Model for Analyzing the Significance and Usefulness of Grouping Portfolios

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Abstract

Many researchers have used a small number of portfolios extracted from a large volume of assets and, therefore, the problem was that these portfolios were first grouped, according to certain characteristics, variables and then these portfolios thus rearranged can ensure that a constant characteristic is maintained for a portfolio that refers to the stops it includes. A series of issues are then presented to identify the fact that a grouping of portfolios is an activity with enormous significance and utility, in the sense that it eliminates the discrepancies, ensures homogeneity of the groups in which the respective portfolios are constituted and thus can be reached. the fact that in the realization of portfolios the idiosyncratic volatility is reduced and it is possible to obtain more accurate estimates of the action of the factors considered in the equations used. Another aspect that is taken into consideration is that it is sometimes observed that portfolio weights are not always deterministic, as they are calculated from a previous data sample or from previous data samples. In this context, the problem arises to recalculate or calculate the market capitalization at the basic moment of the observed price, the number of outstanding shares and then sort those entities and portfolios based on a market ceiling, which is now deterministic. The relationships used are presented and explained in their essence. Time is a determining factor that is largely explicit, using some data and graphical representations, which we have considered. Some critical appraisals are also expressed in connection with CAPM testing, pointing out that not always positive portfolios in parameter space is like looking for linear numbers with arithmetic progression, which is not always the case. The active or passive management of the portfolio is another problem that has been solved within this exposure, in the mentioned article.

1. Introduction

The article meaning and usefulness of grouping portfolios starts from the fact that the data are gathered empirically and the data we record. It is based on the fact that there is an additional variety of a portfolio, in quite a large number, with variations ranging from the factors of influence, market conditions, price conditions and many more.

The following is a careful presentation of how portfolios are periodically balanced, in order to maintain the specific value of the portfolio considered and therefore, if the distribution of stocks changes over time, then the selected portfolios no longer maintains the absolute variables found, but only on some

which, in this sense, become relative. The flattening of these aspects that arise as a result of the evolution of portfolios over time is essential in the study we do on the portfolio.

The following article highlights the time variation model, considering the CAPM market model, which must be based on an independent observation sample and identically distributed from a community, i.e. the sample used to be representative for the entire mass of portfolios that we have in mind. There are presented on the basis of graphs and figures how this method of groupings of portfolios can be ensured, which in the end have a special significance, are useful for market research and the results obtained are clearly superior in these conditions.

The article also makes a critical appraisal regarding the CAPM testing, in the sense that a number of researchers have expressed opinions showing that not always such a used model leads to the best results and some solutions can be found in view. The normality or elliptic symmetry is crucial for the derivation of the CAPM, but the normal distribution is strongly rejected by the data we use. The CAPM has only one power in the application, that is in cross-section and therefore the testing must be wider and more complex. In the final part of the article, details are made regarding the active and passive management of a portfolio. Both variants are analyzed, a series of relationships are used to ensure the estimation of correlations and based on this the estimation of portfolios on the capital market is realistic and very precise.

2. Literature review

Adam *et al.* (2008), as well as Anghel *et al.* (2016) have analyzed a number of issues regarding the selection of the portfolio of financial instruments. A similar theme is addressed by Hagstromer & Binner (2009). Aldrich & Kung (2009), as well as Anghelache & Anghelache (2009) highlighted the methods of asset analysis. Amenc & Le Sourd (2003), as well as Anghelache & Anghel (2014) presented models of portfolio analysis. Anghel *et al.* (2019) they approached elements regarding the dynamic models. The dynamics of the portfolio of financial instruments is a topic that has been analyzed by Anghelache *et al.* (2016). Anghelache *et al.* (2015), as well as Iacob (2019) they emphasized the importance of using the econometric instrument in the complex analysis of the economic field. Badiu *et al.* (2019), but also Lucas & Siegmann (2008) they studied aspects of profitability and risk. Justiniano *et al.* (2010), as well as Okhrimenko & Manaenko (2014) they studied elements regarding the factors of influence in the adoption of investment decisions. Kini *et al.* (2009) they referred to the accuracy of estimating the evolution of portfolios. Liu (2007) analyzed the use of stochastic processes in the selection phase of the portfolio.

3. Methodology of research. Results and discussions

In the case of FM, many researchers have used a small number of portfolios extracted from a large volume of assets. There are a number of advantages to this approach. One advantage is that we should not focus on survival biases that arise when we analyze individual stocks, because we can only include actions that are still in business and trading. If we follow a company throughout its life cycle, and that includes the failure in the end, then the purchasing and holding efficiency of this company is very low. In practice, a sample is selected that excludes stocks that do not enter the sample, which do not form a representative sample of the stocks. By using portfolios, one can in principle maintain a constant characteristic for a portfolio by rotating the stocks it includes. The practice is to sort the stocks according to the characteristic observed at a certain date and then to form portfolios based on the quantiles of the characteristic distribution, that is, of the stocks in the respective distribution. Portfolios are periodically rebalanced to maintain its characteristic value. However, if the distribution of stocks changes over time, then the portfolios chosen do not maintain absolute characteristics, but only the relative ones.

The size or market capitalization has changed considerably over time. Thus, through the Dow Jones index any large portfolio is evaluated at market conditions. In the case of beta sorting portfolios, there is a smaller problem with the global trend, but there are some trends and variations over time in terms of beta level and dispersion.

Another econometric problem that the grouping of the portfolio alleviates is the error in the variables problem. This is because cross-section mediation reduces variation. Let's suppose that:

$$\hat{\beta}_i = \beta_i + \eta_{Ti} \tag{1}$$

Where η_{Ti} is a zero mean error due to the time series estimation of $\hat{\beta}_i$, beta on stock i. Usually, η_{Ti} is small in the sense that $var(\eta_{Ti}) \leq cT^{-1}$ for some values of the constant c.

If we consider a portfolio p with fixed weights $(w_{pi}, i = 1, \dots, n)$, such that $\sum_{i=1}^{n} w_{pi} = 1$ where the number of base assets n is large, the beta version of the portfolio is the weighted average of the component beta assets, ie $\hat{\beta}_p = \sum_{i=1}^{n} w_{pi} \hat{\beta}_i$. Therefore, we can formulate the relationship:

$$\hat{\beta}_p = \sum_{i=1}^{n} w_{pi} \beta_i + \sum_{i=1}^{n} w_{pi} \eta_{Ti} = \beta_P + \sum_{i=1}^{n} w_{pi} \eta_{Ti} = \beta_P + \eta_{PTi} \tag{2}$$

and the measurement error in the beta portfolio η_{pTi} has zero mean and has a variation $var(\hat{\beta}_p) = \sum_{i=1}^{n} w_{pi}^2 var(\eta_{Ti})$ (assuming that individual errors are uncorrelated).

If $w_{pi} = 1/n$ and $\sigma_{\eta Ti}^2 = \sigma_\eta^2/T$ for all i, then the measurement error in beta portfolio has variance σ_η^2/nT, which is smaller in size than the measurement error in the individual stock supplied n, which is large. Suppose they are considered $p = 1, \dots, N$ portfolios. Through Cauchy-Schwarz inequality, $cov(\hat{\beta}_p, \hat{\beta}_q)$ is reduced in size. This quantity can be exactly zero if we assume that the idiosyncratic errors are not correlated in the section and if the portfolios do not overlap, so $(i: w_{pi} \neq 0, w_{qi} \neq 0, p \neq q)$ this is empty. If portfolios are formed by considering individual stocks on some characteristics and then dividing them into N subsets, which do not overlap, then this last condition is satisfied.

A second econometric problem that results from grouping portfolios is whether the regression from the second stage can have satisfactory properties. We specify that the linear regression, the necessary condition of the existence of the slope estimates, is that the regressors have a positive variation. Moreover, standard errors decrease with the variability of the regressors. Given this, it is desirable for the portfolio grouping to maintain variations, that is, if $p = 1, \dots, P$, we should maximize the relationship:

$$\sum_{p-1}^{P} (\hat{\beta}_p - \overline{\hat{\beta}})^2 \tag{3}$$

One way to do this is to form beta portfolios, that is, to sort estimated beta and then to build stock portfolios with high beta levels. If we had formed portfolios at random, for example alphabetically, then we would find that this variation is smaller, which is used as a form of portfolio grouping. This is a convenient procedure, which has a solid foundation. However, there is one view that holds that grouping portfolios is not the best approach, Roll (1977).

Specifically, the portfolio grouping procedure, on a large scale, can support a theory even when it is false. This is due to the fact that the individual deviations of the assets from the linearity can be canceled in the formation of portfolios.

Brennan *et al.* (1998) are for the use of individual stocks. In the literature it is claimed that the realization of portfolios reduces the idiosyncratic volatility and allows more precise estimations of the action of the factors. It can be stated analytically and empirically that lower standard errors of beta portfolio estimates do not lead to lower standard errors of cross-sectional coefficient estimates. The standard errors of the risk premiums are determined by the cross-sectional distributions of the influence of the factors and the residual risk. The portfolios cancel the evolution by reducing the dispersion, which leads to large standard errors.

Connor & Korajczyck (1988) developed the principal asymptotic component (APC) method precisely to manage a large cross-section of individual stocks. Their methodology is widely followed and developed in industry. More recent work by Bai & Ng (2002), Connor *et al.* (2012), Fan *et al.* (2015) and Pesaran & Yamagata (2017) are studies that work with individual stocks and we believe it will stimulate more studies using individual assets.

$$R_{it} = \gamma_{0t} + \gamma_{1t} \beta_i + \gamma_{2t} S_i + \eta_{it} \tag{4}$$

If we assume that at the individual level, where S, is a variable that affects the expected profitability and η_{it} is a certain term, then we can write:

$$\sum_{i=1}^{n} w_{pi} R_{it} = \gamma_{0t} + \gamma_{1t} \sum_{i=1}^{n} w_{pi} \beta_i + \gamma_{2t} \sum_{i=1}^{n} w_{pi} S_i + \sum_{i=1}^{n} w_{pi} \eta_{it} \tag{5}$$

It is possible, though $S_1 \neq 0, S_2 \neq 0$ the amount $\sum_{i=1}^{n} w_{pi} S_i$ to be zero or close to zero, the power of the test must be reduced. Every economist knows that aggregating accounts and accumulating them over time helps reduce the risk of detection for fraudulent transactions.

A final consideration, which is sometimes overlooked, is that portfolio weights are not deterministic, as they are calculated from a sample of data. In statistics, this aspect is called an estimator and is usually concerned about the consequences of this estimation for further estimation and testing. The segregation of the portfolio formation period from the estimation period is designed to minimize the influence of portfolio formation on the subsequent results. Suppose we calculate the market capitalization, the moment t = 0 based on the observed price and the number of outstanding shares, and then we sort the firms into portfolios based on their market cap thus determined. One can equally weigh the first 5% of the companies in a single portfolio, the equal weight of the second big company of 5% in the second portfolio, etc. In this case, we will consider that the weights are themselves random variables determined by the specific realization of the price in the zero period, respectively:

$$\widehat{w}_{pi} = w_{pi}(P_{10}, \ldots, P_{n0})$$

(6)

that is to say $w_{pi} = \frac{1}{n_p}$ if $P_{i0} \in [P_{(p)0}, P_{(p+1)0}]$ and $w_{pi} = 0$.

Otherwise when P_{i0} is an uncertain value or quantity of the empirical distribution of the price zero time (for simplicity, we equate the size with the price here, ie the number of shares is constant). If $\hat{\beta}_i$ is estimated from data from period B, then:

$$\hat{\beta}_p = \sum_{i=1}^{n} w_{pi} \hat{\beta}_i = \sum_{i=1}^{n} w_{pi} \beta_i + \sum_{i=1}^{n} w_{pi} \eta_{Ti} + \sum_{i=1}^{n} (\widehat{w}_{pi} - w_{pi}) \beta_i + \sum_{i=1}^{n} (\widehat{w}_{pi} - w_{pi}) \eta_{Ti} =$$
$$\beta_p + \sum_{i=1}^{n} w_{pi} \eta_{Ti} + \sum_{i=1}^{n} (\widehat{w}_{pi} - w_{pi}) \beta_i + \sum_{i=1}^{n} (\widehat{w}_{pi} - w_{pi}) \eta_{Ti}$$

(7)

The question is: should we consider the uncertainty introduced by estimating portfolio weights, and if we do, does it matter? An argument can be whether we should calculate everything conditional $P_{10}, \ldots, P_{n0}$, as if they were ancillary statistics (Cocs & Hinkley, 1979). It is clear that there are no ancillary statistics, because the marginal distribution $P_{10}, \ldots, P_{n0}$ it certainly depends on the parameters. This aspect is more difficult to evaluate theoretically. Indeed, grouping portfolios is an example of a particularly harmless economy, because the main thinking and justification does not use a complete model.

In conclusion, the portfolio grouping method has advantages and disadvantages. To some extent it is a solution to certain statistical problems, which can now be addressed by other techniques within a complete or partially articulated model for individual stock returns. However, it is a practical and convenient method that is widely used and reflects industry practices.

In the case of a time variation model, the starting point of the market model and the CAPM testing is that we have an independent and identically distributed sample of observations from a fixed community. This setting is vital for the development of statistical inference. Most of the practical implementations recognize the variation of time by working with short windows of five or ten years. Some authors have emphasized the variation estimated over time. Fernandez (2015) calculated "beta" for 3813 companies using 60 monthly returns per day. It is found that the historical "betas" change from day to day, only 2780 companies out of the 3813 had all the positive estimation points and the historical betas depend on the market index. Different suppliers in the "beta" industry offer completely different values for this significant stock specific parameter.

We will present the estimated values for IBM (compared to the S & P500) calculated from the daily returns on stocks using a five year window over a period. During this period there was a substantial variation of IBM's "betas" with an overall upward trend, indicating that it became a riskier stock. On the other hand, the "alpha" corresponding to IBM is very small and fluctuate over zero and below zero, without a definite trend.

This phenomenon is not limited to individual stocks. In the presence of the time variation of the parameters, the usual tests are inconsistent.
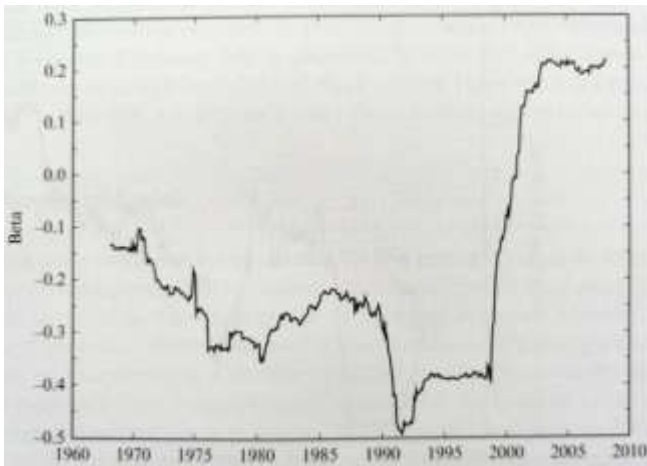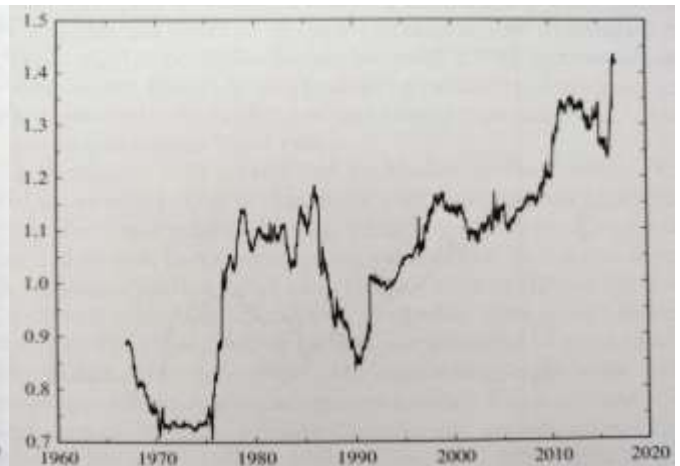
Figure 1. IBM's time variation   *Figure 2*. IBM's beta time variation

We are going to consider a more general framework in which the time variation is explicitly taken into account. Suppose in the interval considered:

$$Z_{it} = \alpha_{it} + \beta_{it}Z_{mt} + \varepsilon_{it} \qquad (8)$$

Where α_{it}, β_{it} it varies over time, but otherwise the regression conditions are satisfied, i.e. $E(\varepsilon_{it}|Z_{mt}) = 0$. We establish that $\sigma_{it}^2 = var(\varepsilon_{it}|Z_{mt})$ varies in time and actively.

If we use previous data to estimate the parameters and build the Wald statistic, we can treat the parameters as fixed in the estimation window. This model does not seem realistic, especially since the beginning and closing periods are arbitrary. The realistic model is to assume that the parameters evolve slowly. Suppose:

$$\alpha_{it} = \alpha_i(t/T), \ \beta_{it} = \beta_i(t/T) \qquad (9)$$

Where α_i and β_i are continuous functions.

This can be reconciled by testing whether $\alpha_i(u) = 0$, for $u \in [0,1]$. Please note that it is not necessary to provide a complete specification for σ_{it}^2, because a robust inference of heterostedasticity can be constructed.

An alternative framework is the model of the unobserved, shaped components:

$$\alpha_{it} = \alpha_{i,t-1} + \eta_{it}, \ \beta_{it} = \beta_{i,t-1} + \xi_{it} \qquad (10)$$

where η_{it}, ξ_{it} are shocks with zero mean and variance $\sigma_\eta^2, \sigma_\xi^2$ (Harvey, 1998).

In this framework, the problem is to test whether $\sigma_\eta^2 = 0$ compared to the general alternative. A final approach is to specify a generalized multivariate dynamic model of autoregressive conditional heterostedasticity (GRACH) or a stochastic volatility model.
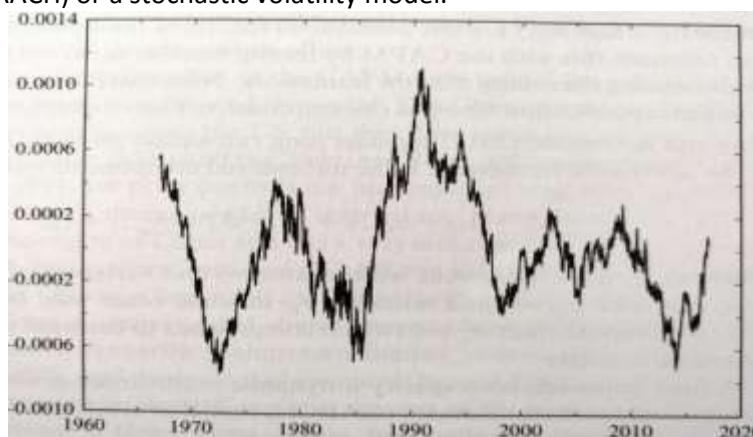

Figure 3. IBM's alpha time variation

There are some practical aspects that we will briefly reconsider. What assets should be included in the samples? Most studies consider portfolios, but there are a number of studies that use individual assets. What sample frequency should be used? Most studies use monthly data, but there are a growing number of studies using a higher frequency, such as daily or weekly. How long should a time series be considered? Most studies consider data from five years or ten years, at one point, to alleviate non-stationary problems. What market portfolio should be used? CRSP indices are widely used in academic studies, but they are not traded, or there are no futures contracts traded on these indices. What risk-free rate should be used? Most studies take the three-year T bill rate.

There are a number of anomalies well established in the literature. Keim (1983) documented the size effect. This is because companies with low market capitalization seem to be making abnormally positive profits $(\alpha > 0)$, while big companies get abnormal negative profits $(\alpha < 0)$. Banz (1981) documented the value effect. This is because companies with high values compared to the market value seem to obtain abnormally positive profits $(\alpha > 0)$, while companies with small values of value relative to market value (also known as growth stocks) have $\alpha < 0$. Typical values are the dividend at price (D/P) and the ratio of book value to market value (B / M). In accounting, the book value is the value of an asset according to the balance of its balance sheet account. For assets, the value is based on the initial cost of the asset, less any depreciation, amortization or depreciation costs incurred relative to the asset. Traditionally, the company's carrying amount represents total assets less intangible assets and liabilities. However, in practice, depending on the source of calculation, the carrying amount may sometimes include goodwill or intangible assets or both. The inert value of the workforce, part of the intellectual capital of a company, is always ignored. When intangible assets and goodwill are explicitly excluded, the size is often specified to be the tangible book value. Jegadeesh & Titman (1993) documented the effect of the moment.

Most classic CAPM studies have been conducted on the US market. The Chinese economy and the stock market are now very close to the US, but they have some special features. The Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange (SZSE), although launched in 1990 and 1991, however, prices are implemented and separate classes of shares are established as a means of capital restraint. However, Hong Kong has a different institutional framework under the principle of one country, two systems. The Hong Kong Stock Exchange (SEHK) was established in 1891 as an open free market, with no restrictions on price fluctuations and capital flows. Shares A and Shares H are two major classes of shares issued by Chinese companies. First, A shares are limited to domestic investors, listed on SEHK and denominated in Hong Kong dollars, even with the recent relaxation of capital restrictions. The A and H stock markets remain segmented. Shares A receive a price premium over the same companies and which grant equal ownership rights. The price difference reflects a significant institutional impact on stock markets. Therefore, it is of interest to compare AH shares with double share. Ng (2014) considered that if there is convergence in the prices of AH shares, if there are long-term collaborative relationships between the prices of AH shares, the difference between AH listings in terms of risk and profitability, if the price difference implies inefficiencies, that is, overvaluation of shares A and/or undervaluation of H shares and how the effects of price risks on partially segmented markets are determined.

There are a number of problems with CAPM evidence. The criticism of Roll is well known, which says that the central element of the CAPM forecasts is that the market portfolio is a medium effective variation, but the market portfolio cannot actually be observed, as it includes many assets that are not traded or are rarely traded. Therefore, the CAPM and specific anomalies may not be valid. We will find that asset pricing models based on the multifactor created avoid this criticism by not needing to know the specific factors. Roll criticism is somewhat arbitrary, but it is logically valid. We need to determine how the CAPM will be tested when a certain proxy variable is available for returns on the market. Another analysis is to distinguish betas ex-ante and ex-post and to incorporate CAPM in a dynamic framework. Starting from the possible difference between the ex-post and ex-ante parameters, in a recent paper, Levy & Roll (2010) show that when only small changes of the sample means and standard deviations are made, the observed market portfolio is an average variance, which according to Roll (1977) and assumes that linear CAPM is still possible. They use a new reverse engineering approach. Levy (1997) shows that the probability of obtaining a positive tangent portfolio based on sample parameters converges to zero exponentially with the number of assets. However, at the same time, very small adjustments of the profitability parameters, in the limit of

the estimation error, produce a positive tangent portfolio. Therefore, searching for positive portfolios in the parameter space is like looking for rational numbers on the numeric line. If a point in the parameter space is chosen at random, it almost certainly corresponds to a non-positive portfolio (a rational number). However, close points can be found in the parameter space, corresponding to the positive portfolios (rational numbers). The normality or elliptic symmetry is crucial for the derivation of CAPM, but the normal distribution is strongly denied.

Active portfolio management can be defined as attempts to achieve superior returns through market selection. According to CAPM, such higher yields cannot be obtained, so the existence of a large active management industry seems to contradict this theory. The selection of security involves choosing the wrong securities, trying to buy at low prices or sell at the highest prices or to sell at a low price and then buy back. The moment of the market involves trying to enter the market at times with low prices and going to the top. Hedge funds are often classified according to their self-described investment style: event driven. For example, spin-offs, mergers, bankruptcy reorganizations, etc. Global or macro is also those that invest on the basis of macroeconomic analyzes, usually through major changes in the interest rate. Market neutral are those funds that actively try to avoid major risk factors. In practice, maintaining market neutrality in times of crisis is difficult. The asset management industry is accompanied by conflicts of interest between owners and managers, as the managers are often remunerated according to a definitive method of evaluating the funds that can be manipulated to some extent by them.

We can consider an example and consider the case, since an active manager overseeing a portfolio of $ 5 billion could increase the annual return by 0.1%, his services would be worth up to $ 5 million. Should it invest? The role of chance is also important and one should take into account the image of 10,000 managers whose strategy is to place all assets in a fund, but at the end of each year to use a quarter of it to follow (independently) a single purpose. After 10 years, many of them no longer keep their jobs, but many survivors have been very successful (the probability of 10 successes in a row is approximately $(1/2)^{10} \sim 1/1000$). The probability that no investor will produce is:

$$\log_{M \to \infty} (1 - K^{-n})^M = 0 \tag{11}$$

Passive portfolio management involves tracking a predefined index of securities without any security analysis. There is a big increase in the passive system. It is much cheaper than active management because there are no acquisition and analysis accounts, there are lower transaction costs (less frequent trading) and there is also a generally greater diversification of risks. There are some opinions that this affects price discovery and sets relative prices within a narrow range that does not reflect the fundamentals.

So far, we have clarified the choice of portfolio, we have clarified the two common approaches for CAPM testing based on the maximum likelihood method and the cross-sectional regression method. We also clarified the method of grouping the portfolio and how some of the key statistical issues are addressed. Considering the result of the aggregation that the returns follow a sequence of stationary marked difference after a constant average adjustment and possesses four moments, the result:

$$k_3(r_A) = \frac{1}{\sqrt{A}} k_3(r) + \frac{3}{\sqrt{A}} \sum_{j=1}^{A-1} \left(1 - \frac{j}{A}\right) E(\tilde{r}_t^2 \tilde{r}_{t-j}) \tag{12}$$

where $\tilde{r}_t = [r_t - E(r_t)]/std(r_t)$. For kurtosis we have:

$$k_4(r_A) = \frac{1}{A} k_4(r) + \frac{4}{A} \sum_{j=1}^{A-1} \left(1 - \frac{j}{A}\right) E(\tilde{r}_t^3 \tilde{r}_{t-j}) + \frac{6}{A} \sum_{j=1}^{A-1} \left(1 - \frac{j}{A}\right) [E(\tilde{r}_t^2 \tilde{r}_{t-1}^2) - 1] + \frac{6}{A} \sum_{k=1}^{A-1} \sum_{j=1}^{A-1} \left(1 - \frac{j}{A}\right)\left(1 - \frac{K}{A}\right) E(\tilde{r}_t^2 \tilde{r}_{t-j} \tilde{r}_{t-k}), \quad j \neq k \tag{13}$$

If we consider the weekly returns:

$$(r_1 + r_2 + r_3 + r_4 + r_5)^3 = r_1^3 + r_2^3 + r_3^3 + r_4^3 + r_5^3 + 3r_1^2 r_2 + 3r_2^2 r_3 + 3r_3^2 r_4 + 3r_4^2 r_5 + 3r_1 r_2^2 +$$
$$3r_2 r_3^2 + 3r_3 r_4^2 + 3r_4 r_5^2 + 3r_1^2 r_3 + 3r_2^2 r_4 + 3r_3^2 r_5 + 3r_1 r_3^2 + 3r_2 r_4^2 + 3r_3 r_5^2 + 3r_1^2 r_4 + 3r_2^2 r_5 +$$
$$3r_1^2 r_4 + 3r_2 r_5^2 + 3r_1^2 r_5 + 3r_1 r_5^2 + 6r_1 r_2 r_3 + 6r_2 r_3 r_4 + 6r_3 r_4 r_5 + 6r_1 r_2 r_4 + 6r_2 r_3 r_5 + 6r_1 r_2 r_5 +$$
$$6r_1 r_3 r_4 + 6r_2 r_4 r_5 + +6r_1 r_3 r_5 + 6r_1 r_4 r_5 \tag{14}$$

If we have a waiting martingale difference sequence, it is reduced to:

$$5E(r_1^3) + 12E(r_2^2 r_1) + 9E(r_3^2 r_1) + 6E(r_4^2 r_1) + 3E(r_5^2 r_1) \tag{15}$$

In general:

$$E(r_A^3) = AE(r_t^3) + 3(A-1)E(r_t^2 r_{t-1}) + 3(A-2)E(r_t^2 r_{t-2}) + \cdots + 3E(r_t^2 r_{t+1-A}) \tag{16}$$

This shows that the rate at which the aggregation works is the same, but the constants reflect the more complicated dependency structure possible in the martingale difference sequence. Therefore, the aggregation process should produce low frequency yields with less inclination and kurtosis, at least when A is high. We consider the two regression estimators with cross-section with betas. We only consider where $X_i = (1, \beta_i)^T$ și $\Gamma = (\gamma_0, \gamma_1)^T$.

Suppose:

$$\hat{\beta}_i = \frac{\sum_{t=1}^T (R_{mt} - \overline{R}_m) R_{it}}{\sum_{t=1}^T (R_{mt} - \overline{R}_m)^2} = \frac{1}{\hat{\sigma}_m^2} \frac{1}{T} \sum_{t=1}^T (R_{mt} - \overline{R}_m) R_{it} \tag{17}$$

where $\hat{\sigma}_m^2 = \sum_{t=1}^T (R_{mt} - \overline{R}_m)^2 / T$.

Next we can formulate:

$$Z_t = X\Gamma + u_t = \hat{X}\Gamma + v_t, \quad \overline{Z} = \hat{X}\Gamma + \overline{v} \tag{18}$$

where $v_t = (X - \hat{X})\Gamma + u_t$ și $\overline{v} = (X - \hat{X})\Gamma + \overline{u}$. We have:

$$\hat{\Gamma}_t = \Gamma + (\hat{X}^T W \hat{X})^{-1} \hat{X}^T W v_t, \quad \hat{\Gamma} = \Gamma + (\hat{X}^T W \hat{X})^{-1} \hat{X}^T W v \tag{19}$$

but $\hat{X} \xrightarrow{P} X$ cu $T \to \infty$, we have:

$$\hat{X}^T W \hat{X} \xrightarrow{P} X^T W X \tag{20}$$

$$\sqrt{T}(\hat{\Gamma} - \Gamma) = (X^T W X)^{-1} X^T W \overline{v}_t + \mathbb{R}_T, \tag{21}$$

Where the term remains $\mathbb{R}_T \xrightarrow{P} 0$ with $T \to \infty$. The essential problem is obtaining $var(\overline{v})$, which leads to the relationship:

$$\sqrt{T}\overline{v} = \sqrt{T}\overline{u} - \sqrt{T}(\overline{X} - X)\Gamma \tag{22}$$

and both terms generally contribute and might correlate with each other because they use the same time series. It was calculated $var(u_t)$ and consequently we have the relationship:

$$(X^T W X)^{-1} X^T W \sqrt{T}\overline{v} = (X^T W X)^{-1} X^T W \sqrt{T}\overline{e} - \sqrt{T}(\overline{\Gamma} - \Gamma) - (X^T W X)^{-1} X^T W \sqrt{T}(\hat{\beta} - \beta)\gamma_1 \sim AU_T \tag{23}$$

where A is the matrix $2 \times (2N+2)$ and U_T it is $(2N+2) \times 1$ a vector of stochastic variables:

$$A = \left[(X^T W X)^{-1} X^T W, -1, -1, \frac{\gamma_1}{\sigma_m^2} (X^T W X)^{-1} X^T W \right]$$

$$U_T = \begin{pmatrix} \frac{1}{\sqrt{T}} \sum_{t=1}^T e_t \\ \frac{1}{\sqrt{T}} \sum_{t=1}^T (\Gamma_t - \Gamma) \\ \frac{1}{\sqrt{T}} \sum_{t=1}^T [R_{mt} - E(R_{mt})] e_t \end{pmatrix}, \tag{24}$$

where $\hat{\sigma}_m^2 = var(R_{mt})$. Under the null hypothesis, U_T is zero, and the covariance matrix a U_T is the main diagonal.

$$\Lambda = \begin{pmatrix} \Omega_\varepsilon & 0 & 0 \\ 0 & \Pi & 0 \\ 0 & 0 & \sigma_m^2 \Omega_\varepsilon \end{pmatrix} \tag{25}$$

In this case $\Pi = diag(\sigma_0^2, \sigma_m^2 + \sigma_0^2)$ where $\sigma_0^2 = var(R_{0t})$ because $var(\gamma_{1t}) = var(R_{mt} - R_{0t}) = \sigma_m^2 + \sigma_0^2$ and beta zero portfolio is not correlated with market performance. Because we assume that $E(e_t e_t^T | \Gamma_1, \ldots, \Gamma_T) = \Omega_\varepsilon$ either does not depend on Γ_t and so on $E\{[R_{mt} - E(R_{mt})] e_t e_t^T\} = E[R_{mt} - E(R_{mt})] \Omega_\varepsilon = 0$. In the case of time series the hetrostedasticity is not true, and the limiting variation is more complicated.

Use a different period B for estimating it β from the C period for the cross section regression can ensure the structure of the main diagonal and under hetrostedasticity.

The definition of the next covariance matrix is:

$$\phi^* = \lim_{T \to \infty} var\left(\sqrt{T}v\right) = \lim_{T \to \infty} var\left(\sqrt{T}\bar{u}\right) + \lim_{T \to \infty} var\left[\sqrt{T}(\hat{\beta} - \beta)\gamma_1\right] = A\Lambda A^T = \phi + \frac{\gamma_1^2}{\sigma_m^2}\Omega_\varepsilon$$

(26)

Under the null hypothesis with $T \to \infty$ with N fixed:

$$\sqrt{T}(\Gamma - \Gamma) => N(0, \Upsilon)$$
$$\Upsilon = (X^T W X)^{-1} X^T W \phi^* W X (X^T W X)^{-1}$$

(27)

Also, we can show that standard FM errors are not taken into account. We have:

$$\hat{\Gamma}_t - \hat{\Gamma} = \left(\hat{X}^T W \hat{X}\right)^{-1} \hat{X}^T W (v_t - \bar{v}) = \left(\hat{X}^T W \hat{X}\right)^{-1} \hat{X}^T W (u_t - \bar{u})$$

(28)

Furthermore:

$$\hat{\Psi} = \frac{1}{T}\sum_{t=1}^{T}\left(\hat{\Gamma}_t - \hat{\Gamma}\right)\left(\hat{\Gamma}_t - \hat{\Gamma}\right)^T \xrightarrow{P} \psi$$

(29)

Such that $\hat{\Psi}$ ignores the contribution from the preliminary estimate. Instead, we can estimate constantly Υ through the relationship:

$$\hat{\Upsilon} = \hat{\Psi} + \frac{\hat{\gamma}_1^2}{\hat{\sigma}_m^2}(\hat{X}^T W \hat{X})^{-1}\hat{X}^T W \hat{\Omega}_\varepsilon W \hat{X}(\hat{X}^T W \hat{X})^{-1}$$

(30)

The portfolio grouping approach essentially uses a large section n to produce a smaller section of assets for testing the size N. Suppose the market model is of the form:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it},$$

(31)

Portfolio $R_{it} = \sum_{i=1}^{n} w_{pi} R_{it}$ where $\sum_{i=1}^{n} w_{pi} = 1$, satisfy the relationship:

$$R_{pt} = \alpha_p + \beta_p R_{mt} + \varepsilon_{pt},$$

(32)

Where $\beta_p = \sum_{i=1}^{n} w_{pi}\beta_i$ and $e_{pt} = \sum_{i=1}^{n} w_{pi} e_{it}$.

We build portfolios $p = 1, 2, \dots, N$ which do not overlap. If the portfolios are well diversified, for example we say the share exal to assets $n_p = n/P$, then the portfolio error variation $\sigma_p^2 = var(e_{pt})$ is small when n is large. In this case $\hat{\beta}_p$ can be considered to be $\sqrt{nT}$ consistent. It turns out he provided n, $T \to \infty$ with application.

$$(X^T W X)^{-1} X^T W \sqrt{T}v = (X^T W X)^{-1} X^T W \sqrt{T}\bar{e} - \sqrt{T}(\Gamma - \Gamma) + \mathbb{R}_T,$$

(33)

Where standard FM errors are consistent.

4. Conclusions

Some theoretical and practical conclusions are drawn from the ones presented in this article. Thus, first of all, grouping portfolios is a necessity that is required whenever we want to study in the capital market the assets that are on the market.

The size or market capitalization evolves from time to time and thus, the Dow Jones index shows that any large portfolio is evaluated under market conditions and thus, in the case of beta portfolios, there is a smaller problem in relation to the global trend, but there are trends and variations over time, in terms of the dispersion level, so the dispersion is a little wider. Another conclusive problem is that, from an econometric point of view, the grouping of portfolios mitigates as far as possible the error regarding the variables used and the way in which they were chosen, correlatively. On the other hand, it is obvious that this grouping of portfolios has a special significance in the study of portfolios in the capital market, ensuring

a correction of any errors that can be obtained if we use a large number of portfolios, with variations and factors of influence special. The relationships in this direction are those that give meaning and can be used in both static and dynamic models.

Another conclusion is that the time variation model must be used with care, as independent and identically distributed observations must lead to the same result. This setting, this grouping is important in the context in which the statistical inference of the results obtained in the study of portfolios and the capital market must be ensured.

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