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Optimizing the Earliness and Tardiness Penalties in the Single-machine Scheduling Problems with Focus on the Just in Time

Ahmad Jafarnejad [a], Seyed Mehdi Abtahi [b]*, Sayyed Mohammad Reza Davoodi [b]

[a] Professor, Department of Management, Tehran University, Tehran, Iran
[b*] Ph.D. Student, Department of Management, Zand Institute of Higher Education, Shiraz, Iran
[b] Ph.D. Student, Department of Management, Dehaghan Branch, Islamic Azad University, Isfahan, Iran
*Corresponding author. E-mail addresses: abtahi.m@ut.ac.ir

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Abstract
The single-machine scheduling problem is the basis of scheduling theories. Because some of the production activities are completed earlier or later than standard time, they create several earliness and tardiness penalties. On the other hand, it is necessary to consider just in time philosophy and its importance in the scheduling problems. Therefore, this study seeks to develop a goal function for optimizing single-machine scheduling problems that combines both earliness and tardiness penalties. This study also is determined to minimize the earliness and tardiness penalties in this area. The restricted due date also has multiple effects on the optimum time. This is why that the present study also was aimed to examine the single-machine scheduling problems with focus on the earliness and tardiness penalties that considering restricted due date simultaneously. The proposed model of this study is an efficient method for understanding the importance of just in time in the scheduling problems.
Keywords: Just in time, Earliness, Tardiness, Penalty, Common due date

1. Introduction
Everybody schedules his/her activities so exactly that will be able to complete them in the accessible time. Development of industrial world can make many resources critical. The machines and manpower usually use critical resources in the production and service activities. This is why that creation of scheduling plans for these resources results in more efficiency and productivity. Scheduling efforts refer to the different activities and efforts that can be shown through related diagrams and creative algorithms. The study of single-machine scheduling problems is one of the fundamental issues in the scheduling science and literature. The single-machine scheduling problem is one of the simplest scheduling problems in which each set of n tasks should be processed on a machine. Therefore, the purpose of this process is to find sequence of the activities that have an answer or an optimized region [13]. Indeed, this method seeks to schedule the activities in the most suitable conditions for optimizing some of the system goals. These goals are usually in conflict with each other and one of them may have

superiority to others in some conditions. For example, minimizing due date is one of these goals. On the other hand, minimizing the tardiness penalties can be considered as another goal. There are some cases that these two goals are considered simultaneously [17]. There are several reasons that why single-machine scheduling problems should be considered. On the other hand, it is not simple to measure their calculations because of complexity of several production machines in the scheduling problems. Therefore, it is economical to analyze the single-machine scheduling problems for understanding them and their structures. On the other hand, it is should be attended that single-machine scheduling is one of the most important methods among other scheduling problems that can create bottleneck in the production environment. There is a machine for servicing in the single-machine scheduling and the activities servicing should be done through entering to this machine. Also activities processing is done on a machine in a same time. Every work includes a time processing, a due date time, and some other characteristics [17].
Due date scheduling is an important issue in the literature review. The reason is competitiveness of the factories. These factories introduce different and exclusive products and their customers expect the customized products and should be delivered in a predetermined time. There are several methods that examine these conditions and necessities such as just in time systems, lean management, and concurrent engineering. Just in time is one of these important methods. This method refers to the condition in which a predetermined quantities of products should are delivered to the customers in the especial time. The activities are seemed accurate and the delivery tardiness and earliness are considered in this section. The due date time can be considered as a common due date in the just in time production. For example, a set of activities need simultaneous assembling in the higher stages of production. Additionally, the common due date depends on many demands and applications. For example, it can be expressed that when a customer orders a product or service, the company should deliver his product-service in the certain time. The earliness in the due date refers to the conditions in which some activities may finished in earlier time considering common due date. On the other hand, some activities may be finished earlier than its standard time. Each of tardiness and earliness has its own penalties. Although activities tardiness results in several penalties such as inventory and maintenance penalties, but earliness results in the customer dissatisfaction and less organizational reputation. There are several tardiness and earliness situations in the single-machine scheduling problems. These make several penalties. This is why that the single-machine scheduling problems are considered as one of the most applicable and famous problems of scheduling science with regard to the tardiness and earliness penalty. Regarding this problem in the recent years results in several attractive solutions. Also it is should be remembered that if these can be combined with common due date, a comprehensive range of the single-machine scheduling problems will be included.

2. Literature review

Probably Sidney is the first author who studied single-machine scheduling problems with focus on the tardiness and earliness penalties. He introduced the concepts of tardiness and earliness maximization and minimization and also examined differentiated due dates. He also presented a cost function for examining the model tardiness and earliness. On the other hand, he presented an efficient algorithm and showed optimum time for order-oriented activities [16]. Lakshminaryanan et al. introduced a similar method and focused on the improved algorithm and considering different times [12]. Seidmann et al. examined due date problems of autonomous works and introduced an especial sequence for minimizing the tardiness and earliness penalties. They also indicated that it is necessary to consider different works optimum due date, task completion time, and expected time for examining customer perspective [15]. Kanet is one of the pioneers that examined time problems. He introduced the minimization of the total time deviations for evaluating this work and presented a new multi-stage algorithm for solving problems in different times. He also introduced assembly scheduling of the certain works in certain time [11]. Most of the studies that have been done in terms of single-machine tardiness and earliness focused on this fact that all of the scheduled works should are accessible in a certain time. Series of hypotheses have been introduced such as due date and its costs by Backer and Scudder [2]. It is possible to consider the evaluation and effects of different scheduling in the optimum sequence of the works for determining inventory quantity and other related costs in the tardiness and customer dissatisfaction. Kanet is one of the pioneers of unrestricted types of single-machine scheduling problems. He introduced single-machine scheduling problems for minimizing unweight total tardiness and earliness in the due date area. Due date times are increased so much that achieve their total processing times [11]. Hall examined the multi-machine scheduling of Kanet and also studied optimum conditions and their present optimum solutions for these problems [8]. Hall and Posner examined and developed the primary studies of Kanet and considered them as hypotheses and indicated that tardiness and earliness are similar in the processing. They also consider similar tardiness and earliness penalties. They examined optimum conditions and showed that the problem is totally NP-hard shape. They also showed a binomial algorithm for this purpose [9]. Cheng and Gupta examined parallel machines with focus on the tardiness and earliness penalties [5].

Just in time production is determined to produce necessary and crucial materials. This is why that the activities which are finished earlier than standard time results in penalties in the just in time. Therefore, a normal and favorable scheduling in the just in time systems is one that all of the activities are completed in a certain time. Ventura and Radhakrishnan revised the single-machine scheduling tardiness and earliness in the activities with different processing and due date times. The purpose of this process is to minimize total absolute variations between completion time and relative due date time [18]. Hino et al. introduced single-machine scheduling problems with respect to the common due date times for minimizing tardiness and earliness penalties in the present works and also examined Tabu search algorithm and genetic algorithm for investigating optimum solution behavior. Their proposed method was appropriate in helping tardiness and earliness problems in the restricted due date [10]. Feldmann and Biskup examined single-machine scheduling problems with respect to the tardiness and earliness penalties in a common due date and also indicated that threshold of accepting is an efficient algorithm for this purpose [7]. Chang et al. introduced a combinative genetic algorithm for single-machine scheduling problems and also developed a goal function for minimizing total weight of the tardiness and earliness penalties [4]. Ying introduced the efficient and effective algorithm of Recovering beam search for solving single-machine scheduling problems with respect to the due date and tardiness and earliness penalties [19]. Fang and Lin presented an optimum distribution of works on the parallel machines for determining equipment sequence and every machines processing speed with a function goal that includes tardiness and earliness penalties and also showed two algorithms for solving this problem [6]. Behnamian and Zandieh introduced the scheduling model for minimizing total linear tardiness and earliness in the hybrid flow shops scheduling problem with regard to the expected and down times [3]. Allaoua and Osmane used a new genetic algorithm with dynamic planning ration for single-machine scheduling problems and minimizing tardiness and earliness penalties in the common due date [1].

3. Problem formulation

In order to formulize the problem, it was assumed that there are n works for processing on a machine at zero time and each of these works need a series of operations for processing. P_i refers to the work processing time (i: 1, 2, …, n) that was predetermined. It is necessary to indicate that tardiness of works is not allowed. If completion time of a work (C_i) is less than its due date time (d), then this work has tardiness and its value can be calculated through this formula $E_i = d - C_i$. If completion time is over than common due date, then there is earliness and its value can be calculated through this formula: $T_i = C_i - d$. On the other hand, tardiness and earliness of every work is showed through α_i and β_i. As indicated in the past sections, the purpose of this goal function is to minimize total tardiness and earliness penalties. This function has been presented in the following section.

$$f(s) = \sum_{i=1}^{n} \alpha_i E_i + \sum_{i=1}^{n} \beta_i T_i$$

$$s.t$$

$$E_i \geq 0 \qquad \mathrm{i} = 1,\ldots \mathrm{n} \qquad (1)$$

$$\mathrm{E_i} \geq d - c_i \qquad \mathrm{i} = 1,\ldots \mathrm{n} \qquad (2)$$

$$\mathrm{T_i} \geq 0 \qquad \mathrm{i} = 1,\ldots \mathrm{n} \qquad (3)$$

$$\mathrm{T_i} \geq c_i - d \qquad \mathrm{i} = 1,\ldots \mathrm{n} \qquad (4)$$

"S" refers to the works practical scheduling. In order to determine sequence time and scheduling plan, it is necessary to consider sequence as an order in which works should be processed. On the other hand, the S scheduling includes all of the necessary information for works construction. This includes works sequence, primary starting time, and common due date. If the due date is over than total works processing time $\left(\sum_{i=1}^{n} P_i \leq d \right)$, so its goal function $f(s^*)$ will be expected with respect to the optimum scheduling time (s) and works sequence in the range of due date. The common due date time (d) is considered unrestrictedly in this situation. All in all, if the sequence is considered without its common due date, this refers to the unrestricted condition.

3.1. The characteristics of restricted and unrestricted problems

Some of the characteristics of unrestricted due date problems have been indicated in the following section.

1) Down time is not considered during scheduling time.

2) The scheduling is V shape in which the works with tardiness are ranked based on the P_i/α_i and works with earliness are ranked based on the P_i/β_i. The following formula shows this condition.

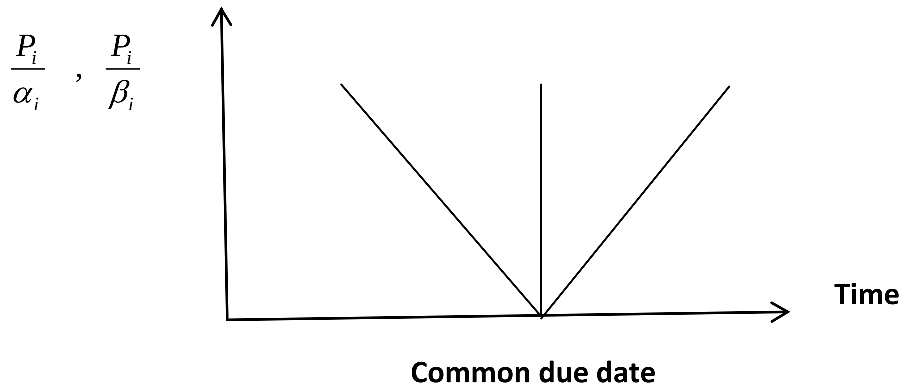

Fig 1: The characteristics of V shape diagrams

3) Each work has its own due date.
4) When the b work will be finished in its due date in the sequence that b is the least integer number is the following adjective.

$$\sum_{i=1}^{b} (\alpha_i + \beta_i) \geq \sum_{i=1}^{n} \beta_i$$

It is should be remembered that the first and second characteristics include restricted conditions, but the third and fourth ones do not include restricted type. An especial and solvable polynomial algorithm is shown for an unrestricted due date in which $\alpha_i = \beta_i = 1$. On the other hand, it is possible to solve the problems in which $\alpha_i \neq \beta_i$. More descriptions are available in [14]. If different costs of every work is considered, the due date problem will be a NP-hard one. Hall and posner indicates that if the tardiness and earliness penalties are similar, the problem can be solved through a dynamic planning algorithm ($\alpha_i = \beta_i$) [9]. The unrestricted due date and without considering cost limitation is NP-hard problem. Also the problem will be clearer if the polynomial algorithm is $p_i/\beta_i \geq p_k/\beta_k$, $p_i/\alpha_i \geq p_k/\alpha_k$. In all of the cases, the problem is NP-hard.

3.2. Developing a mathematical model

It is necessary to develop a mathematical model for restricted common due date problems in which penalties are different. It is necessary to develop a linear programing in order to achieve an optimum scheduling (S^*). In the model, S_i and X_{ik} are variables that indicate optimum scheduling (S^*). S_i is the time for beginning the work of i and X_{ik} will be 1 if the work i is finished before work k and otherwise it will be zero. This condition can be shown as following:

1) X_{ik} will be 1, if work i is finished before work k.
2) Otherwise it will be 0.

In this case, the purpose is determining optimum time in which goal function can minimize f (s). This can be shown as following.

$$f(s) = \sum_{i=1}^{n} \alpha_i E_i + \sum_{i=1}^{n} \beta_i T_i \quad (1)$$

s.t

$$T_i \geq s_i + P_i - d \qquad i = 1,...n \quad (2)$$

$$E_i \geq d - S_i - P_i \qquad i = 1,...n \quad (3)$$

$$S_i + P_i \leq S_k + R(1 - \chi_{ik}) \qquad i = 1,...n-1, K = i+1,...n \quad (4)$$

$$S_k + P_k \leq S_i + RX_{ik} \qquad i = 1,...n-1, K = i+1,...n \quad (5)$$

$$T_i, E_i, S_i \geq 0 \qquad i = 1,...n \quad (6)$$

$$X_{ik} \in \{0,1\}, \qquad i = 1,...n-1, k = i+1,...n \quad (7)$$

In the above model, the values of tardiness and earliness are indicated by limitations 2 and 3 and also the limitations 4 and5 show its start time. Also it is necessary to remember that R is considered as a big value. The $S_i + P_i \leq S_k$ refers to the situation in which work i is placed in the sequence before work k and $X_{ik=}$ 1. Because of addition of R, the fifth limitation by $X_{ik=}$ 1 will be an unrestricted. On the other hand, $X_{ik=}$ 0 refers to the situation in which the fifth limitation is $S_k + P_k \leq S_i$ and the fourth one is unlimited.

4. The results of calculation

The mathematical model that has been developed and introduced in the later section was tested in a travels bag factory. This problem has been solved through Gams software. This problem has been used for several purposes.

Table 1: The assumed data for 8 works

n	1	2	3	4	5	6	7	8
P_i	8	2	17	5	14	15	4	5
α_i	3	9	3	8	6	6	6	5
β_i	15	8	7	8	8	10	4	13

Table 2: The goal function values for different due dates

Due date	1	3	6	9	12	15	18	21	22	25	27
Goal Function	1164	1122	771	636	517	386	275	190	171	171	171

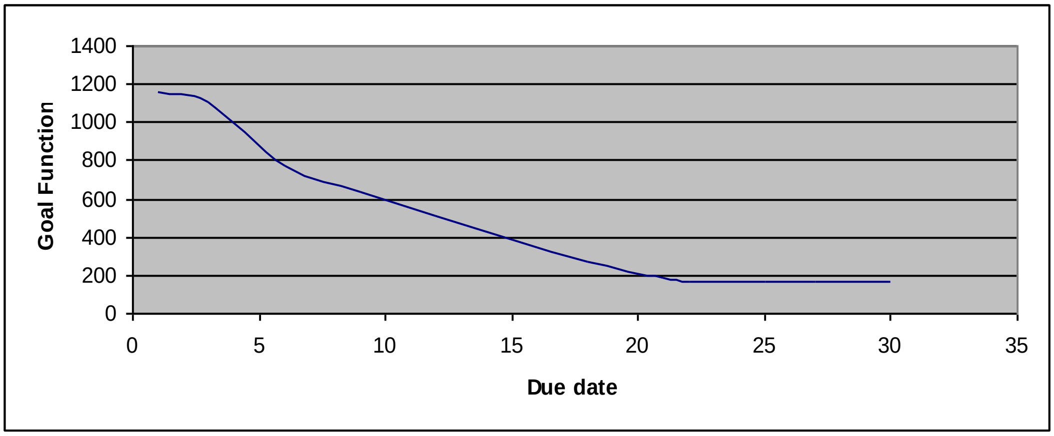

Fig 2: Goal function values for different due dates

As indicated in the later model, goal function has descending order for increasing values of a. Also goal function has different slopes for different values. Also it is unrestricted for d≥22 that its goal function value is 171 in this condition. With regard to the complexity and importance of this problem from judgmental perspective, it is necessary to use modeling and ration of this problem as a basis for solving sample works with high value through a Meta heuristic problem.

5. Conclusion and empirical suggestions

The model that has been examined in this study is a comprehensive model that can be used many industrial factories. As indicated in the later sections, there is an especial process time. Also there are different values of tardiness and earliness that result in its solution difficulty. As indicated in the later sections, increase in the works frequency leads to increase its limitations. It also results in more difficulties in the model solution. Since our model is a restricted due date model and it influences optimum answer, so it is very important and critical. Therefore, the model and ration of this problem can be used in many works through creative methods for problem-solving. Several empirical suggestions have been presented in the following section.

- It is assumed in this study that all of the works are started in the zero time. But some factories may are forced to start their activities in the nonzero times.
- Our study assumes that machines start time is zero, but many machines may have nonzero starting times.
- Any down time has not considered in our study, it is suggested for future authors that present these methods in solving these restricted problems.
- Our model is focused on the single-machine scheduling problems. It is suggested that the future authors and researchers develop methods for solving multi-machine problems.
- Finally, it is should be remembered that the reliable and convergent data can be efficient in these situations.

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