International Journal of Academic Research in Business and Social Sciences
October 2012, Vol. 2, No. 10
ISSN: 2222-6990

A Study on Non-Linear and Chaotic Behavior of Iran's Economic Growth

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Abstract

Non-linear dynamic systems show different behaviors so that they can be applied to interpretation many of the seemingly random economic phenomena. The chaos theory provides a new approach to investigating the changes trend of non-linear dynamic systems in monetary and financial markets. This paper investigates the behavior of economic growth in Iran using the chaos theory, BDS and bootstrap test, maximum Lyapunov exponent, the Hurst exponent and phase space reconstruction. The aim of this study is to examine whether economic growth has an independent identical distribution or follows from a non-linear (random or chaotic) process. The results of BDS test and the Hurst exponent indicates that economic growth has a non-linear process. The results of two chaotic tests includes maximum Lyapunov exponent and phase space reconstruction also confirmed the existence of chaos in GDP.

Keywords: Chaos theory, BDS and bootstrap test, The Hurst exponent, Maximum Lyapunov exponent, Phase space

Introduction

The aim of the chaos theory is to identifying the ways of diagnosis discipline lies in very complex systems that in case of success allows that the future trend of their movements to be predicted contrary to previous belief. In the chaos theory, it is expressed that the complex systems have merely chaotic apparent and consequently seems random and irregular, while in reality they are function of a specified flow with a specific mathematical formula; Hence the issue of chaos in mathematics is usually raised as deterministic chaos that is formed based on the theory of non-linear growth with feedback.

So far this view has been exist that economic time series especially macroeconomic series as well as monetary and financial markets series follow a random process and therefore their changes are not predictable. The remarkable improvements in computational tools in recent decades have provided the possibility of applying the theories based on existence of seemingly random regular or chaotic non-linear models. In fact, the chaos theory allows a more detailed study of very complex behavioral characteristics of economic variables that is not feasible with usual tools. In the traditional economic and econometrics literature, a random behavior is considered for most economic variables. The result of such assumption is that variations of these variables are not predictable. In fact, the chaos theory provides the possibility of discovery the pattern and complex regulation governing the behavior of such variables and use them to prediction of future trend in short term. Despite the expansion of various methods in the literature of econometrics and computational methods to explore the chaotic process, it can't be claimed yet that these methods are well able to distinguish a linear process with random disturbances from a regular (chaotic) non-linear process. But despite the such shortcomings in empirical researches, we can generally conclude that due to the possibility of existence of the chaos process in economic series, it is insufficient to applying the standard and common method in econometrics, that is using the linear models in estimation and prediction of this series and in some cases can be followed by misleading conclusions. In terms of the economic stabilization policies, we can also conclude that such policies should be done more carefully; Because, if there be the chaotic process in some macroeconomic series, applying some inadequate and premature policies may lead to the disruption and irregularity in the trend of variables and made the complex conditions governing them substantially more complicated and therefore incontrollable.

Application of chaos model in macroeconomic models is important from three aspects. First, to explain the business cycles, in the presence of chaotic processes in macroeconomic variables, would be no longer need to assume the existence of exogenous shocks. The mere existence of certain non-linear relationships in the model can leads to fluctuations in the production. Second, restrictive monetary policies in order to control the price level (which in classical view are considered favorable policies due to their ineffectiveness on the level of production), in the presence of chaotic processes in the model can make serious irregulars in the state of economy. Third, the issue of full employment and achieving it, can be dependent to short-term targeting strategy for applying monetary policy, either asymptotic or in the case of extent periods. This point is very important in terms of difference between classical and Keynesian view in macroeconomics, since so far the main and long-term properties of the models was attributed to classics, and issues related to short-term fluctuations was attributed to Keynesian. Providing the policy strategies was also made taking with respect to same traditional and common way, such that the fiscal and monetary policies were only recommended in the short term and with the specific assumptions and conditions. As a result, discussions related to the impact of stabilization policies on production level are not only do not limited to the short term, but also are considered as an integral part of the main properties of macroeconomic models.

The empirical researches that is done so far using the mentioned test methods to determining the existence of chaos in economic series has had no consistent and coordinated results. For example, Frank and Gencay showed the existence of non-linear chaotic process in Japan's gross domestic product. Also, existence of chaos in the Spain's currency market was demonstrated by Gecen and Erkal. However Brock and sayer failed to show the existence of chaos in America's gross national product. Or Moshiri and others failed to prove the existence of chaos in total consumption of America and Canada. These different results can be somewhat attributed to the use of different methods with different null hypotheses.

In this study we are now looking for answers to this question that whether the economic growth as one of the most important macroeconomic variables in Iran follows from a random and linear process or has a non-linear and chaotic process. Also how can modeling the dynamics of non-linear variables. To achieve this goal, in the beginning we mentions the theoretical bases and then investigates the behavior of economic growth in Iran, according to the BDS criteria, Lyapunov exponent and Hurst exponent.

Theoretical Bases

In this paper, we use two methods of Largest Lyapunov Exponent (LLE) and phase space reconstruction to check the existence of chaos in the GDP time series and use R/S test analysis or Hurst exponent (HE) and BDS test to distinguish whether series has random or non-random process. In following we mention the theoretical bases of each of the above methods.

The BDS test

The BDS test is an initial step to determine whether the time series process has observations with independent identical distribution. Brock, Dechert and Scheinkman (1987) presented a test with IID assumption based on Grasberger and Prokachia correlation integral (BDS). The null hypotheses of this test is based on that time series to be IID against the hypothesis that time series is linear or non-linear correlated. In this approach, in the first, m predetermined values is considered for a time series $\{x_t ; t = 1, \dots, T\}$. For each pair of points, the probability of that the distance between these two points is less than or equal to ε is a fixed number that is shown as $c_1(\epsilon)$. Similarly, the probability of that the number of points in m-dimensional space are placed at a distance of less than or equal to ε is defined as $c_m(\epsilon)$ where m is the same embedded dimension or the number of consecutive points in targeted set.

$$\begin{aligned} &P_1 = P(\|x_t - x_s\| < \epsilon) \rightarrow c_1(\epsilon) \\ &P_2 = P(\|x_{t-1} - x_{s-1}\| < \epsilon, \|x_t - x_s\| < \|) \rightarrow c_2(\epsilon) \\ &\vdots \\ &P_m = P(\|x_{t-m+1} - x_{s-m+1}\| < \epsilon, \dots, \|x_t - x_s\| < \epsilon) \rightarrow c_m(\epsilon) \end{aligned} \quad (1)$$

Where, x_t^m and x_s^m are time series with m predetermined values. When the observations, $\{x_t\}$ has independent identical distribution, then we will have $p_m = p_1^m$, means that despite the independence of observations condition, we can write:

$$c_m(\epsilon) = c_1^m(\epsilon) \quad (2)$$

Where, $C_m(\epsilon)$ is the Correlation Integral or Correlation Sum or the number of points in m-dimensional space that has a less than ϵ small and defined amount distance from each other. If the time series is the result of a random process, with increase of embedded dimension, the points in m-dimensional space, will be scattered in all directions, but if the series is the result of a regular process, the points is absorbed toward a subset of the absorbent space. In this situation, with the increase of embedded dimension, the absorbent dimension in the state space does not exceed from a range and will be a number smaller than m. The correlation dimensions are given by $\mathrm{D^m = \lim_{\varepsilon \to \infty} \frac{\log C_m}{\log \varepsilon}}$.

In a chaotic system for a certain amount of ε, with increase of m, the number of points which has a distance less than ε in state space, is reduced and as a result, the value of correlation dimension is converged to a saturation limit, while in a random system, with increase of m, D^m also increases.

When we use the sample data, we can't observe $c_1(\epsilon)$ and $c_m(\epsilon)$ directly, but they can be estimated from sample. Thus, the existence of errors in the above relationship can be expected. Whatever the error is much, the error is generated with less likely by random sample changes. The BDS test provides a formal basis to judgments about the size of this error. To estimate the probability of a given dimension, we investigate all possible extractive sets from sample, and count the number of sets that satisfy the condition ϵ. The ratio of the number of sets that satisfy the condition to total number of sets, gives the estimation of probability. If we have a sample with n observations from the series x_t, then we can write:

$$C(\epsilon, m) = \frac{1}{T_m(T_m - 1)} \sum_{i,j=1}^{T_m} H\left(\left\| X_i^m - X_j^m \right\|\right) \qquad (3)$$

This equation is the same correlation integral from dimension m that measures the spatial correlation between T scattered points in m-dimensional space and selects a part of m-dimensional binary points among them, (x_t^m, x_s^m), that their distance is less than or equal to ε. $\mathrm{T_m = T - m + 1}$ is the number of m previous values of the sequence $X_t^m = (X_t X_t^m, X_{t-1}, \dots, X_{t-m+1})$, that has been made from sample with following lengths:

$$T; H(\|X - Y\|) = \prod_{s=1}^{m} H(|X_s - Y_s|) \qquad (4)$$

Where, m is the embedded dimension and H is the Heaviside function:

$$H\left(\left\|X_i - X_j\right\|\right) = \begin{cases} 1 & if\ \left\|X_i - X_j\right\| < \epsilon \\ 0 & if\ \left\|X_i - X_j\right\| \geq \epsilon \end{cases} \qquad (5)$$

This sample estimation is used to make the independence test statistic.

$$b_{m,T}(\epsilon) = c_{m,T}(\epsilon) - c_{1,T}^{m}(\epsilon) \quad (6)$$

The second term eliminates the m-1 last observation from sample, so that the number of terms of two statistics $c_{m,T}$ and $b_{m,T}$ becomes equal. Indeed, Brock and others in 1996 showed that:

$$\sqrt{T-m+1}\,\frac{b_{(m,T)}(\epsilon)}{\sigma_{m,T}(\epsilon)} \to N(0,\sigma_m^2) \;\; with \;\; T_m \to \infty \quad (7)$$

That is $b_{m,T}$ is convergent to the standard normal distribution with zero mean and variance σ_m^2.

$$\sigma_{m,T}(\epsilon) = 4(k^m + 2\sum_{j=1}^{m-1} k^{m-j} c_1^{2j} + (m-1)^2 c_1^{2m} - m^2 k c_1^{2m-2}) \quad (8)$$

Where c_1 can be estimated using $c_{1,m}$, k is the probability that each ternary of observations is placed in a distance less than or equal to ϵ. Thus, BDS statistic is written as follows:

$$W(\epsilon,m) = \frac{T_m^{\frac{1}{2}}\left[C(\epsilon,m) - \left(C(\epsilon,1)\right)^m\right]}{\sigma_m(\epsilon)} \quad (9)$$

Under null hypotheses, x_t has normal independent identical distribution with zero mean and unit variance. Note that $W(\epsilon,m)$ is a function of the two unknown parameters, the embedded dimension m and the radius ϵ. Considering the characteristics of a small sample for BDS statistic, there is a significant relationship between the choice of m and ε. For a given m, ε can not be too small because otherwise there won't be enough number of pairs of point X_i and X_j that their maximum distance be less than or equal to ε (the necessary condition for calculating the correlation integral). These small values of ε leads to a gradient that is close to m, due to the problem of noise,(noise chaos), (Brock and others, 1987). On the other hand, ϵ should not be too large, since the correlation integral includes many observations.

The Hurst test

The Hurst exponent is a suitable tool for diagnosis of a non-random time series from a random series, regardless of its distribution. The Hurst test and study method was also gradually extended to other phenomena that seems random in appearance but may have a regular pattern. Test method is as follows[1]: Consider a time series $x = x_1, \ldots, x_n$. First, the scale of data is changed or in other words is normalized as follow:

[1]. Edgar Peter's book is a good source for this test and its applications in financial markets.

$$Z_r = (x_r - x_m) \ , r = 1, \dots, n \qquad (10)$$

Where, x_m is the mean of series. In the next step, a new time series is calculated as follow:

$$Y_r = (Z_1 + Z_r) \ , r = 2, \dots, n \qquad (11)$$

Since Z has zero mean, the last value of Y, namely Y_n, will be zero always. The modified scope will be equal to:

$$R_n = \max(Y_1, \dots, Y_n) - \min\ (Y_1, \dots, Y_n) \qquad (12)$$

Obviously, since Y has zero mean, it's maximum value is always greater than or equal to zero and the minimum value is always less than or equal to zero. Therefore, the modified Scope (R_n) will be always non-negative. Hurst defined the following relationship using the half rule in the statistics[2]:

$$\left(\frac{R}{S}\right)_n = a.n^H \qquad (13)$$

Where, R is the scale revised scope, S is the standard deviation of time series, a is a constant, n is the number of observations and H is the Hurst exponent. The above formula can be rewritten approximately as follows:

$$\log(\frac{R}{S})_n = \log a + H \log(n) \qquad (14)$$

In practice, one can estimate the coefficient of the Hurst exponent (H) by running a regression. According to the Hurst results, if the value of the Hurst exponent is equal to 0.5, implies that there is an independent process; If it is placed between 0.5 and 1, implies that the time series is durable with long-term memory. Finally, if the Hurst exponent is equal to a positive value but less than 0.5, implies the temporality of the process. The studies have shown that the many of the series in nature and some economic series especially in the capital market are not random and have relatively long-term memory and duration.

Maximum Lyapunov Exponent

The Lyapunov exponent test is based on the characteristic of chaotic series that the neighboring points in this series is separated from each other over time and become divergent to each other. The Lyapunov exponent measures this divergence by an exponential function. Calculating the Lyapunov exponent is conducted through measuring the amount of stretching or bending that occurs in the motion of system. In fact, in this way, the average rate that two-point transmission paths, that were initially close together, exponentially away from each other, is calculated. If the calculated Lyapunov largest exponent has a positive amount, then the system has a chaotic behavior, and vice versa. It can be shown that the Lyapunov exponent is representable as follows:

[2]. This rule is defined based on the Einstein rule. Under this rule, the distance which is traveled by a random element, is a function of square root of time that is spent to measuring it, that is $R = T^{.5}$, where, R is the traveled distance and T is the time index.

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \log \left| \frac{df^n(x_0)}{dx_0} \right| \quad (15)$$

λ is known as Lyapunov exponent, where the term inside of || is the derivative of the function f. For the chaotic series, the sign of Lyapunov exponent is positive and is negative otherwise.

Evaluation of economic growth in Iran

The results of BDS test for the GDP growth time series are shown in table 1 for embedded dimensions 2 through 6. The Z statistics (BDS test divided by standard deviation) is applied to evaluating the null hypotheses of test. The large amounts of this statistics or small amount of the test probability (zero) rejects the null hypotheses (the data generating process is IID) and accepts the opposite hypothesis that time series has general dependence (random and chaotic non-linear process). However, the result of this test mentioned in table 1, just confirms that Iran's economic growth has non-linear process and it is not confirmed yet that this time series has chaotic process.

Table1. The results of the BDS test by selecting ϵ (Fraction of Pairs)

Dimension	BDS Statistic	Std. Error	z-Statistic	Normal Prob.	Bootstrap Prob.
2	0.155586	0.010231	15.20704	0.0000	0.0000
3	0.249941	0.016491	15.15657	0.0000	0.0000
4	0.304186	0.019916	15.27349	0.0000	0.0000
5	0.325591	0.021055	15.46362	0.0000	0.0000
6	0.332400	0.020599	16.13670	0.0000	0.0000
Raw epsilon		194836.2			
Pairs within epsilon		1916.000	V-Statistic	0.708580	
Triples within epsilon		75734.00	V-Statistic	0.538618	
Dimension	C(m,n)	c(m,n)	C(1,n-(m-1))	c(1,n-(m-1))	c(1,n-(m-1))^k
2	868.0000	0.680784	924.0000	0.724706	0.525199
3	815.0000	0.665306	914.0000	0.746122	0.415365
4	772.0000	0.656463	906.0000	0.770408	0.352276
5	732.0000	0.648936	900.0000	0.797872	0.323346
6	696.0000	0.643848	890.0000	0.823312	0.311448

In this paper, the Hurst exponent test is also used to ensure that time series process is nonlinear. According to the expressed algorithm, the estimated value for Hurst exponent for GDP is exactly 0.98 indicates that this time series is durable with a long-term memory as well as have a nonrandom process. The estimation of largest Lyapunov exponent (LLE) is also presented in table 2. The amount of LLE is positive for all dimensions between 2 through 11 that provide a strong evidence that the time series of economic growth is chaotic.

Phase Space Reconstruction

In order to study the geometric and dynamic properties of a given system, we can use from description of the state space. But in many practical processes, we can rarely measure all dynamic variables in the system and the scalar series of system observations is available only. The dynamics governing these processes is not specified directly from this data series. So, one of the most basic steps in analysis of time series obtained from a nonlinear process, is state space reconstruction with limited dimensions using this series, so that is equivalent to the state space of data generating process.

Table 2 . LLE for different embedded dimensions

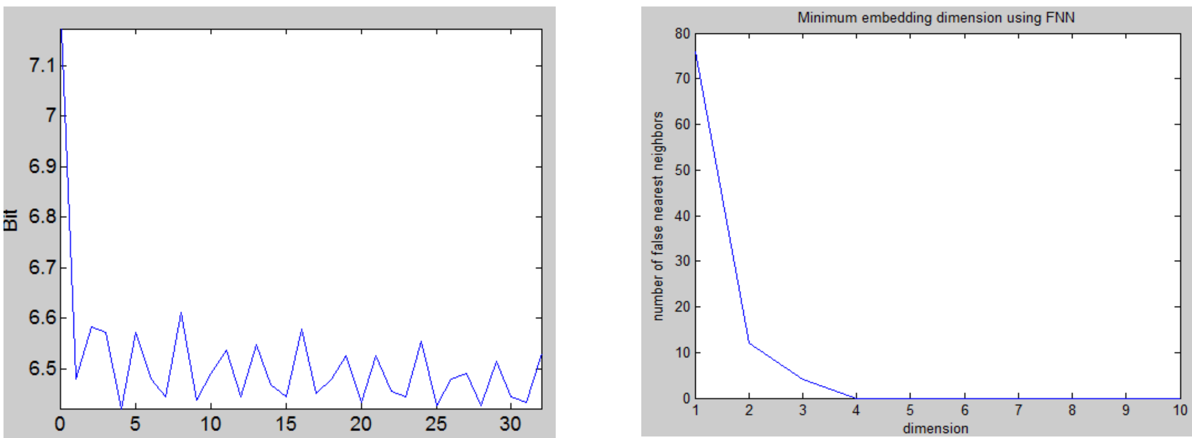
Embedding Dimension	Maximum LE
2	4.33
3	3.95
4	3.71
5	3.42
6	3.24
7	3.08
8	2.93
9	2.74
10	2.60
11	2.49

The issue of state space reconstruction of time series can be solved by the surrounded theory. In fact, the points on the absorbent of system have a one-to-one relationship with the measurements performed for the dynamical variables of system. On the other hand, these points contain complete information about the current state of the system. Thus, the existence of a one-to-one relationship means that a state of the phase space is identifiable by carried out measurements. Hence, it is need to seek a mapping from absorbent of system to reconstructed space, such that this mapping be one-to-one and can maintain the information of the system. This actually is the conceptual definition of embedded dimension. Based on the Taken theory, if the time series is obtained from a given dynamic system, then there are a scalar m represents the embedded dimension and a scalar τ represents delay time (the delay is optional) and a function f, so that:

$$y(t+1) = f(y(t), y(t-\tau), \ldots, y(t-(m-1)\tau) \qquad (16)$$

If the data in time series are chaotic, then the function f is necessarily nonlinear. In this paper, minimum embedded dimensions have been obtained using FNN function in the Matlab environment. As is observed in figure 1.b the optimum embedded dimension is 4. However, due to that the tolerance value of FNN method is considered equal to 10, so the minimum embedded dimension is considered where the number of false neighbors be close to tolerance value. As can be seen in the figure, this value is equal to 2.

To obtain the delay time of the first minimum point, the mutual information function is same τ that in figure 1.a this amount is equal to 2. Having two parameters τ and m, the phase space is reconstructed that this is shown in figure 3. As it is seen in the figure, we have a specific pattern on GDP in reconstructed phase space suggests that there is chaos in the process of economic growth.



(a) (b)

Fig.1 (a) Mutual information function for delay time. (b) Minimum embedding dimension using Cao's method.

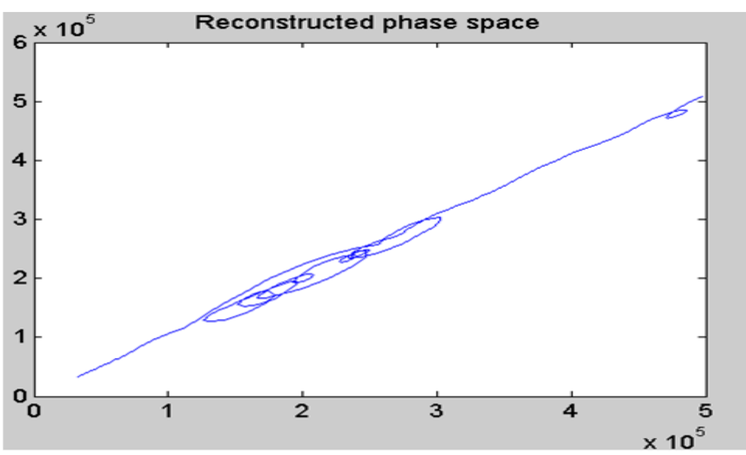


Figure 2 . Phase space reconstruction

Summary and Conclusions

As was demonstrated in this paper, GDP growth could follows a chaotic process. The results of BDS test and Hurst exponent confirmed nonlinearity of the process. The positive sign of Lyapunov exponent and phase space reconstruction, confirms that time series is chaotic. The remarkable point in hear is lake of GDP data. Having more data, the result of phase space reconstruction will be lead to more accurate directions. Having a time series contains a number of $10^{2+0.4d}$ data points; the correlation dimension can also be used as an additional test to examine the existence of chaos in the time series.

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