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Mnemonics of Basic Differentiation and Integration for Trigonometric Functions

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Abstract
Calculus has always been issue of underachievement performance for public university students. Insufficient background of SPM Additional Mathematics affected the students' performance in Mathematics at university level. One of the main reasons that contribute to this problem is the lack of understanding of the basic concept in differentiation and integration. Two innovative techniques of basic differentiation and integration for trigonometric functions by using mnemonic chart are developed in this paper. Many students face difficulties to memorize the formulas which unable for them to solve the given problems. These proposed techniques minimize students' problem in memorizing formulas as well as improve teaching method for educators. In addition, these new technique is not only can be advantageous for university students in Calculus course, but also for Matriculation and STPM students in Mathematics subject. This technique emphasizes square and triangle shape to be used for Original Rule and Chain Rule. All formulas can be derived from clockwise and anticlockwise direction. In the future, further research need to be done whether mnemonics are effective and useful to academic programmes such as Science- based programmes and also for secondary high schools.
Keywords: Differentiation, Integration, Calculus, Mnemonics, Trigonometric Functions

1.0 Introduction

There are some concerns of many lecturer on the passing rate for Calculus course on full-time diploma students'. According to Eng et al. (2008), Calculus 1 is one of the high failure rate course among other Mathematics courses for students of Universiti Teknologi MARA (UiTM). He also reported that, one of the factors that contribute to the high failure rate in this course is due to the lack of understanding the basic concept of Calculus. Suresh (2003) mentioned that, the high failure rate course for engineering students is Calculus followed by Physics and Statistics. As stated in Salleh & Zakaria (2011), the major reason to the decline in students' performance in Calculus is the existing gap of Mathematics knowledge at university level. This gap is due to the deterioration of Mathematics performance at secondary schools and the mismatch of teaching and learning culture between secondary schools and university. As traditional methods of teaching Mathematics have been found to be ineffective due to lack of performance in certain topics of Mathematics, an innovative change needs to be implemented. According to Eng et al. (2010) & Eng et al. (2013), the lecturer must explore and be creative in teaching instead of simply giving them the solutions to mathematical problems, lecturers could train their students to actively work for alternative solutions which help them to think creatively.

Due to the lack performance in certain topic as mentioned in Eng et al. (2013), this study will focus on improving the way that students memorize formulas of basic differentiation and integration for Calculus 1 course at university level and also for Mathematics subject at Matriculation and STPM level. Under the subtopic of basic differentiation and integration for trigonometric functions, most of the students rarely able to remember all the formulas effectively. Students do not have the interest and face difficulties to memorize the formulas. In traditional way, student have to memorize the formula given in textbook and this have been practiced on campus over the years before. As an educator, genuine desire to minimize students' difficulties in learning Calculus is strongly high. In UiTM Tapah Campus, the challenge facing those who charged with teaching Calculus course to Computer Science and Applied Science's student.

There has been many research studied on importance of diagram in the solution of mathematical problem (Diezmann & English, 2001; Norvick & Hurley, 2001). Diagrams has been identified as one of the most effective strategies that have been proposed to improve efficiency in mathematical problem solving (Hembree, 1992; Uesaka, Manalo & Ichikawa, 2007). Generating diagrams to solve mathematics problems can help learners in numerous ways (Stylianou, 2010). Visual mnemonic is one of the techniques that present the information using diagram. The role of mnemonic in learning mathematics is memory enhancing instructional strategy that provide a visual or verbal prompt for students who may have some difficulty to remember important information (DeLashmutt, 2007).

This paper proposed two new innovative techniques to memorize formula of basic differentiation and integration for trigonometric functions by using mnemonic chart. Hence, this is an alternative way which more interactive instead of memorize the formulas given in the textbook. The objective of this paper are:

1) To develop mnemonics of basic differentiation and integration for trigonometric functions.
2) To improve method of learning and teaching for students and educators respectively.

2.0 Literature Review

Research interest in mnemonics has been continually increasing since the 1960s. Masachika Nakane has developed a wide range mnemonics namely "Yodai" which means "the essence of structure". The yodai system consist of verbal mediators (phrases, sentences, rhymes, song and etc). This system has been used in learning mathematics (arithmetic, algebra, trigonometri, geometry, and calculus), science (organic chemistry, physics, and biology) and English (McDaniel & Pressly, 1987).There are numerous types of mnemonics such as visulatizations, keyword, acronym, acrostics, peg word, loci and etc (Heather & Gibson, 2009). Mnemonic have been used in many disciplines.

DeLashmutt (2007) has investigated the effective of mnemonic in classroom. He has discovered mnemonic may serve important tools in classroom. Many teachers found that there are many benefits of the mnemonic such as students can memorize the concept quickly and able to retain the information for longer period. Scruggs & Mastropieri (1991) found that the students preferred mnemonic instruction over traditional instructional methods. This is one of learning strategies to improve students' memory. This mnemonic learning strategies also have been applied for students with learning and behavioral problems. It is because the main problem this student have is memory for academic content (Scruggs & Mastropieri, 2000). Mnemonic has well-known applied in subject chemistry. With the help of mnemonic, many chemistry aspects, sequence of elements, rules, names of compounds and their reactivity can be easily memorized.

About 53% Psychology students used mnemonic in preparing final examination to sustain examination performance (Gruneberg, 2006). Schoen (1996) have developed monopoly board game that applied the mnemonic system which called *mnemopoly.* This game is to created loci visualization for introductory physiological students. He has compared the recollection of words either *mnemopoly* with the method of loci or phonetic peg system. By using the mnemonic, the subject involving verbal material is easily to remember through visual imaginary (Bellezza & Reddy, 1978).

Within nursing or medication education, mnemonic devices is important for nurses and nursing students to learn about disease process. Gibson (2009) studied the important of mnemonic in nursing in order to assist student to understand the nursing framework effectively. He found that by applying mnemonic in nursing schools have increased the students' interest and confident level to express the concept in the organizing framework. Fernandes & Speer (2002) created a mnemonic to aid learners in memorizing sequential information regarding neonatal resuscitation.

In addition, mnemonic have been utilized by many professional mathematics. There are some visual aid study in trigonometric functions has been produced to help student visualize the relationship of the change in the function as the angle change (Shimberg, 1934 & Henry, 1950). There are three methods of teaching mnemonics has been introduced by DeLashmutt (2007): keyword, pegword and letter strategies. He used keyword mnemonic to place numerator and denominator for fraction number. While for pegword mnemonic is useful for improper fraction. Another example of teaching mnemonic in mathematics is mnemonic chart as in Figure 1 that written by anonymous. This is to help students to remember abundance of Trigonometric

Identities. This technique contain hexagonal figure with function one side and co-function on the other side and a 1 in the middle. The mnemonic also has been used in spherical and hyperbolic trigonometric consist of formula relating to side and angle of a triangle (Conway & Ryba, 2016).

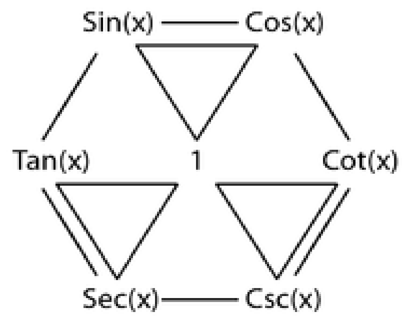


Fig. 1: Mnemonics chart for trigonometric identity (Magic Hexagon)

In recent years, several of teaching and learning strategies to improve students' difficulties in learning Calculus have been extensively studied by many researchers and educators (Eng et al., 2013). The traditional or innovative methods of teaching are critically examined, evaluated and some modifications in the delivery of knowledge is suggested. As such, the strengths and weaknesses of each teaching methodology are identified and probable modifications that can be included in traditional methods are suggested. The innovation technique brings several benefit. The students are revived from their passivity of merely listening to a lecture and instead become attentive and engaged.

3.0 Methodology

Presented in this section the traditional and innovative technique to solve basic of differentiation and integration for trigonometric functions. Two innovative techniques are discussed thoroughly. First technique is for the Original Rule while the second technique for the Chain Rule.

3.1 Traditional Technique

In this section, the review of trigonometric functions is explained in details. Recall, the derivative of function f defined for all real numbers x by

$$f(x) = \sin(x).$$

The function $\sin(x)$ means the sine of the angle whose radian measure is x. Similar to the other trigonometric functions cos, tan, csc, sec and cot. The derivative of $\sin(x)$ can be solved by using definition of differentiation. The result is getting by calculating the limit as h approaches to zero.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\left(\frac{\sin(x+h) - \sin(x)}{h}\right).$$

Using angle formula $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ we have

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h},$$
$$= \lim_{h\to 0}\left[\frac{\sin(x)(\cos(h) - 1)}{h} + \frac{\sin(h)\cos(x)}{h}\right],$$
$$= \lim_{h\to 0}\left[\frac{\cos(h) - 1}{h}\bullet\sin(x) + \frac{\sin(h)}{h}\bullet\cos(x)\right].$$

On the basis of numerical and graphical evidence, that

$$\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1 \text{ and } \lim_{\theta\to 0}\frac{\cos(\theta) - 1}{\theta} = 0.$$

Hence,

$$= \lim_{h\to 0}\left[0\times\sin(x) + 1\times\cos(x)\right] = \cos(x).$$

So, we have proved the formula for the derivative of the sine function:

$$\frac{d}{dx}\sin(x) = \cos(x).$$

The derivative of the remaining trigonometric functions also can be obtained by using the same procedure as differentiation for sin(x) by using definition of differentiation. The summarization of all differentiation formulas of trigonometric functions as given in Table 1. This only valid when x is measured in radian.

Differentiation	Integration
$\frac{d}{dx}\sin(x)=\cos(x)$ $\frac{d}{dx}\cos(x)=-\sin(x)$	$\int\cos(x)dx=\sin(x)+c$ $\int\sin(x)dx=-\cos(x)+c$
$\frac{d}{dx}\tan(x)=\sec^2(x)$ $\frac{d}{dx}\sec(x)=\sec(x)\tan(x)$	$\int\sec^2(x)dx=\tan(x)+c$ $\int\sec(x)\tan(x)dx=\sec(x)+c$
$\frac{d}{dx}\cot(x)=-\csc^2(x)$ $\frac{d}{dx}\csc(x)=-\cot(x)\csc(x)$	$\int\csc^2(x)dx=-\cot(x)+c$ $\int\cot(x)\csc(x)dx=-\csc(x)+c$

Table 1: Differentiation and integration formula for trigonometric function

Whenever the radian measure is no longer as *x,* suppose that $y=\sin(u)$, where u is a differentiable function of x, then by the Chain Rule,

$$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}=\cos(u)\frac{du}{dx} \; .$$

Thus,

$$\frac{d}{dx}\sin(u)=\cos(u)\frac{du}{dx} \; .$$

For example, given $u=2x$, $\frac{du}{dx}=2$.

Thus,

$$\frac{d}{dx}\sin(2x)=\cos(2x)\bullet 2=2\cos(2x) .$$

Note that, by using Chain Rule, the derivative of outer function is multiply by the derivative of the inner function.

In this paper, an alternative way to memorize formulas in Table 1 is developed by using mnemonic chart. This study aid, offer more interactive and mistake-free whenever students loss their memory in remembering the formulas.

3.2 Innovative technique

Two innovative techniques of differentiation and integration for trigonometric function are designed in this section. First technique are developed when x is measured in radian (Original Rule) and second technique for the Chain Rule when the radian is no longer as x but defined as u.

3.2.1 Mnemonic for Original Rule

The first proposed technique for the case radian measure is x, for example $f(x) = \sin(x)$. The mnemonic technique for original rule are shown in Figure 2. The process of derivation the formulas are discussed in details.

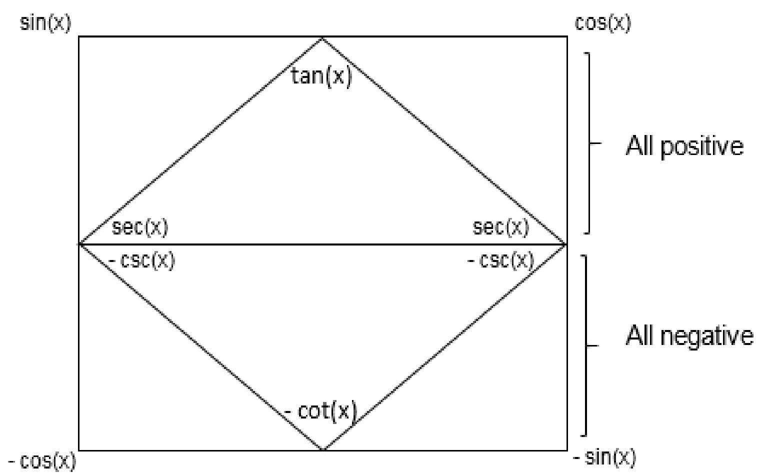


Fig. 2: Mnemonic of basic differentiation and integration for Trigonometric Function (Original Rule)

Step 1: Draw a square and two triangles inside the square.

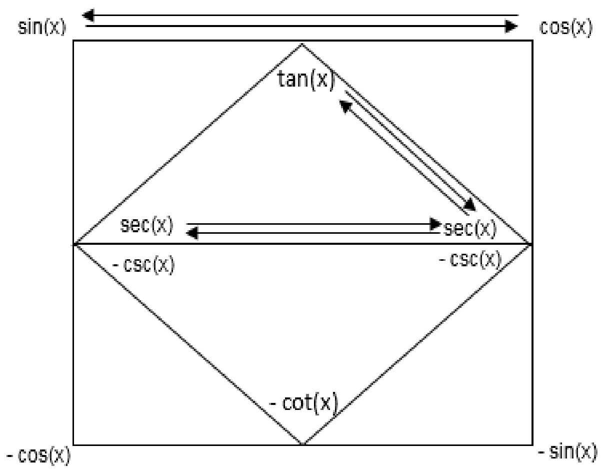
Step 2: We divided into two parts. Positive sign for the top-half square and top triangle while negative sign for the bottom-half square and bottom triangle.

For the top: From the left, the function sine of x and cosine of x are placed at the top corner of square. While the inside triangle we placed tangent of x at the top corner and secant of x at the right and left corner.

For the bottom: From the left, the function negative cosine of x and negative sine of x are placed at the bottom corner of square. While the inside triangle we placed negative cotangent of x at the bottom corner and negative cosecant of x at the right and left corner.

Step 3: We can get differentiation formulas by clockwise direction. Remember, for the triangle, we must differentiate for single function only.

Step 4: We can get integration formulas by counterclockwise direction. However, for the triangle, remember, we must integrate product of two functions. The summarization of step 3 and 4 can be visualized in Figure 3 as follows:



$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$
$$\int \cos(x)dx = \sin(x) + c$$
$$\int \sec^2(x)dx = \tan(x) + c$$

Fig. 3: Mnemonic Process of basic differentiation and integration for Trigonometric Function (Original Rule)

The collection of the differentiation and integration formulas given in Table 2.

Differentiation (Clockwise)	Integration (Anti-clockwise)
$\frac{d}{dx}\sin(x)=\cos(x)$ $\frac{d}{dx}\cos(x)=-\sin(x)$ $\frac{d}{dx}-\sin(x)=-\cos(x)$ $\frac{d}{dx}-\cos(x)=\sin(x)$	$\int\cos(x)dx=\sin(x)+c$ $\int\sin(x)dx=-\cos(x)+c$ $\int-\cos(x)dx=-\sin(x)+c$ $\int-\sin(x)dx=\cos(x)+c$
Differentiation (differentiate for single function only)	Integration (integrate of 2 function)
$\frac{d}{dx}\tan(x)=\sec^2(x)$ $\frac{d}{dx}\sec(x)=\sec(x)\tan(x)$	$\int\sec^2(x)dx=\tan(x)+c$ $\int\sec(x)\tan(x)dx=\sec(x)+c$
$\frac{d}{dx}\text{-}\cot(x)=\csc^2(x)=\frac{d}{dx}\cot(x)=\text{-}\csc^2(x)$ $\frac{d}{dx}\text{-}\csc(x)=\text{-}\cot(x).\text{-}\csc(x)$ $=\frac{d}{dx}\csc(x)=\text{-}\cot(x)\csc(x)$	$\int\csc^2(x)dx=-\cot(x)+c$ $\int-\cot(x).-\csc(x)dx=-\csc(x)+c$ $=\int\cot(x)\csc(x)dx=-\csc(x)+c$

Table 2: Formulas of basic differentiation and integration for trigonometric functions

3.2.2 Mnemonic for Chain Rule

The second proposed technique for the case radian measure is not x but defined as u, where u is a differentiable function of x, for example $f(u)=\sin(u)$. The mnemonic technique for Chain Rule is shown in Figure 4. The process of derivation the formulas are discussed in details.

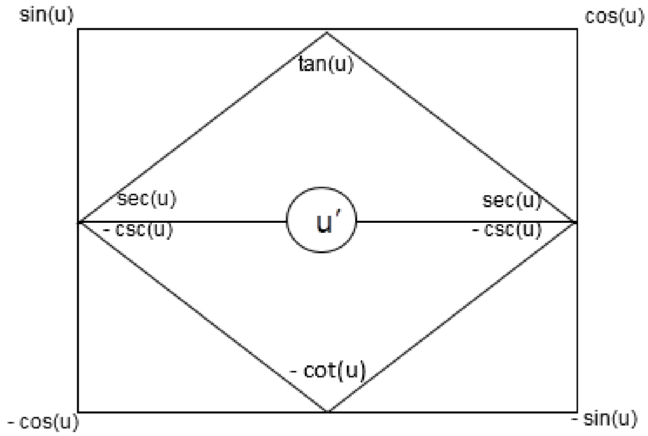


Figure 4: Mnemonic of basic differentiation and integration for trigonometric functions (Chain Rule)

Step 1 and **Step 2** follow the previous steps in original rule but now we write the functions in terms of u. As example sine of u (no longer in terms of *x*)

Step 3: Follow the previous step 3 but in addition we need to **multiply** the derivative of the given function with the u'. The arrow of u' is moving clockwise direction (above), it means we need to multiply the function with u'.

Note: **Clockwise – Differentiation – Multiply**

$$\frac{d}{dx}\sin(u) = \cos(u) \bullet u'$$

Step 4: Follow the previous step 4 but in addition we need to **divide** the derivative of the given function with the u'. The arrow of u' is moving counterclockwise direction (below), it means we need to divide the function with u'.

Note: **Counterclockwise – Integration – Divide**

$$\frac{d}{dx}\sin(u) = -\frac{\cos(u)}{u'} + c$$

The summarization of the step 3 and 4 can be seen in Figure 5.

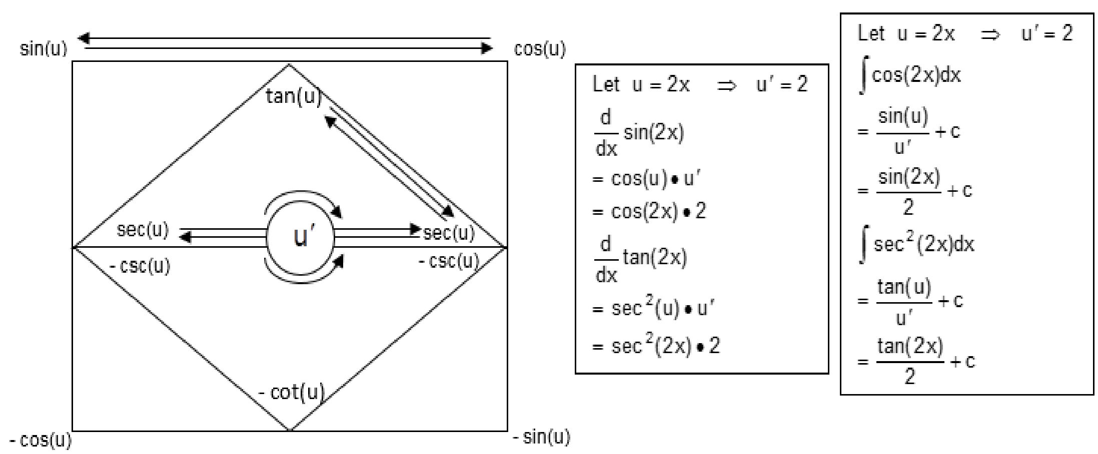


Fig. 5: Mnemonic process of basic differentiation and integration for trigonometric functions (Chain Rule)

4.0 Conclusion and Recommendation

Mnemonic device involve reorganizing information and advantageous for students to become more active learners. If students master the practice on how it works correctly, they can solve the given problem successfully. Weak students and excellent students can perform well equally. Differentiation and integration for trigonometric functions is not only studied by university calculus student but also in Mathematics subject for Matriculation and STPM students. This technique can be advantageous for wide range level of education and in the hope it will be widely used in area of education.

One of the goals of this study is to provide alternative technique which is more interactive for learning and teaching strategy. Indirectly, this would increase students' comprehension test score. On the recommendation, further research need to be conducted to see whether this proposed techniques are useful and effective in learning for first year diploma students(i.e: science-based programmes) and high school students.

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