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# Conditional Heavy Tails, Volatility Clustering and Asset Prices of the Precious Metal

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**Abstract:** The world palladium demand has increased steadily and dramatically in the years of 21th century. However, its spot price still has not reached the peak level observed in January 17, 2001. In 2008, a single-day increase of the palladium spot price has exceeded 37%, which witnesses significant risk for investments in the world palladium market. In this paper, we apply the GARCH model with heavy-tailed distributions into the palladium spot returns series for risk management purpose. We compare empirical performance of the Student's *t* distribution and the normal reciprocal inverse Gaussian (NRIG) distribution. Our results show the newly-developed distribution, the NRIG, cannot outperform the older fashion one, the Student's *t* distribution. Nevertheless, our results do demonstrate that it is important to incorporate conditional heavy tails for precious metals' spot return modelling.
**Key words**: Student's *t* distribution, GARCH model, palladium

## 1. Introduction

Palladium is a rare element widely-used in automotive, chemical, electrical, jewelry and dental industries. Of the four precious metals, palladium is the least known. As one of the investment vehicles, the world palladium price has drawn a lot of attentions over the world. On the supply side, the two countries, Russia and South Africa, account for more than 75% of annual global mine supplies. On the demand side, recent increases mainly come from rapidly growing gasoline-powered vehicle production in the U.S., China, India and elsewhere, which requires palladium in the catalytic converters to control exhaust emissions. Also, a recent introduction of a United States palladium coin is likely to draw an endorsement of palladium as a bullion alternative and increases the demand. World demand for palladium increased from 100 tons in 1990 to nearly 350 tons in 2010. The global production of palladium from mines was slightly more than 250 tons in 2010 according to the United States Geological Survey.

Recently, although the palladium spot price is growing in trend, the price becomes very volatile. Many market participants are hesitant to investment in the palladium, although many analysts claim there are increasing demands. Thus, people are demanding effective quantitative risk

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management tools. In this paper, we follow this demand trend and try to investigate an existing widely-used model, the generalized autoregressive conditional heteroscedasticity (GARCH) model, in risk management of palladium spot returns. We follow the work in Guo (2017a) and compare two types of heavy-tailed distribution, the Student's *t* distribution and the normal reciprocal inverse Gaussian (NRIG) distribution, under the GARCH framework. We are interested in if the two distributions have differences in empirical performance of quantifying palladium spot volatilities.

**Literature Review**

The GARCH models have been adopted to investigate the palladium market in the recent decade. Adrangi and Chatrath (2002) provided evidence of nonlinear dependencies in palladium and platinum futures markets and found that ARCH-type processes, with controls for seasonality and contract-maturity effects, generally explain the nonlinearities in the data. Diaz (2015) investigated the spots prices of the two scarce precious metals, platinum and palladium. Diaz found intermediate memory in the return structures of both precious metals, which implies the instability of platinum and palladium returns' persistency in the long run. Moreover, Diaz showed both the ARFIMA-FIGARCH and the ARFIMA-FIAPARCH models confirm long-memory properties in the volatility of the two spot prices and the leverage effects phenomenon is not also present based on the ARFIMA-APARCH and ARFIMA-FIAPARCH models. Auer (2015) used dummy-augmented GARCH models to investigate the impact of the specific calendar day on the conditional means of palladium returns. Auer illustrated that during the period from July 1996 to August 2013 there is no significant impact of the specific calendar day observed. Lucey and Li (2014) analyzed what and when precious metals could act as safe havens using the US data. Lucey and Li showed that for the period examined silver, platinum and palladium could act as a safe haven but gold could not. Lucey and Li provided evidence that at times palladium could serve as the strongest and safest haven among the four precious metals.

All the above literature did not consider conditional heavy tails and thus cannot be directly used for risk management purpose. Here, we want to develop a quantitative risk management tool based on the GARCH framework. We consider two different heavy-tailed distributions, the Student's *t* distribution as in Bollerslev (1987) and the NRIG distribution as in Guo (2017b, 2017c). Most of the existing studies on the GARCH models with heavy-tailed distributions have been focusing on the US data. For instance, Tavares, et al. (2007) investigated the heavy tails and asymmetric effect on stocks returns volatility in the GARCH framework, and found the Student's *t* and the stable Paretian distribution clearly outperform the Gaussian distribution in fitting S&P 500 returns. Su and Hung (2011) studied a range of stocks returns in the NYSE market during the period of the U.S. Subprime mortgage crisis, and show that the GARCH model with normal, generalized error distribution (GED) and skewed normal distributions provide accurate VaR estimates.

In this paper, we follow the model framework in Guo (2017a) and compare the empirical performance of the Student's *t* distribution and the NRIG distribution. The remaining sections of the paper are organized as follows. In Section 2, we discuss the models. Section 3 summarizes the data. The estimation results are in Section 4. Section 5 concludes.

**2. The Models**

Here, we list a simple GARCH(1,1) process as:

$$\varepsilon_t = \mu + \sigma_t e_t \quad (2.1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2.2)$$

where the three positive numbers $\alpha_0$, $\alpha_1$ and $\beta_1$ are the parameters of the process and $\alpha_1 + \beta_1 < 1$. The assumption of a constant mean return $\mu$ is purely for simplification and reflects that the focus of the paper is on dynamics of return volatility instead of dynamics of returns. The variable $e_t$ is identically and independently distributed (*i.i.d.*). Two types of heavy-tailed distributions are considered: the Student's *t* and the normal reciprocal inverse Gaussian (NRIG) distributions. The density function of the standard Student's *t* distribution with $\nu$ degrees of freedom is given by:

$$f(e_t \mid \psi_{t-1}) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^2}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \ \nu > 4. \quad (2.3)$$

where $\psi_{t-1}$ denotes the $\sigma$-field generated by all the available information up through time $t-1$.

The NRIG is a special class of the widely-used generalized hyperbolic distribution. The generalized hyperbolic distribution is specified as in Prause (1999):

$$f(e_t \mid \lambda, \mu, \alpha, \beta, \delta) = \frac{(\sqrt{\alpha^2 - \beta^2} / \delta)^{\lambda} K_{\lambda-1/2}(\alpha\sqrt{\delta^2 + (e_t - \mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2 + (e_t - \mu)^2} / \alpha)^{1/2-\lambda} K_{\lambda}(\delta\sqrt{\alpha^2 - \beta^2})} \exp(\beta(e_t - \mu)), \quad (2.4)$$

where $K_{\lambda}(\cdot)$ is the modified Bessel function of the third kind and index $\lambda \in \mathbb{R}$ and: $\delta > 0$, $0 \le |\beta| < \alpha$. When $\lambda = \frac{1}{2}$, we have the normalized NRIG distribution as:

$$f(\varepsilon_t \mid \psi_{t-1}) = \frac{\alpha K_0(\sqrt{(\alpha^2 - 1)^2 + \frac{\alpha^2 \varepsilon_t^2}{\sigma_t^2}})}{\pi \sigma_t} \exp(\alpha^2 - 1). \quad (2.5)$$

## 3. Data and Summary Statistics

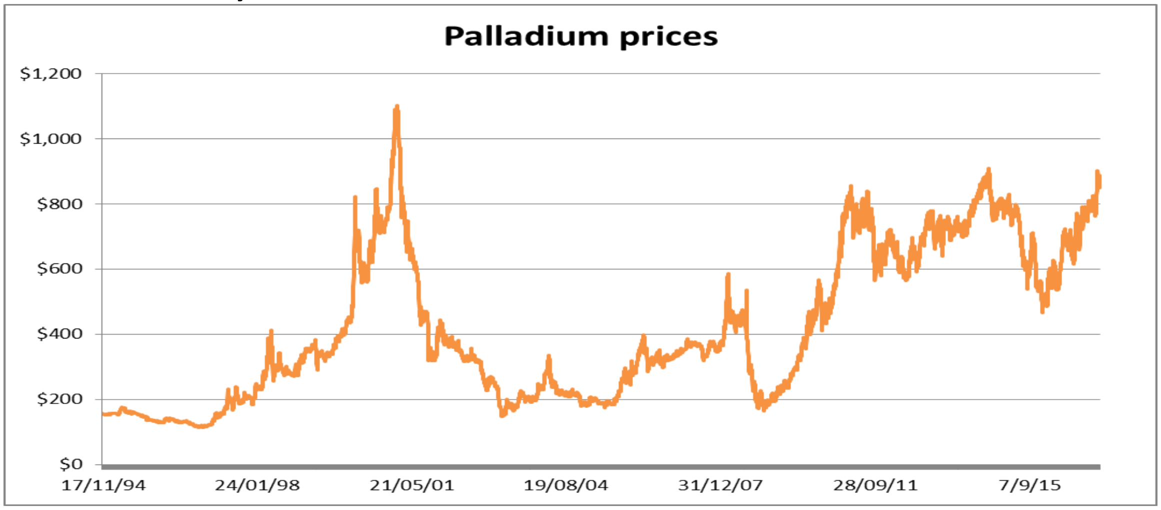


**Figure 1**: Daily palladium spot prices

We collected the data from the London Platinum and Palladium Market (LPPM). The LPPM is the most important over-the-counter trading market for platinum and palladium and one of the world's major commodity trading associations. The trade in LPPM was established in the early 20th century, typically by existing dealers of gold and silver. The data includes the period from November 17, 1994 to June 30, 2017 and in total 6459 observations. Figure 1 illustrates the daily palladium spot prices in the LPPM. We can the palladium spot prices have never researched the peak level of $1102.5 per ounce in January 27, 2001 in the last decade. Figure 2 illustrates the dynamics of the palladium spot returns. There are significant volatility clustering phenomenon and two huge positive and negative spikes are observed in the recent financial crisis.

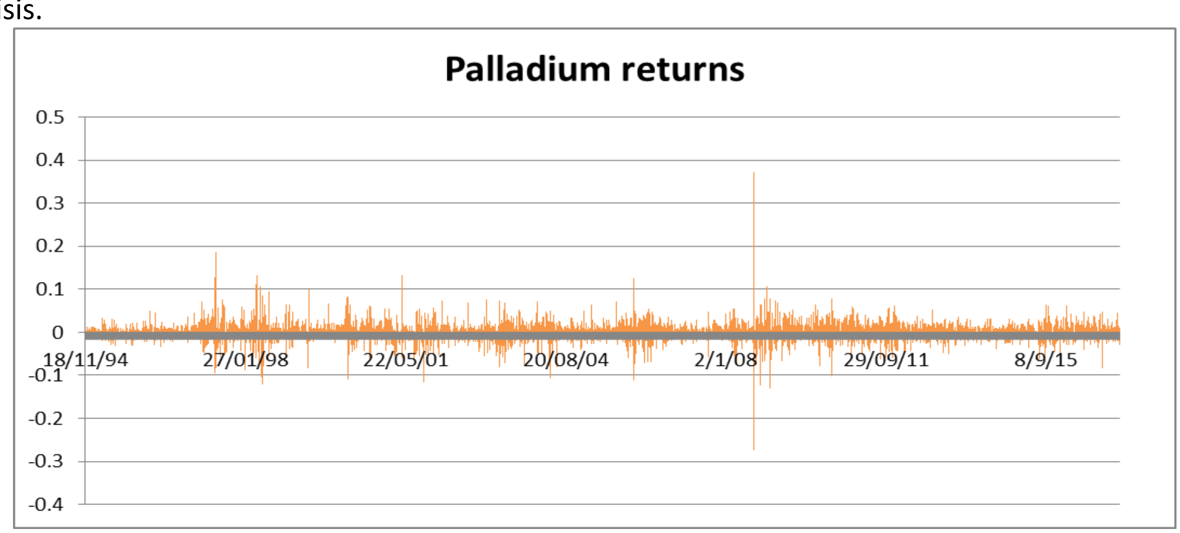


**Figure 2**: Daily palladium spot returns

The summary statistics of the data is presented in Table 1. The data present the standard set of well-known stylized facts of asset prices series: non-normality, limited evidence of short-term predictability and strong evidence of predictability in volatility. All series are presented in daily percentage growth rates/returns. The Bera–Jarque test conclusively rejects normality of raw returns in all series, which confirms our assumption that the model selected should account for the heavy-tail phenomenon. The smallest test statistic is much higher than the 5% critical value of 5.99. The market index is negatively skewed and has fat tails. The asymptotic SE of the skewness statistic under the null of normality is $\sqrt{6/T}$, and the SE of the kurtosis statistic is $\sqrt{24/T}$, where $T$ is the number of observations. The data exhibits statistically significant heavy tails.

| *Series* | *Obs.* | *Mean* | *Std.* | *Skewness* | *Kurtosis* | *BJ* | *Q(5)* | *$Q^{ARCH}$(5)* | *$Q^2$(5)* |
|---|---|---|---|---|---|---|---|---|---|
| Palladium spot returns | 6459 | 0.05% | 2.01% | 0.66* | 28.84** | 91.7** | 10.14* | 7.13* | 41.28** |

**Table 1:** Summary statistics. BJ is the Bera-Jarque statistic and is distributed as chi-squared with 2 degrees of freedom, Q(5) is the Ljung-Box Portmanteau statistic, $Q^{ARCH}$(5) is the Ljung-Box Portmanteau statistic adjusted for ARCH effects following Diebold (1986) and $Q^2$(5) is the Ljung-Box test for serial correlation in the squared residuals. The three Q statistics are calculated with 5 lags and are distributed as chi-squared with 5 degrees of freedom.
* and ** denote a skewness, kurtosis, BJ or Q statistically significant at the 5% and 1% level respectively.

We use the Ljung-Box portmanteau, or Q, statistic with five lags to test for serial correlation in the data, and adjust the Q statistic for ARCH models following Diebold (1986). The results that no serial correlation is found confirm our assumption of a constant mean return $\mu$ in Equation (2.1). The evidence of linear dependence in the squared demeaned returns, which is an indication of ARCH effects, is significant.

## 4. Estimation Results

We estimate the GARCH(1,1) model with the heavy-tailed distributions by maximizing the log-likelihood function of equation:

$$\hat{\theta} = \arg\max_{\theta} \sum_{t=1}^{T} \log(f(\varepsilon_t \mid \varepsilon_1, \cdots, \varepsilon_{t-1})) \quad . \tag{4.1}$$

Table 2 reports estimation results of the GARCH(1,1) model with the two types of heavy-tailed distribution for all the daily palladium spot return series. We also include the normal distribution as the benchmark statistical distribution. All the parameters are significantly different from zero. There results show that it is crucial to introduce heavy-tailed distributions into the GARCH framework and the Student's *t* distribution has the best in-sample performance. Since the two distributions has the same number of parameters, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) also indicate the Student's *t* distribution has best empirical performance.

| | alpha1 | beta1 | 1/nu (1/alpha) | log-likelihood | AIC | BIC |
|---|---|---|---|---|---|---|
| **Normal** | 0.052** | 0.897** | | -12486 | 24979 | 24999 |
| **Student's t** | 0.041** | 0.901** | 0.159** | -12115 | 24238 | 24265 |
| **NRIG** | 0.047** | 0.905** | 0.629** | -12172 | 24352 | 24379 |

**Table 2:** Estimation of the GARCH model with heavy-tailed innovations
* and ** denote statistical significance at the 5% and 1% level respectively.

## 5. Conclusion

In the recent decades, the world demand of palladium has increased dramatically. However, its spot price still has not reached the peak level observed in 2001. In 2008, a single-day increase of the palladium spot price has exceeded 37%, which indicates significant risk for investments in the world palladium market. In this paper, we apply the GARCH model with heavy-tailed distributions into the palladium spot returns series for risk management purpose. Our results show the newly-developed distribution, the NRIG, cannot outperform the older fashion one, the Student's *t* distribution. Nevertheless, our results do demonstrate that it is important to incorporate conditional heavy tails for precious metals' spot return modelling.

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