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Applications of Chaos and Fractal Theory on Emerging Capital Markets

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Abstract:
The analysis of capital markets efficiency has attracted a considerable number of studies in empirical finance, but conflicting and inconclusive outcomes have been generated. The aim of this paper is to find any evidence that the selected emergent capital markets (eight emergent European and BRIC markets, namely Hungary, Romania, Estonia, Czech Republic, Brazil, Russia, India and China) abide by a particular evolution pattern (long range dependence) or the random walk hypothesis. In view of attaining the goal of the paper, we employed a methodology based on the interdisciplinary approach to the subject matter under investigation and applied the deterministic chaos and fractal theory. In this paper, the Hurst exponent calculated by the rescaled-range analysis is our measure of long range dependence in the series. We use a "rolling sample" approach to evaluate the Hurst exponent. The results suggest that this fractal exponent may be useful in assessing the stage of stock market inefficiency.

Keywords: chaos, fractal, Hurst exponent, R/S analysis

JEL Classification codes: G10, G14, C58

Introduction

According to the classical theory, phenomena in the fields of economics and finance are characterized by a kind of mechanism subject to accurate measurement, prediction and control. On the other hand, chaos is a deterministic system defined by complex behavior relying on apparently random interaction among elements.

Chaos theory has been first mentioned in Henri Poincaré`s attempts of mathematical modeling of the instability of mechanical systems (in the 1880s). In 1898, the French mathematician, Jacques Hadamard, published a relevant study of the chaotic movement of a particle pointing out that the instability of all its trajectories is given by their exponential

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deviation from one another, with a positive Lyapunov exponent[2]. Edward Lorenz, a meteorologist, was another scientist who experimented with chaos theory. In 1960, while he was working on a weather prediction problem, he noticed that even a slight change to the initial conditions is likely to change dramatically the long-term behavior of a system (this sensitive dependence on initial conditions is also known as the *butterfly effect,* a widely used concept in chaos theory). He introduced a homonymous concept – *Lorentz attractor*[3] – where attractor refers to a set of points in the area of stages leading to the system trajectories, and the attracting pool defines a set of initial conditions leading where the trajectories converge to an attractor. Lorenz`s discovery highlighted that it is downright impossible to render an accurate long-term weather prediction, however it also enabled him to come across other issues later known as chaos theory; it mainly refers to the fact that complex systems seem to run through certain cycles of events (notwithstanding the difficulty of acknowledging a pattern in chaotic systems) although the events are rarely repetitive and identically rendered.

We consider it appropriate to compare the financial market to a fractal since a price graph analysis over a given time span will show their similar structure. Just like the fractal, however, the financial market evinces a sensitive dependence to initial conditions which makes the dynamic market systems hard to predict. It is our opinion that although a system may evince short-term unpredictability, it may however become a long-term deterministic system.

The theory and models regarding capital market operation have initially developed from the assumption that these represent efficient markets. Rational agents quickly assimilate any kind of information that proves relevant to asset pricing and their output and subsequently adjusts the price in accordance with this information. By way of explanation, agents do not benefit from different comparative advantages in the process of information acquisition. To sum up, the efficient market hypothesis refers mainly to three fundamental and highly controversial concepts: efficient markets, random trajectories, rational agents.

Non-linear dynamic systems have also triggered an interest in fractals which have come to be regarded as the geometry of chaos perceived as a non-linear and deterministic dynamic system enabling the emergence of random, unpredictable results. Mandelbrot was the first one to notice the potential use of Brownian motion in the study of motion in other fields, such as: price motion on financial markets. The author introduces the concept of fractal motion as a generalization of Brownian motion (Zerfus 1999). The new capital market theory combines fractals and other concepts from chaos theory with quantitative traditional methods in order to explain and predict market behavior. The fractal market theory (Fractal Market Hypothesis – FMH) (Peters 1994, 1996) is a rendering of efficient markets where the focus is placed on market stability however, instead of market efficiency. Thus, it takes into account the everyday market randomness as well as market anomalies.

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[2] This exponent represents the rate of exponential divergence of two neighboring trajectories. The Lyapunov exponent indicates the extent of trajectory divergence considering the small observation errors, disturbances or differences. A chaotic system requires a positive Lyapunov coefficient.

[3] It refers to the system presentation as a graph showing a given status that should be reached by the system, some kind of balance.

The paper is structured as follows. Section 2 presents the literature review on market efficiency, section 3 describes the methodology used in view of attaining the goal of this paper, the methods we used for the Hurst exponent estimation. In section 4, we describe the data and section 5 covers the empirical results and discusses the implications whether the selected markets are subject to a specific evolution pattern or the random walk hypothesis. Section 6 concludes.

1. Literature review

The literature on market efficiency is vast as the theme is of great interest for both practitioner and academics. Since it is a very intriguing issue, a big part of this literature focuses on seeking long memory dependence in asset returns. The analysis of long memory processes in capital markets has been one of the topics in finance, since the existence of the market memory could implicate the rejection of an efficient market hypothesis. Actually, if the stock returns present long range dependence, the random walk hypothesis is not valid anymore and neither does the market efficiency hypothesis. Cajueiro and Tabak (2004) tested for long-range dependence and efficiency in stock indices for 11 emergent markets along with US and Japan. They adopted a “rolling sample” approach and calculated median Hurst exponents to assess relative efficiency of these equity markets. They suggested that Asian equity markets show greater inefficiency than those of Latin America and developed markets rank first in terms of efficiency.

Wang et al. (2011) gave a demonstrative research in Chinese stock markets by stable software. The results showed that the Chinese stock markets had obviously fractal characteristics and the fractal market hypothesis can deal with this problem very well, and not the efficient market hypothesis.

Granero (2008) et al. (studied the existence of the market memory through Hurst exponent and the R/S analysis. In their paper they discussed the efficiency of this methodology as well as some of its more important modifications to detect the long memory. They also proposed the application of a classical geometrical method with slight changes and they compared both approaches.

Mitra (2012) estimated Hurst exponent of twelve stock index series from across the globe using daily values of for ten years and found that the Hurst exponent value of the full series is around 0.50 confirming market efficiency. But the Hurst exponent value was found to vary widely when the full series was split into smaller series of 60 trading days. Later on, they tried to find a relationship between Hurst exponent value and profitable trading opportunity from these smaller series and found that periods displaying high Hurst exponent had potential to yield better trading profits from a moving average trading rule.

Kale and Butar (2011) made a fractal analysis by conducting rescaled range (R/S) analysis of time series. Simulation study was run to study the distribution properties of the Hurst exponent using first-order autoregressive process.

Carbone et al. (2004) calculated the Hurst exponent of time series by dynamical implementation of a scaling technique: the detrending moving average (DMA). They calculated the exponent Hurst for artificial series, simulating monofractal brownian paths, with assigned Hurst exponents. They next calculated the exponent H for the return of high-frequency series of the German market. They found a much more pronounced time-variability in the local scaling exponent of financial series compared to the artificial ones.

In their paper, Steeb and Andrieu (2005), considered nonlinear dynamical systems with chaotic and hyperchaotic behavior. They investigated the behavior of the Hurst exponent at the transition from chaos to hyperchaos. A two-dimensional coupled logistic map was studied.

Although many methods have been proposed to deal with the determination of Hurst exponent, Cajueiro and Tabak (2004) stated that none of them are suitable for any time series and sometimes when applied to the same time series present conflicting results. In this context, in their paper they present a new method based on the rescaled variance statistics which can be used efficiently to this end.

Matos et al. (2008) used a method of studying the Hurst exponent with time and scale dependency. This approach allowed them to recover the major events affecting worldwide markets and analyze the way those effects propagated through the different scales. The time-scale dependence of the referred measures demonstrates the relevance of entropy measures in distinguishing the several characteristics of market indices: "effects" include early awareness, patterns of evolution as well as comparative behavior distinctions in emergent/established markets.

2. Methodology

2.1. The R/S fractal analysis method – rescaled range analysis

This method, applicable to the study of financial markets, relies on the concept of the Hurst exponent. It was introduced by English hydrologist H.E. Hurst in 1951, based on Einstein's contributions regarding Brownian motion of physical particles, to deal with the problem of reservoir control near Nile River Dam. Hurst (1965) proved that the dynamics of many natural phenomena is given by a randomly changed law. If the data series were perfectly random then these value series would increase simultaneously with the square-root of time enhancement (the $\sqrt{T}$ rule). Hurst introduced the non-dimensional ratio, dividing the data series to the standard deviation of observations to the average, in view of enabling comparison of data selected at different time moments. The result was a redimensioning of the scale hence the method is also called the *R/S* rescaled range analysis. R/S analysis in economy was introduced by Mandelbrot (1972), who argued that this methodology was superior to the autocorrelation, the variance analysis and to the spectral analysis.

In this paper, the Hurst exponent calculated by the R/S analysis is our measure of long range dependence in the series. The R/S method (range / standard deviation) requires an initial dynamic series, standing for the evolution of a natural phenomenon or process.

Let P_t be the price of a stock on a time t and r_t be the logarithmic return denoted by the first difference of logarithmic values of daily prices:

$$r_t = d\log(P_t) = \log(P_t) - \log(P_{t-1})$$
(1)

As financial time series display high degree of non-stationarity, it is a common practice to work with first differenced series than with the original series. Therefore in the first step, we reduce non-stationarity by converting the original series to a returns series taking logarithm returns from successive values of the series.

The R/S statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. The entire data series is divided into several contiguous sub-periods each having *n*-observations and define each sub-period as I_a, $a = 1, 2, \ldots, k$, so $k \cdot n = N$. For each sub-period, the average value of the sub-period is determined. Starting from this assumption, we may determine the following dimension (considering that the series is divided into *a* sub-periods of *n* range):

$$X_{n,a} = \sum_{i=1}^{n} (r_i - \overline{r_{n,a}}) \qquad (2)$$

where $X_{n,a}$ represents the cumulative deviation for each I_a sub-period; r_i represents the *i* component of the dynamic series; $r_{n,a}$ represents the r_i value average on every I_a sub-period.

The range (R) of the cumulative trend adjusted return series for each sub-period I_a is measured by taking differences of maximum and minimum values of $X_{n,a}$:

$$R_{n,a} = Max(X_{n,a}) - Min(X_{n,a}) \qquad (3)$$

Hurst divided the *R* value to the standard deviation of initial observations for each I_a sub-interval, in view of comparing various dynamic series, for instance:

$$S_{n,a} = \sqrt{\frac{\sum_{i=1}^{n}(r_i - \overline{r_{n,a}})^2}{n}} \qquad (4)$$

For every I_a sub-interval, the ratio given below is further determined:

$$(R/S)_{n,a} = \frac{Max(X_{n,a}) - Min(X_{n,a})}{\sqrt{\frac{\sum_{i=1}^{n}(r_i - \overline{r_{n,a}})^2}{n}}} \qquad (5)$$

As there are many contiguous sub-periods, the average R/S value of full series is calculated by averageing R/S values of all individual sub-periods:

$$(R/S)_n = \frac{1}{k} \cdot \sum_{a=1}^{k} (R/S)_{n,a} \qquad (6)$$

The n length is increased and the procedure is repeated until $n = (N-1)/2$.

Hurst noticed that this ratio increases as the number of observations in the initial data series enhances. If the data series were perfectly random, then the ratio would increase proportionally with the square root $\sqrt{N}$of the number of observations. Brownian motion is the primary model for a random walk process. Einstein (1908) found the distance a particle covers increases with respect to time according to the following relation:

$$R = T^{0.5} \qquad (7)$$

where R is the distance covered by the particle in time T (see Peters 1994, 1996).

We can use equation (7) only if the time series we are considering is independent of increasing values of T. To take into account the fact that economic time series systems are not independent with respect to time, Hurst (1965) found a more general form of equation (7). In order to measure the R/S ratio enhancement in dependence on the phenomenon observation time, Hurst employed the following relation:

$$(R/S)_n = c \cdot n^H \qquad (8)$$

where c is a proportionality constant and H represents the Hurst exponent.

Afterwards, the value of Hurst exponent (H) is established by means of regression:

$$\log(\mathrm{R/S})_{\mathrm{n}} = \log(c \cdot n^H) = \log(c) + H \cdot \log(n) \qquad (9)$$

2.2. Hurst exponent: dynamically analysis

Since market efficiency (predictability) seems to evolve over time (Cajueiro and Tabak 2005), we measure this exponent dynamically. In this paper, we use a “rolling sample” approach to evaluate the Hurst exponent and to test for the existence of long-term linear dependence in the stock market volatility. In this framework, we adopt a rolling window of N = 100 observations to compute the extent of inefficiency. The fixed sized window moves one observation at a time and so the calculation of H statistic at each period reveals the behavior of stock market efficiency over time. Since this approach makes available several Hurst exponents for each stock market and it is impossible to compare all of them, then we make statistical inference with the means, medians and other statistical measures of these Hurst exponents in order to compare their market inefficiency degree.

Hurst found that the rescaled range, R/S, for many records in time is very well described by the following empirical relation:

$$(R/S)_n = (n/2)^H \tag{10}$$

Lo (1991) has revealed that the R/S statistic is sensitive to short-range dependence in financial time series. In order to avoid any short-range dependence in mean and in variance of the stock returns, we implemented the rolling R/S statistic on the adjusted time series. More precisely, the original daily returns are filtered in order to avoid any short-range dependence. In line with some previous studies including Cajueiro and Tabak (2004, 2005) and others, we employ the GARCH (1, 1) model in order to filter the stock return time series. Consequently, we implemented the R/S statistic on the standardized residuals. GARCH (1,1) residual can be used to eliminate or reduce linear reliance degree, i.e. autocorrelation. Because linear reliance will deviate from Hurst exponent *H* or easily lead to the first kind of error. By taking the GARCH (1,1)'s residual, we can reduce deviation degree and this may hopefully reduce insignificant degree of the results. This process is usually called the pre white noise treatment or trend expunction method.

The model can be used successfully in volatile situations. GARCH model includes in its equation both terms and the phenomenon of heteroskedasticity. It is also useful if the series are not normally distributed, but rather they have "fat tails". No less important is that confidence intervals may vary over time and therefore more accurate intervals can be obtained by modeling the dispersion of residual returns (Opreana et all, 2012).

GARCH (*Generalized Autoregressive conditional heteroskedasticity*) was proposed by T. Bollereslev in 1986 in the Journal of Econometrics. GARCH (1,1) model includes an equation for mean and one for dispersion, respectively:

$$r_t = \Upsilon x_t + \varepsilon_t \tag{11}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{12}$$

where:

r_t – dependent variable in the current period;

x_t – independent variable in the current period;

Υ – coefficient that shows the influence of the independent variable on the dependent variable;

ε_{t-1} – the residuals in the previous period = news about volatility from the previous period, measured as the lag of the squared residual from the mean equation. It is the "ARCH" term;

σ_{t-1}^2 – variance of the dependent variable in the previous period = last period's forecast variance. It is the "GARCH" term.

In this model and in our study, parameter restrictions are:

- ensure positivity variance: $\omega, \alpha, \beta > 0$;
- reversion to the mean if $\alpha + \beta < 1$. Then the long run variance is $\omega / (1 - \alpha - \beta)$;
- estimation of the parameters by "maximum likelihood".

In the GARCH (1,1) model, described above, the α parameter shows that the residual terms of the previous period acts on dispersion and the β parameter shows that the dispersion of the previous period has influence on current dispersion. In fact, for very large series, GARCH (1,1) can be generalized to GARCH (p, q).

3. Data

The data of this survey represent daily stock market quotes of the most traded indices in the eight emergent capital market countries, four UE countries and the four BRIC countries (BET for Romania, OMX Tallin for Estonia, PX for the Czech Republic, BUX for Hungary, BOVESPA for Brazil, SENSEX for India, RTSI for Russia, and the Shanghai Composite Index for China). The indices are selected as they can best reflect and exhaustively capture all events on a market.

In table 1 we present some market information for these stock markets. The Shanghai Stock Exchange, also the Bombay Stock Exchange and the Sao Paulo Stock Exchange have the market capitalization much higher than the others.

Table 1. Market capitalization

No.	Symbol	Index	Country	Values	Market capitalization (US$ millions)
1	*BOVESPA*	*The* BM&F BOVESPA Stock Exchange	*Brazil*	*2476*	1,227,000
2	RTS	The Russian Trading System Stock Exchange	Russia	2493	816,928
3	SENSEX	The Bombay Stock Exchange	India	2474	1,263,000
4	SSE Composite	The Shanghai Stock Exchange	China	2512	2,547,000
5	BET	The Bucharest Stock Exchange	Romania	2497	16,010
6	PX	The Prague Stock Exchange	Czech Republic	2514	37,340
7	OMX Tallinn	The Tallinn Stock Exchange	Estonia	2744	1,966

8	BUX	The Budapest Stock Exchange	Hungary	2510	20,830

Data has been mainly collected from stock exchange own websites, showing the indices transaction. We have collected daily values over a ten-year time span, between October 2002 – June 2013, in order to reach conclusions based on thorough and accurate data; the ten-year time span thus enabled a proper modeling of the phenomena occurring on the respective capital markets. The data sample consists of an average of 2500 daily values of each index price separately.

4. Results

The aim of the tests we are going to perform is to identify whether the selected markets are subject to a specific evolution pattern or the random walk hypothesis.

4.1. Tests for the hypothesis of independence of instantaneous returns of indices

The Ljung Box test is used to determine the degree of serial correlation for the indices in this study and to compute the autocorrelation coefficients. The autocorrelation coefficients for 12 lags and the partial correlation have been computed for level and first difference return series. As probabilities related to testing are inferior to the level of relevance, the correlation coefficients value being higher, the hypothesis of autocorrelation range to 12 lags is accepted for all stock indices. When autocorrelation coefficients (AC) are close to the value of 1 and slightly go lower, series strongly correlate and become stationary. The results of these tests (for the first difference) are showed below in table 2.

Table 2. Autocorrelation and Q-Statistic for returns (values for the first difference)

	Brazil (BOVESPA)			Russia (RTSI)			India (SENSEX)			China (Shanghai_C_I)		
	AC	Q-stat	Prob	AC	Q-stat	Prob	AC	Q-stat	Prob	AC	Q-stat	Prob
1	-0.001	0.0042	0.949	0.118	34.846	0.000	0.069	11.819	0.001	-0.001	0.0049	0.944
2	-0.038	3.5367	0.171	0.009	35.065	0.000	-0.044	16.568	0.000	-0.011	0.3055	0.858
3	-0.066	14.283	0.003	-0.042	39.510	0.000	-0.009	16.768	0.001	0.046	5.6100	0.132

4	-0.007	14.407	0.006	0.023	40.876	0.000	0.004	16.800	0.002	0.046	10.883	0.028
5	-0.010	14.660	0.012	-0.001	40.881	0.000	-0.033	19.469	0.002	-0.022	12.106	0.033
6	-0.015	15.209	0.019	0.009	41.079	0.000	-0.043	24.064	0.001	-0.037	15.632	0.016
7	-0.036	18.443	0.010	0.032	43.632	0.000	0.013	24.456	0.001	0.012	16.001	0.025
8	0.034	21.372	0.006	-0.073	56.948	0.000	0.058	32.908	0.000	-0.018	16.791	0.032
9	-0.008	21.547	0.010	-0.016	57.602	0.000	0.028	34.913	0.000	0.002	16.798	0.052
10	0.044	26.314	0.003	-0.008	57.753	0.000	0.026	36.595	0.000	0.032	19.408	0.035
11	-0.012	26.688	0.005	0.025	59.382	0.000	-0.020	37.564	0.000	0.025	21.013	0.033
12	0.013	27.089	0.008	0.024	60.779	0.000	0.003	37.579	0.000	0.022	22.263	0.035

	Romania (BET)			Hungary (BUX)			Estonia (OMX)			The Czech Republic (PX)		
	AC	Q-stat	Prob	AC	Q-stat	Prob	AC	Q-stat	Prob	AC	Q-stat	Prob
1	0.089	19.717	0.000	0.058	8.5991	0.003	0.152	63.832	0.000	0.065	10.782	0.001
2	-0.00	19.718	0.000	-0.07	22.835	0.000	0.059	73.298	0.000	-0.08	27.200	0.000

	0			5						1		
3	0.011	20.024	0.000	-0.027	24.647	0.000	0.067	85.766	0.000	-0.049	33.149	0.000
4	-0.033	22.750	0.000	0.068	36.390	0.000	0.017	86.554	0.000	0.033	35.970	0.000
5	0.008	22.893	0.000	0.045	41.507	0.000	0.057	95.602	0.000	0.053	43.133	0.000
6	-0.010	23.123	0.001	-0.038	45.150	0.000	0.044	100.97	0.000	-0.015	43.689	0.000
7	0.035	26.111	0.000	-0.069	57.204	0.000	0.019	101.93	0.000	-0.023	45.056	0.000
8	0.043	30.830	0.000	0.012	57.595	0.000	0.060	111.91	0.000	0.003	45.078	0.000
9	0.012	31.211	0.000	0.056	65.501	0.000	0.072	126.17	0.000	0.012	45.444	0.000
10	-0.021	32.356	0.000	0.003	65.524	0.000	0.051	133.24	0.000	0.033	48.269	0.000
11	0.064	42.511	0.000	-0.032	68.042	0.000	0.043	138.37	0.000	-0.008	48.427	0.000
12	0.019	43.424	0.000	-0.024	69.454	0.000	0.050	145.32	0.000	0.019	49.349	0.000

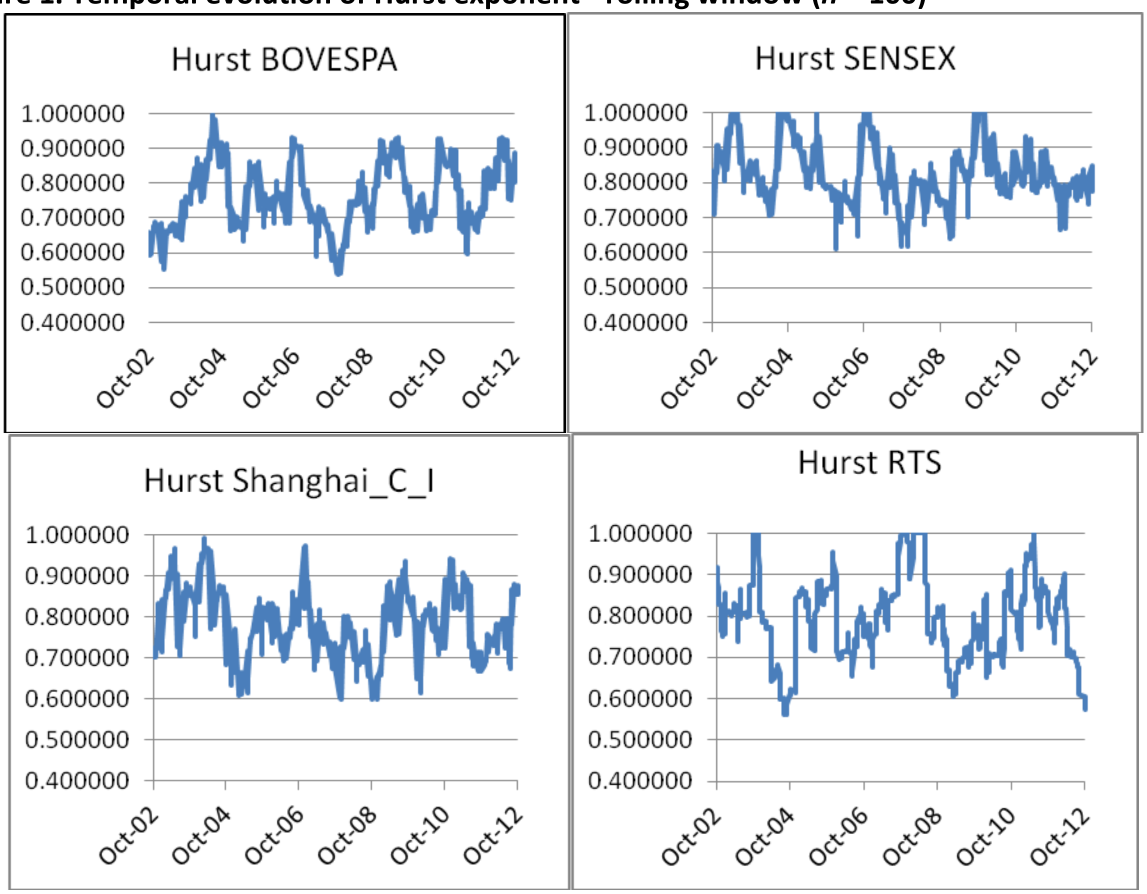
The results we found help us conclude that the first difference denotes stationary time series. Stationarity goes hand in hand with the inefficiency of these capital markets. The linear dependence of these returns is emphasized by significant values of autocorrelation coefficients for level (for example, the autocorrelation coefficient for BET at lag 1 was of 0.998). The Ljung Box test may determine linear dependence, the *p* values being lower than the critical value of 0.05. Therefore the previous returns may be used to forecast future returns. This fact shows that the weak form of the analysed market efficiency does not exist. The *p* values in the table 2 show for the first difference that the null hypothesis is rejected for all markets. The capital market in China highlights a weak efficiency between lags 1 and 3 and at lag 9. The same thing goes for Brazil that indicates weak efficiency between lags 1 and 2.

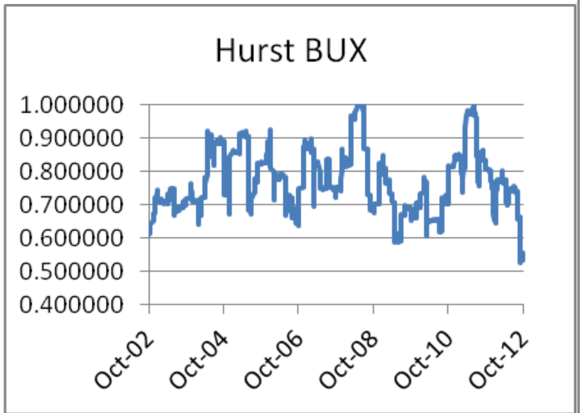
4.2. Dynamically analysis of Hurst exponent

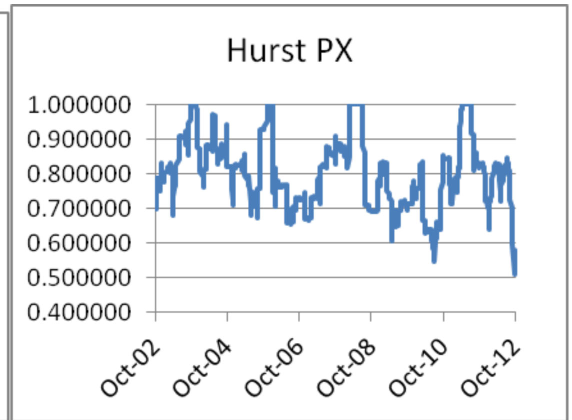
In order to identify the existence of long-term linear dependence in the stock market volatility over time, the Hurst exponent is calculated, in the present paper, also by a rolling window of N = 100 observations to compute the extent of inefficiency. In order to avoid any short-range dependence in mean and in variance of the stock returns, we implemented the rolling R/S statistic on the adjusted time series. We employ the GARCH (1, 1) model in order to filter the stock return time series. Consequently, we implemented the R/S statistic on the standardized residuals.

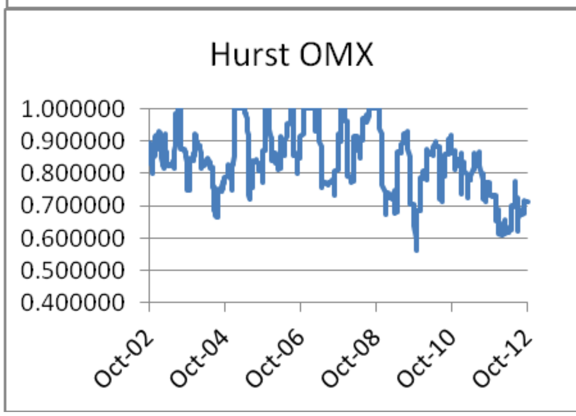
The graph of the temporal evolution of Hurst exponent, dynamically calculated as rolling window applied on GARCH(1,1); residuals are shown in figure 1.

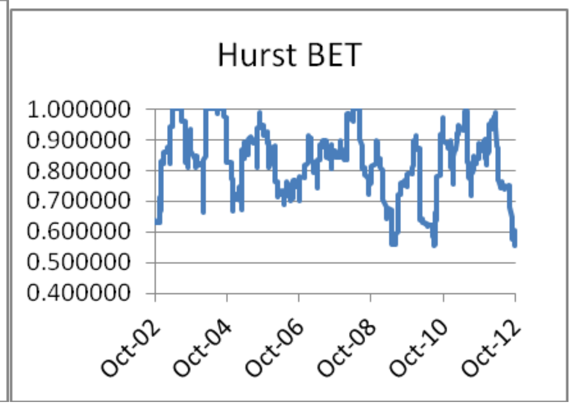
Figure 1. Temporal evolution of Hurst exponent - rolling window (*n* = 100)











These Hurst exponents are well above 0.5 and for most time series seem to be time varying. Since this approach makes available several Hurst exponents for each stock market and it is impossible to compare all of them, then we make statistical inference with the means, medians and other statistical measures of these Hurst exponents in order to compare their market inefficiency degree. In table 3 we present descriptive statistics for these Hurst exponents for each one of the eight indices.

Table 3. Descriptive statistics for the rolling Hurst exponent

	BOVESPA	RTS	SENSEX	SHANGHAI_C_I	PX	OMX	BUX	BET
Mean	0.764504	0.796242	0.826510	0.784442	0.803978	0.856661	0.774454	0.831968
Median	0.751462	0.797735	0.809438	0.783255	0.798973	0.848520	0.758812	0.839114
Max.	0.999465	0.999990	0.999900	0.990748	0.999990	0.999990	0.999990	0.999990
Min.	0.537406	0.560947	0.609136	0.598364	0.548511	0.560822	0.586712	0.555354

Std. Dev.	0.094804	0.100779	0.085905	0.081840	0.100345	0.089753	0.093019	0.106964
Skewness	0.123388	0.183799	0.356255	0.037734	0.267403	-0.05138	0.496084	-0.30969
Kurtosis	2.310615	2.686552	2.779016	2.581860	2.494181	2.634756	2.708911	2.541243
Jarque-Bera	53.07885	23.10441	55.09382	17.87315	53.64521	14.25268	105.8439	58.81604
Prob.	0.000000	0.000010	0.000000	0.000131	0.000000	0.000804	0.000000	0.000000
Sum	1816.461	1891.870	1963.787	1863.834	1910.252	2035.426	1840.104	1976.756
Sum Sq. Dev.	21.34588	24.12126	17.52672	15.90725	23.91415	19.13213	20.54998	27.17305
Obs.	2376	2376	2376	2376	2376	2376	2376	2376

From this table, we can see that the mean values of Hurst exponent are ranging from 0.7645 (Brazil) to 0.8566 (Estonia) while the median value of Hurst exponent are ranging from 0.7514 (Brazil) to 0.8485 (Estonia). As well, we can point out that the Hurst exponents are always above 0.5 indicating the presence of long-range dependence on stock market returns. In addition, all the indices daily returns exhibits a persistent behavior. Also, the highest standard deviation of the Hurst exponent is attributed to Romania's market while the lowest standard deviation is a feature of China's Hurst exponent. Furthermore, the skewness and the kurtosis statistics reveal that the Hurst exponents are not normally distributed.

Kurtosis indicator shows that the coefficient series evince a fluctuation to a lesser degree that one pertaining to a normal distribution (k=3), and the *H* exponent series distribution for all eight indicators is *platykurtic*. Likewise, we cannot conclude that the profitability series evince a normal distribution, by means of the Jarque-Bera test. Almost all markets evince significant deviations from normality.

To ensure that the variation of the Hurst statistics over time is not due to noise, this preliminary finding is supported by several normality test results. In particular, we have employed diverse normality tests, namely Lilliefors (D), Cramer-von Mises (W2), Watson (U2), Anderson-Darling (A2), Kolmogorov-Smirnov and Shapiro–Wilk (W) tests. These tests are based on the comparison between the empirical distribution and the specified theoretical distribution function (the normal distribution, in our case). Another means of testing normality was to design two additional diagrams: QQ-plot and Detrended Normal QQ-plot. A QQ-plot charts observed values against a known distribution, in this case a normal distribution. If our distribution is normal, the plot would have observations distributed closely around the straight line. The Detrended Normal QQ-plot shows the differences between the observed and expected values of a normal distribution. If the distribution is normal, the points should cluster in a horizontal band around zero with no pattern.

All these normality test results are reported in table 4 and figure 2.

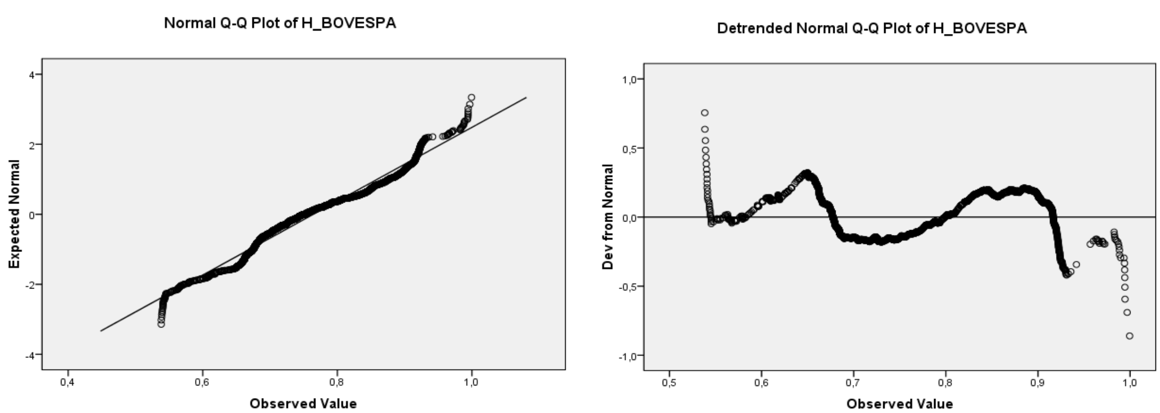
Table 4. Normality tests for the rolling Hurst exponent

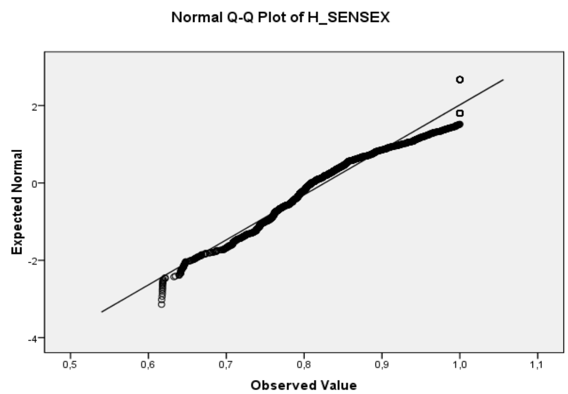
	BOVESPA	RTS	SENSEX	SHANGHAI_C_I	PX	OMX	BUX	BET
Lilliefors (D)	0.070597	0.047061	0.079144	0.037442	0.050357	0.063643	0.068608	0.059497
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cramer-von Mises (W2)	4.297629	0.925023	4.615094	0.496045	1.143984	1.298103	2.560419	1.523489
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Watson (U2)	4.173634	0.843836	4.133489	0.490342	1.002495	1.289110	2.179425	1.282532
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Anderson-Darling	25.07288	10.01606	28.79900	2.820075	11.51470	15.36982	15.51670	14.80033

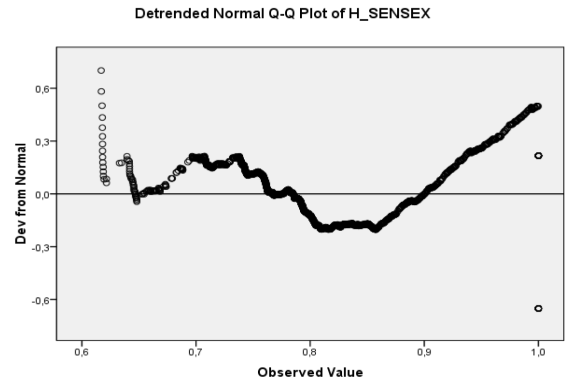
(A2)								
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Kolmogorov-Smirnov[a]	0.070	0.047	0.081	0.036	0.053	0.077	0.075	0.058
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Shapiro-Wilk	0.974	0.979	0.964	0.994	0.974	0.964	0.970	0.970
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

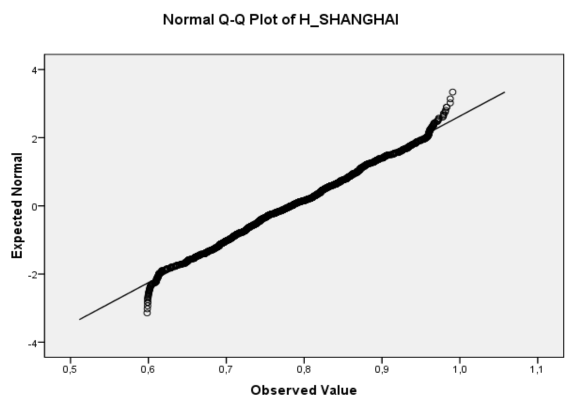
a. Lilliefors Significance Correction

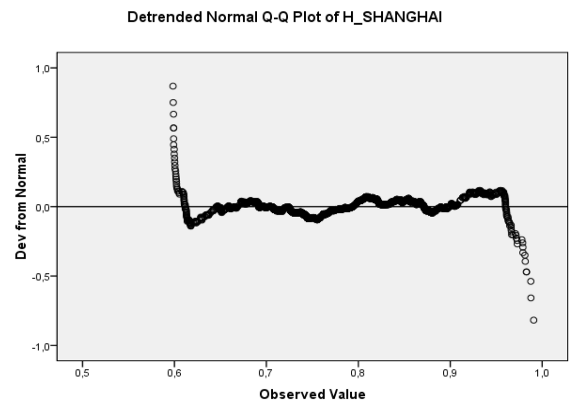
Figure 2. QQ-plot and Detrended Normal QQ-plot for the rolling Hurst exponent

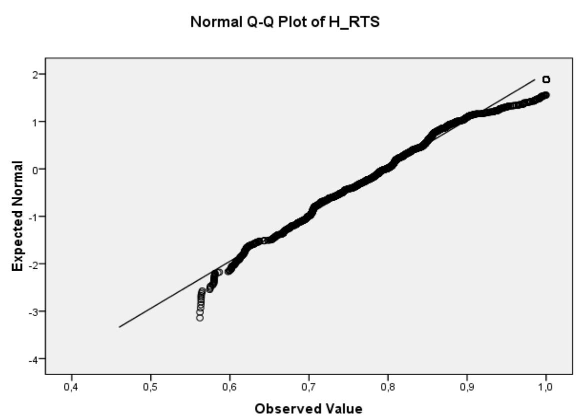


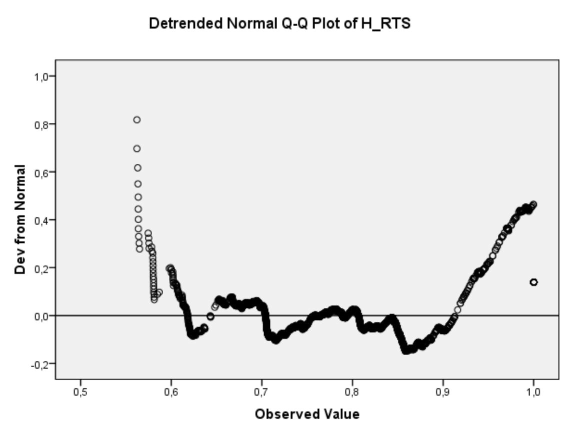

Normal Q-Q Plot of H_SENSEX
Expected Normal
Observed Value

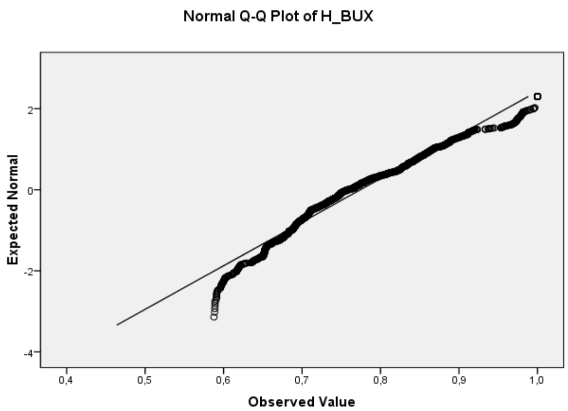

Detrended Normal Q-Q Plot of H_SENSEX
Dev from Normal
Observed Value

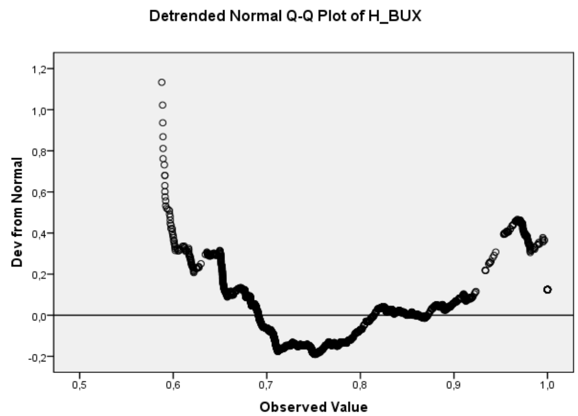

Normal Q-Q Plot of H_SHANGHAI
Expected Normal
Observed Value

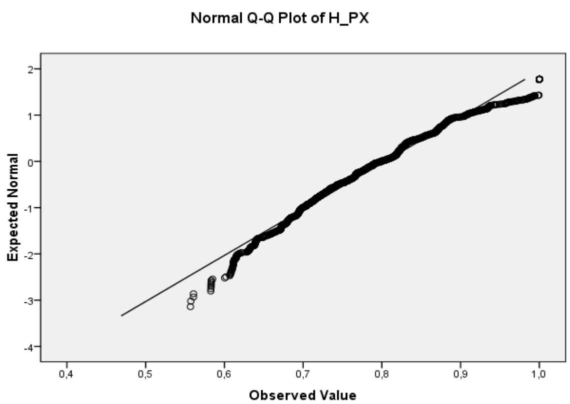

Detrended Normal Q-Q Plot of H_SHANGHAI
Dev from Normal
Observed Value

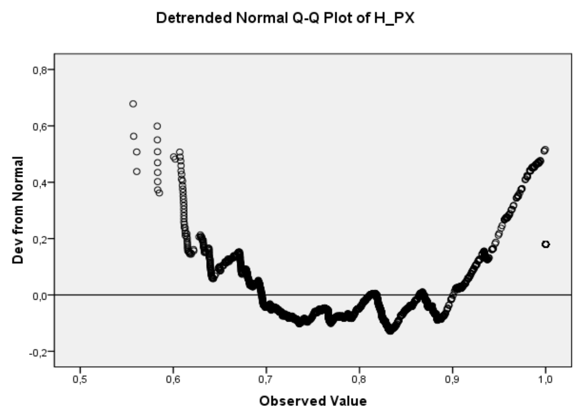

Normal Q-Q Plot of H_RTS
Expected Normal
Observed Value

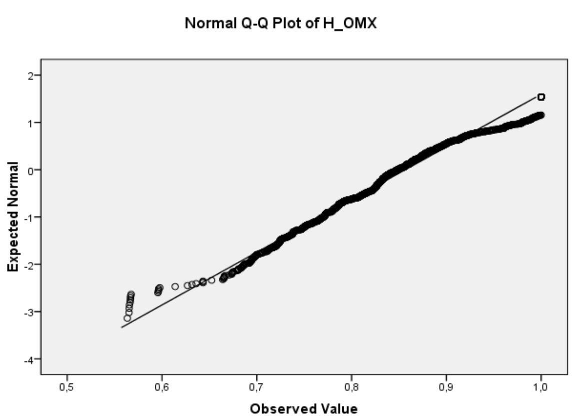

Detrended Normal Q-Q Plot of H_RTS
Dev from Normal
Observed Value

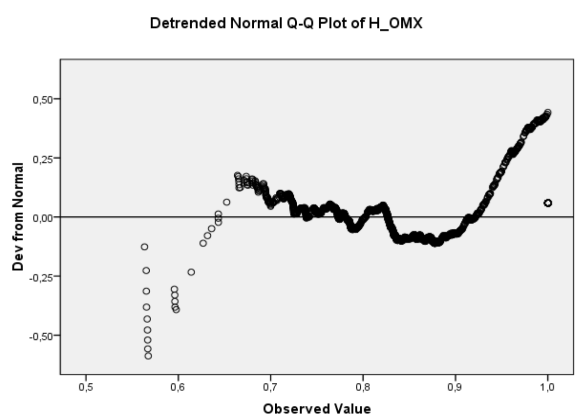

Normal Q-Q Plot of H_BUX
Expected Normal
Observed Value


Detrended Normal Q-Q Plot of H_BUX
Dev from Normal
Observed Value


Normal Q-Q Plot of H_PX
Expected Normal
Observed Value


Detrended Normal Q-Q Plot of H_PX
Dev from Normal
Observed Value


Normal Q-Q Plot of H_OMX
Expected Normal
Observed Value


Detrended Normal Q-Q Plot of H_OMX
Dev from Normal
Observed Value

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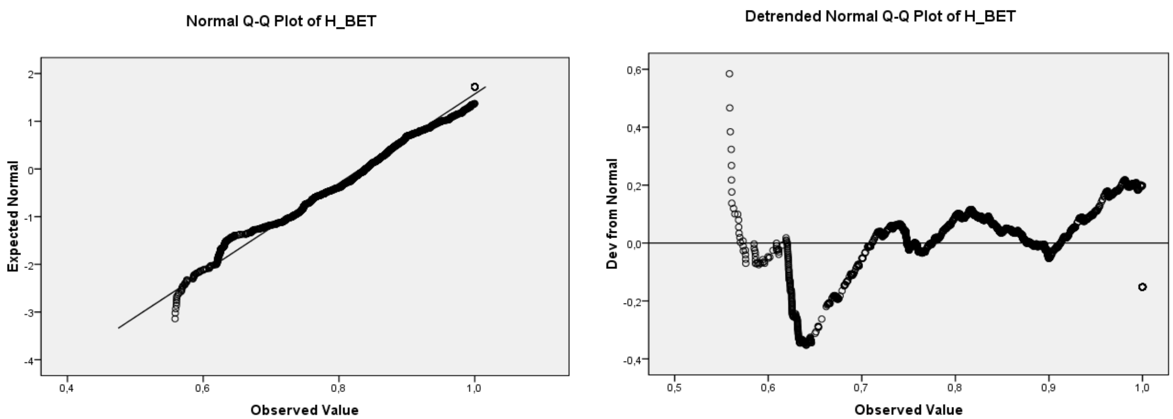

A picture emergent from this table is that the Gaussian distribution for Hurst exponent time series is strongly rejected. Also, our plots indicate some deviation from normal especially at the lower and higher end. Our overall conclusion is that this distribution of these Hurst exponents is not normal and therefore we should compare the medians of the Hurst exponents to compare these indices.

In table 5, we rank these stock markets using medians for the calculated Hurst exponents. Subsequent to the classification presented in the table 5, according to the median value of the Hurst exponent, we may notice that the financial markets in Brazil and Hungary are closest to the random walk hypothesis, a characteristic of efficient market hypothesis, thus we can say that they are the less inefficient stock markets. On the other hand, the financial markets in Estonia, Romania and India prove to be the furthest from the random walk hypothesis, accurately observing the fractal Brownian motion, and thus being the most inefficient markets. These results reinforce our previous findings and suggest that this fractal exponent may be useful in assessing the stage of stock market inefficiency.

Table 5. Classification of capital markets according to median value of Hurst exponent

	Brazil	Russia	India	China	Hungary	Czech Republic	Estonia	Romania
Hurst median	0.751462	0.797735	0.809438	0.783255	0.758812	0.798973	0.848520	0.839114
classification	**8**	**5**	**3**	**6**	**7**	**4**	**1**	**2**

As mentioned above, the Hurst exponent behavior exhibits large deviation from the Gaussian distribution. We can therefore employ the nonparametric tests (nonparametric tests do not make assumptions about a specific distribution) for equality of medians for the different sample markets in order to test whether these rankings are meaningful. Results are summarized in table 6 and they suggest the significance of the difference between the median values of Hurst by all the selected nonparametric tests at the 1% significance level, and therefore our ranking is meaningful.

Table 6. Nonparametric tests for equality of medians between series

Method	df	Value	Probability
Med. Chi-square	7	1174.502	0.0000
Adj. Med. Chi-square	7	1171.253	0.0000
Kruskal-Wallis	7	1689.508	0.0000
Kruskal-Wallis (tie-adj.)	7	1689.755	0.0000
van der Waerden	7	1677.458	0.0000

5. Conclusions

The application of independence tests on the logarithmic profitability of the index series shows that almost all markets evince significant deviations from normality. Given the correlation of profitability and the uncommon distribution, we are against the idea that these temporal series may be subject to a random pattern. Furthermore, the existence of a poor informational efficiency for the eight emergent capital markets is highly questionable.

Subsequent to the calculation of the Hurst exponent for the stock indices subject to analysis, we have noticed that the rolling Hurst exponents are always above 0.5 indicating the presence of long-range dependence on stock market returns. In addition, all the indices daily returns exhibits a persistent behavior. According to the median value of the Hurst exponent, the financial markets of Brazil and Hungary prove to be the closest to the random walk hypothesis, a characteristic of efficient market hypothesis, so we can say that they are the less inefficient stock markets. On the other hand, the financial markets of Estonia, Romania and India prove to be the furthest to the random walk hypothesis, as they accurately observe the fractal Brownian motion, thus being the most inefficient markets.

To sum up, the tests performed in this paper indicate that the yields of these emergent stock markets is a persistent series submitting to fractal distribution. Fractal market means that there exists memory, enhancement and continuity in the variation of stock market. The changes in asset prices increase and continue on the basis of previous state.

To conclude, the capital market represents a system with non-linear self-adjustment mechanisms that may be determined by a series of potentially random time functions, which explain the changes in the stock returns. In the context of a non-linear market model, the stock return evolution may be determined not only by informational, systematic or arbitrary, but also by the non-linear dynamics of the market itself (intrinsic dynamics) (Ghilic-Micu 2002).

Likewise, a variety of other unpredictable sources may disturb the stock exchange system, hence any self-adjusting system becomes much more intricate than a mere logistics function.

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