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Frequency Population Growth Rate

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Abstract

The Solow growth model assumes that labor force grows exponentially. That is not a realistic assumption. In generalized logistic equations that describes more accurately population growth. Economic growth is not a smooth process. Real GDP has fluctuations in the growth rate. We call these fluctuations business cycles. Business cycle theory came about from the failures of classical economics in being able to illuminate on the causes of the Great Depression. The logistic growth model to explain changes in population growth rates are not. In this paper a new analysis of the population growth rate in the frequency space is described with mathematical logic and economic reasoning, so that, firstly, to a higher level of capital per capita, or at least equal to the Solow growth model reaches Second, the limits of saturation (Carrying-Capacity) is not, and ultimately, population growth rates have an impact on long-term per capita amounts. The initial classic assumption is changed in this article based on the available frequencies in the population growth equation.

Keywords: Solow Growth Model, Population Growth, The Fourier series, Frequency

Introduction

Cycling populations have received much attention from economic and mathematical demographers (Easterlin, 1961; Lee, 1974; Samuelson, 1976; Frauenthal and Swick, 1983; Wachter and Lee, 1989). Population analysts have modeled those using nonlinear wave equations with cohort and labor force feedback mechanisms, studying the form and strength of the feedback needed to sustain population cycles.

In this paper a new analysis of the population growth rate in the frequency space is described with mathematical logic and economic reasoning, so that, firstly, to a higher level of capital per capita, or at least equal to the Solow growth model reaches Second, the limits of saturation (Carrying-Capacity) is not, and ultimately, population growth rates have an impact on long-term per capita amounts.

Samuelson (1976)argues that the first article entitled "An Economist's Non-linear Model of Self-generated fertility Waves" that we created a wave of fertility rates in the form of a nonlinear mathematical model can be expressed. Also in the equation to explain the frequency of paper Shone and Kim (1997) entitled "Exploring cyclic net reproduction "used. So that the general model of pure breeding and calving sine functions with any frequency (**The Fourier series**) is expressed.

Change is the logistic growth model for this is the course we consider three cases in which only the third case, the long-term population growth rates converge to a fixed rate to a fixed rate , other states converge is not. In this proposed model we obtain the same result as that of the Solow's economic growth. This happens when the economic power is increased with the possibility of negative population growth rate in the steady-state.

The Model

Since the early contributions to the topic, business cycles have been considered as essentially connected with the development of capitalist economies. In the early 1980s the issue of the relationship between growth and cycles was addressed within a market clearing environment by the real business cycle (RBC) literature.

The current definiton of business cycles is that they are simply movements about trend in GDP. Contrary to economic thinking of 20 years ago, business cycles are not all alike. They vary in both amplitude and frequency. This characteristic makes prediction and stabilization of cycles very di cult. We can use some basic statistics to separate the trend in GDP from the cyclical fluctuations. One such exercise would be to run a simple linear regression on GDP data. The regression line would be the trend in GDP, and the residuals would represent the cycles. This approach to identifying business cycles places no limitations on the length, nature, of phases of the cycles. Another common technique is to use the Solow growth model to identify business cycles. Remember that output in the neoclassical growth models growing at a constant rate. The model fails to produce any fluctuations in economic growth in steady-state. However, we have seen that, if proper calibrated we can do a good job of matching the trend in GDP over time. One way to measure business cycles is to measure the difference between the Solow generated GDP and actual GDP. Any differences are considered cycles, and are often called "Solow Residuals" .Although business cycles do not follow a deterministic pattern, they do exhibit a number of empirical regularities . From Kydland and Prescott we see that the standard deviation in output is 1.71% will the standard deviation in investment is about 8.3%. Thus investment is about 5 times more volatile than in output.

By introducing Fourier series, it is possible to determine the birth trajectory when the time trajectory of net reduction follows virtually any function. Let any cyclic reproductive function with period *T* be represented by the exponentiated Fourier series. An exponentiated sinusoidal net maternity function is considered in detail, as populations with cyclically varying net maternity are of particular interest because of their connection to the Easterlin hypothesis. The dynamics of the model are largely determined by the ratio of the population's generation length *(A)* to the period of cyclist (T), and relatively simple expressions are found for the phase difference and relative amplification of the birth and net reproduction functions. More generally, an analytical expression for a population's birth trajectory is derived that applies whenever net reproductively can be written as an exponentiated Fourier series. In the cyclic model, Easterly's inverse relationship between cohort size and cohort fertility holds whenever the phase difference is zero. At other phase differences, the birth-reproduction equations have the form of predator-prey equations. The present analytical approach may thus be relevant to analyses of interacting populations(Sochen and Kim,1997).

Efforts to model populations with changing vital rates have been impeded by the lack of closed form relationships between vital rates and the resulting births. Sinusoid ally fluctuating vital rates were studied by Coale (1972) and Tuljapurkar (1985) using a Fourier series approach to the birth function. To obtain an approximate solution, however, they needed to assume a small amplitude of oscillation and consider only the first harmonic of the Fourier series.

The explanation was given about the Fourier series, it is possible that the net reproduction function is a different frequency and direction of these frequencies is a function of birth in a stable condition. As a result of the exponential Fourier series with period T and frequency ω is as follows(Sochen and Kim,1997):

$$R(t) = exp\left[\frac{1}{2}a_0 + \sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)\right] \quad (1)$$

Now, the net production function and a simplified mathematical equations with respect to the relationship that exists between the birth of the path can be achieved, so that we can come to the following function:

$$g(t) = exp\left[\frac{1}{2}\left\{\frac{a_0 t}{a} + \sum_m a_m \left[\sin(m\omega t)\cot\left(\frac{1}{2}m\omega A\right) + \cos(m\omega t) - 1\right] + \sum_m b_m \left[\sin(m\omega t) + (1-\cos(m\omega t))cott\left(\frac{1}{2}m\omega A\right)\right]\right\}\right]$$

(2)

Change is here in the logistic growth model and the frequency equation is applied to the growth of our workforce. It is for this we consider the three cases, the third mode is the only long-term population growth rates converge to a fixed rate to a fixed rate , other states are not converging.

$$L(t) = [\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)] + \boldsymbol{ae^{nt}} \quad (5)$$

Frequency is limited by the number of sentences.

$$\dot{L}(t) = \frac{\partial L}{\partial t} = \sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t) + ane^{nt} \quad (6)$$

$$n(t) = \frac{L(t)}{L(t)} \underline{\ } \frac{\sum_m c_m cos(m\omega t)+\sum_m d_m sin(m\omega t)+ane^{nt}}{\sum_m a_m cos(m\omega t)+\sum_m b_m sin(m\omega t)++ae^{nt}}$$

Proof that long-term in the equation is convergent to a constant growth rate is given below:

$$\lim_{t\to\infty} n(t) = \lim_{t\to\infty} F(t,\omega,m,n) =$$

$$\lim_{t\to\infty} \frac{\sum_m c_m cos(m\omega t) + \sum_m d_m \sin(m\omega t) + ane^{nt}}{\sum_m a_m cos(m\omega t) + \sum_m b_m \sin(m\omega t) + +\boldsymbol{ae^{nt}}} = \quad (7)$$

$$\lim_{t\to\infty} \frac{\boldsymbol{ae^{nt}}\left(\frac{\boldsymbol{\sum_m c_m cos(m\omega t) + \sum_m d_m sin(m\omega t)}}{\boldsymbol{ae^{nt}}} + \boldsymbol{n}\right)}{\boldsymbol{ae^{nt}}\left(\frac{\sum_m a_m cos(m\omega t) + \sum_m b_m sin(m\omega t)}{\boldsymbol{ae^{nt}}} + 1\right)} = n$$

We here note that the number of clauses m limit is considered. This result is very accurate is this model, the frequency rate of population growth in the long run to a growth rate of fixed convergent, but during the period of transition population growth rate oscillations is faced with the oscillations of the cosine and sine terms can be explained and this analysis is more consistent with economic realities and facts.

$$n(t) = \frac{L(t)}{L(t)} \underline{\ } \frac{\sum_m c_m cos(m\omega t)+\sum_m d_m sin(m\omega t)+ane^{nt}}{\sum_m a_m cos(m\omega t)+\sum_m b_m sin(m\omega t)++ae^{nt}} = F(t,\omega,m,n)$$

Samuelson's Business Cycle Model

As mentioned, Samuelson chose to model in discrete time, and he chose a second order process. There is a basic time period unit, and all variables are dated, either flow, such as income, investment, and savings, attributed to periods, or stocks, such as capital, attributed to moments of time.

In this model we have:

$$S_t = sY_{t-1}$$
$$C_t = (1-s).Y_{t-1} = cY_{t-1}$$
$$K_t = a.Y_{t-1}$$
$$I_t = K_t - K_{t-1} = a.(Y_{t-1} - Y_{t-2})$$
$$Y_t = C_t + I_t$$

we readily obtain the reduced form recurrence equation in the income variable alone:

$Y_t = (a + c).Y_{t-1} - a.Y_{t-2}$

It is second order, as we see, and hence capable of generating growth or cycles. However, like Phillips's model, it is linear.

The general solution is:

$Y_t = A{\gamma_1}^t + B{\gamma_2}^t$

where

$\gamma_{1,2} = \frac{(a+c)}{2} \pm \frac{1}{2}\sqrt{(a+c)^2 - 4a}$

When $(a+c)^2 < 4a$, then, become complex conjugates. If so, it is more convenient to write the general solution as:

$Y_t = \mu^t\big(A sin(\omega t) + B cos(\omega t)\big)$

$\mu = \sqrt{a}$

$\omega = \arccos(\frac{a+c}{2\sqrt{a}})$

We see that with complex conjugate roots, the solution is the product of a power function and a stationary trigonometric oscillation. This is the case of primary interest in connection with business cycle theory.

Conclusion

The attitude of the population growth equation in frequency space analysis Solow growth model - Swan merits is valuable, that here it is:

1 - the capital per capita is obtained from this frequency. Because the equation obtained Solow - Swan, with its resolution of the equation of capital per capita is surely contain statements sine and cosine, since population growth rate in this equation is contained in this sentence.

2 - The per capita production is obtained from this frequency. Because the frequency is capital per capita.

3 - The per capita consumption is obtained from the frequency model. Because the per capita production is the frequency.

4- The fact that the rate of population growth and economic realities associated with fluctuations over time and sometimes increasing and sometimes taken to reduce these fluctuations and repeated, With this type of equation is decisive, because the sine and cosine equations with oscillatory rates are.

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