al. (2023) state that during hands-on activities, teachers are required to accommodate the diverse learning requirements of their students while simultaneously ensuring their engagement, which is more difficult by low motivation levels and diverse student abilities. (Fischer, et al., 2023). This factor can be the challenge for the teacher to make sure the engagement happens during the activity.

Engineering Design Process (EDP)

Academic research extensively recognises the Engineering Design Process (EDP) as an effective framework for fostering creative problem solving, particularly in an applied context. (Xi et al., 2024). EDP actively engages students in applying mathematical principles to real-world scenarios, thereby enhancing their proficiency in tackling complex mathematical problems According to YT et al. (2021), EDP enhances students' comprehension of abstract concepts by integrating mathematics into a systematic problem-solving framework that prioritises practical and hands-on experiences.

Researchers have introduced various models to clarify the structure of EDP. Cunnigham (2009) and Cunningham et al. (2018) outlined five key stages: enquire, strategise, develop, and improve. These stages are cyclical, giving students the flexibility to engage in different stages based on their individual needs. Moore et al. (2014) argued that engineering practice inherently involves mathematics, bridging theoretical knowledge with practical applications, thereby providing meaningful learning experiences in the classroom. This approach integrated mathematics with the engineering design process, creating a practical context that deepens students's understanding of mathematical principles and strengthens their problem-solving skills (Maiorca & Sohlmann, 2016). Through this method, students connect their mathematical learning to tangible outcomes, making abstract concepts easier, more accessible, and more relevant.

Table 1
Application of the Engineering Design Process (EDP) in the HO5E Module

ASK
Objective: Frame the problem of challenge that students need to solve.
Activity :
- The teacher presents the scenarios : You are given a sealed and opaque container that contains an unknown number of sweets (x). Your task is to determine how many sweets you have in total after receiving additional sweets and confirm your calculations.

Guiding Questions:
"What do we know about the number of sweets in the container?"
"What information do we need to calculate the number of sweets?"

IMAGINE
Objective: Brainstorm and discuss possible approaches to solving the problem.
Activity :
- Students hypothesize the total number of sweets based on receiving additional sweets and discuss how to represent mathematically

PLAN
Objective: Develop a strategy to calculate the total number of sweets and represent the situation algebraically.
Activity :
- Students plan how to represent the unknown quantity with a variable (e.g x) and construct expression for the total sweets after receiving additional quantities (e.g x+5, x+4)

CREATE
Objective : Implement the plan and solve the problem
Activity :
- Each group receives an opaque container and a set number of additional sweets.
- Students calculate and state their predicted total number of sweets using their planned algebraic expression.
- They open the container, count the actual number of sweets, and confirm or revise their calculations.

TEST
Objective : Verify and analyse the accuracy of the prediction and calculations
Activity :
- Students compare their calculated totals with the actual number of sweets.
- They identify whether their algebraic expressions correctly represent the situations for other given quantities.

IMPROVE
Objective : Refine their understanding and representation of algebraic concepts
Activity:
- Students complete a worksheet that includes various scenarios, including unknown quantities, additional amounts, and their total.
- They refine their use of algebraic expressions to consistently represent these scenarios

accurately.
SHARE **Objective :** Communicate findings and reflect on the learning process **Activity:** ● Students present their findings to the class, explaining how they used the algebraic expressions to represent the total number of sweets and confirm their predictions. ● Group discuss the role of variables, constants, and expressions in solving the problem.
EVALUATE **Objective:** Reflect on the process and solidify understanding. **Activity:** ● The teacher emphasises key takeaways, such as the definition of variables with constant and changing values, algebraic terms and algebraic expressions. ● Students reflect on what they learnt about representing and solving problems involving unknown quantities.

The BSCS 5E Instructional Framework

Building on development psychology and early research learning cycles (Atkin & Karplus, 1962), BSCS Science Learning (2023) introduced the BSCS 5E instructional model (Bybee et al., 2006) to effectively organise science instruction and curriculum, aiming for deep understanding and long-term retention of key concepts and skills. Engaging, exploring, explaining, and evaluating are the 5E stages. They come from earlier learning cycle models, such as the one by Atkin and Karplus (1962), which had similar stages of exploring, explaining, and elaborating.This teaching technique has been linked to deeper understanding, creativity, teamwork, better communication, and intellectual risk-taking. According to Makar and Fielding-Well (2018), Model 5E is an effective tool for teaching algebra and other abstract mathematical concepts because it improves both procedural fluency and conceptual understanding through experiential and collaborative learning. Panoura (2018) encourages teachers to implement strategies that fully immerse students in mathematical concepts, promoting critical thinking and problem solving.

Table 2
Stages of 5E instructional Framework

Stages	Summary
Engagement	*The teacher or curriculum task engages students' existing knowledge and fosters involvement with an original idea through brief tasks that provoke interest and memory recall. These activities must link prior knowledge to current experiences, reveal pre-existing concepts, and guide students' cognition towards the desired learning objectives of the current tasks.*
Exploration	*Students can share a variety of experiences through exploration activities, which helps them figure out what they believe and what they don't understand while also encouraging conceptual change. When students do these activities, they can do lab exercises that help them use what they've learnt, come up with new ideas, think about problems and possibilities, and plan and carry out preliminary studies.*
Explanation	*The explanation stage focuses students on essential aspects of their engagement and exploration activities, allowing them to demonstrate their conceptual understanding, abilities, or behaviours. It provides educators the ability to immediately present a topic, technique, or skill. In this stage, students articulate their understanding, and explanations from the teacher or curriculum deepen their comprehension, making this stage crucial for reinforcing learning.*
Elaboration	*Teachers motivate students to expand their knowledge and enhance their skills through diverse experiences. As they engage in new opportunities, students strengthen their understanding and abilities, which they further apply by taking part in additional activities*
Evaluation	*The evaluation stage prompts students to reflect on their understanding and skills while also giving teachers the chance to assess their progress in relation to the learning objectives.*

Theoretical Framework

The HO5E module is grounded in established educational theories that emphasise active learning and conceptual understanding in algebra. Thus, the section elucidates the theoretical foundations that inform the module’s approach to utilising manipulativs in algebra instruction.

Contructivist learning theory posits that learners construct knowledge through experiences and reflection (Piaget, 1970). This approach emphasises active engagement and the importance of prior knowledge in learning new concepts. In the HO5E module, manipulatives are employed to facilitate active learning, allowing students to build upon their prior knowledge and construct new understandings of algebraic concepts.Piaget (1970) proposed that learners progress through stages of cognitive development, moving from concrete operational to formal operational thinking, where abstract concepts become possible. The module leverages manipulatives to support students in the concrete operational stage, providing tangible experiences that bridge to abstract algebraic concepts, aiding the transition to formal operational thinking.

Vygostsky ZPD describes the difference between what learners can do without help and what they can achieve with guidance (Vygotsky, 1978). The HO5E module incorporates collaborative activities and teacher facilitation, aligning with the ZPD by offering scaffolding that enables learners to solve problems slightly beyond their independent capabilities.

In conclusion, these theoretical frameworks provide a robust foundation for the HO5E module. In summary, this module emphasises manipulatives with Piaget's (1970) assertion that concrete experiences are essential for cognitive development. By providing students with hands-on experiences, the module supports students's construction of knowledge, a key principle of constructivist learning theory. Furthermore, through collaborative problem-solving tasks, the module leverages Vygotsky's (1978) concept of the zone of proximal development, enabling students to achieve higher levels of understanding with peer and teacher support.

Conceptual Framework

The hands-on activity-based learning model, as applied in Module HO5E, is rooted in constructivist educational theory that emphasises the active involvement of students and the manipulation of concrete objects to understand abstract mathematical concepts (Piaget, 1970; Vygotsky, 1978). This model emphasises student-centred learning, strengthening conceptual understanding through social interaction, and a collaborative approach to solving mathematical problems, making it one of the most effective strategies in teaching algebra (Bybee et al., 2006; Cramer et al., 2002).

Based on the literature, hands-on activities help students develop a deep understanding of mathematical concepts, especially algebra, by utilising physical manipulation and visual representation. This activity allows students to identify patterns and generalisations (Mason et al., 2008). Understanding the relationship between algebraic symbols and their concrete meaning (Carraher et al., 2008).

Build confidence in solving problems through concrete experiences (Heddens, 1986). Active learning also involves hands-on activities, where students actively participate in the process of discovery, discussion, and reflection on their learning (Rahman et al., 2022). Through manipulatives such as algebra tiles, students can understand the process of solving equations and translate abstract concepts into visual representations.

Conceptual Framework

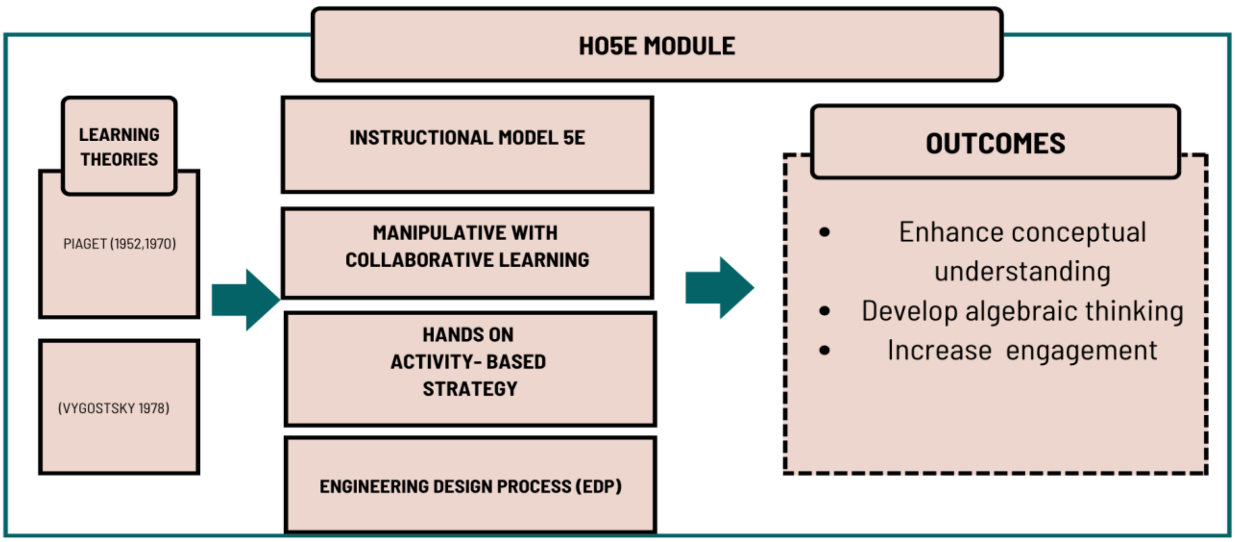

Figure 1 Conceptual framework of the study

Execution of the Module

This section provides a concise overview of the HO5E module on the teaching and learning of a particular unit in the Algebra chapter.

Key Components of the Module

To maximise its effectiveness, the module incorporates several educational frameworks and resources:

1) The Standard Curriculum and Assessment Document (DSKP) for KSSM Mathematics.
2) Hands-on activity-based strategy
3) Activity using manipulatives with collaborative learning
4) 5E instructional model lesson plan
5) Application of the Engineering Design Process (EDP)

Objective of the Module

The HO5E module is designed to enhance students understanding of basic algebra, engagement, and proficiency in algebra through integration of hands-on activities using manipulatives, framed within the 5E instructional model. The objectives of this module align with contemporary educational theories and aim to address critical challenges in algebra education, such as students difficulty in visualising abstract concepts and mastering foundational skills.

Learning Area: Relations and Algebra
Title: Algebraic Expressions
Content Standard:
5.1 Variables and Algebraic Expressions

Learning Standards:
- 5.1.1 Use letters to represent unknown quantities and determine whether the variable has a fixed or changing value, providing justification.
- 5.1.2 Derive algebraic expressions based on arithmetic expressions that represent a situation.
- 5.1.3 When given the value of the variable, determine the value of an algebraic expression and relate it to a suitable situation.

Learning Objectives:
By the end of the lesson, students will be able to:
- Identify variables with fixed values and variables with changing values.
- Derive algebraic expressions involving addition operations.
- State the value of an algebraic expression after substituting the value of the term x.

Module Summary

The HO-5E module enables students to tackle algebraic thinking using EDP as well as enhance engagement in early parts of secondary mathematics education. This module is based on experiential learning using hands-on activities and 5E instructional strategies. This module incorporates student-centred learning through manipulatives. This module aligns with Vygotsky's social constructivist philosophy, emphasising the importance of learning through interaction and collaboration. Manipulatives support students learning by enabling them to transition from hands-on, concrete experiences to abstract reasoning. The 5E model provides a systematic structure for this module, which fosters deeper conceptual understanding. This model guides students from hands-on, experiential learning towards grasping abstract mathematical concepts.

Conclusion

This research makes a significant contribution to the development of mathematics education theory and practice. The study theoretically reinforces the notion that teaching through hands-on activities can effectively bridge the gap between conceptual understanding and procedural competence in algebra instruction. Unlike traditional approaches that tend to be abstract and teacher-centred, this module adds a new layer to constructivist learning theory by proving how dynamic visualisation through manipulatives and technology supports the development of deeper algebraic thinking among students. In addition, this study contributes to the theory of self-determination (Self-Determination Theory), where hands-on elements and collaborative learning are proven to increase students' intrinsic motivation and confidence in mathematics, an aspect that is critical but often neglected in traditional pedagogy.

In terms of context, this research has clear practical implications for engineering mathematics education in Malaysia and globally. In addressing the issue of weak mastery of basic algebra, which is recognised as a major challenge in the development of advanced mathematics. This study offers a structured solution that is relevant to the educational needs of the 21st century. The HO5E module not only helps teachers develop more inclusive and effective teaching methods but also contributes to the STEM and Continuing Mathematics Education agenda in the context of competitive human resource development. Furthermore, this study strengthens the role of mathematics education in achieving Sustainable Development Goal 4 (SDG 4), which is quality education that is inclusive and equitable, by providing an approach that can be adapted in various educational contexts, whether in developing or developed countries. Therefore, this research not only enriches the existing academic discourse but also has the potential to be a key reference in the formation of mathematics teaching policies and practices at the global level.

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