Long Memory Behavior in the Returns of the Mexican Stock Market: Arfima Models and Value at Risk Estimation

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Abstract

Models previously applied to the case of emerging markets have neglected to study the presence of long memory of asset returns taking into account autoregressive fractionally integrated models and different distribution alternatives. To analyze volatility and the persistence of long memory in the returns of the Mexican stock market, as well as to determine more efficient alternatives for VaR analysis, this work applies models from the ARCH family with autoregressive fractionally integrated moving average (ARFIMA) for the mean equation; these models are estimated under alternative assumptions of normal, student-t, and skewed student-t distributions of the error term. Backtestig is used to validate the efficiency of the alternative VaR estimates; these correspond to a one day ahead investment horizon. Daily returns data for the period January 1983 to December 2009 are used to carry out the corresponding econometric analysis.

Keywords: Long Memory, VaR Analysis, Arfima modeling, Mexican Stock Market

Introduction

Research about the long memory behavior from stock markets has recently emphasized problems related to the changing correlation of prices over time, as well as those concerned with its implications on the stochastic behavior of returns. The potential presence of long memory suggests that current information is highly correlated with past information at different levels; that is, stock retunes data reflects time dependency in the generation of information flows to the market so that distant returns impact current returns. This facilitates prediction and opens the possibility to obtain speculative returns, contrary to assertions from the efficient market hypothesis. Another important implication concerning the existence of long memory in asset returns series is about the application of risk analysis models to estimate potential losses, which is the case of Value at Risk (VaR). In this regard, identifying the presence of long memory in financial assets series must aid in producing more conservative and precise estimations in VaR analysis.

Several models and empirical approaches have been applied. However, they have mostly dealt with the case of developed markets. Furthermore, models previously applied to the case of emerging markets have neglected to study the presence of long memory on asset returns taking into account autoregressive fractionally integrated models and different distribution alternatives. This study overcomes those limitations. In order to analyze volatility and the persistence of long memory in the returns of the Mexican stock market, this work applies models from the ARCH family with autoregressive fractionally integrated moving average (ARFIMA) for the mean equation. Analyses presented are compared with models estimated under alternative assumptions of normal, student-t, and skewed student-t distributions of the error term. Due to recommendations from regulatory authorities, derived from the Bassel Committee agreements, VaR has become the most applied model to assess potential losses from investment. However, there is potential tail risk in the use of VaR since conventional models neglect to take into account valuable information from the tails of a distribution of returns of financial series, which can convey to sizable losses or profits. Therefore using VaR to determine minimum capital requirements from banks or simply for investment decision making may lead to erroneous decisions, if a VaR model produces too many incorrect predictions due to the use of incorrect distributions. Thus, to determine more efficient alternatives for VaR analyses this work employs ARCH models with different distributions assumptions. Backtestig, which allows comparing actual profits and losses with VaR measures, is used to validate their efficiency. VaR estimates correspond to one day ahead investment horizon. Daily returns data for the period January 1983 to December 2009 are used to carry out the corresponding econometric analysis. The rest of the paper is organized as follows. Section II presents a review of the literature, emphasizing long memory studies about emerging markets. Section III describes the methodology. Section IV focuses on the empirical application and analysis of results. The paper ends with a brief section of conclusions.

Long Memory: Review Of Previous Studies

To determine the presence of long memory in stock market returns, ARMA time series models with fractional integration (ARFIMA), advanced by Granger (1980) and Granger and Joyeux (1980) have been widely used in the financial literature. The empirical evidence from multiple studies shows mixed results for the case of mature markets. Along this line of research Huang and Yang (1999) dealing with the NYSE and NASDAQ indexes, using intraday daily data and applying a modified R/S technique, confirm the presence of long memory in these two markets. Applying FIGARCH and HYGARCH specifications, Conrad (2007) finds significant long memory effects in the New York Stock Exchange (NYSE). Finally, Cuñado, Gil-Alana and Pérez de Gracia (2008) explore the behavior of the S&P 500 for the period August 1928 to December 2006; their results suggest that the squared returns exhibit a long memory behavior; their evidence also shows that volatility tend to be more persistent in bear markets than in bull markets.

In the case of other developed capital markets, Andreano (2005) applying the Bolllerslev and Jubinski (1999) methodology finds evidence of long memory in the returns from the Milam stock market for a sample covering the period January 1999 to September 2004. Tolvi (2003) also reports evidence of long memory for the case of the Finnish market. Lillo and Farmer (2004) prove for the London stock market that the signs and order of its series comply with a long memory process. Finally, Gil-Alana (2006) demonstrates the presence of long memory for six developed markets: EOE (Amsterdam), DAX (Frankfurt, Hang Seng (Hong Kong), FTSE 100 (London), S&P 500 (New York), CAC 40 (Paris, Singapore All Shares, and the Nikkei (Japan)

Opposite results are reported by Mills (1993) examines the U.K. stock exchange; using Lo's (1991) extension of the rescaled range (R/S) statistic and fractional ARIMA models he finds some evidence uncovering long-range dependence but results are not convincing. Confirming Mill´s results applying similar methodology, Jacobsen (1996) shows that none of the return series of indices of five European countries, and from the United States and Japan exhibits long term dependence. Lo (1991), and Cheung and Lai (1995), Yamasaki *et al* (2005), and Wang *et a*l (2006) also do not find evidence of long memory in a sample of shares from the United States. Similarly, Lobato and Savin (1998) find that S&P returns have short memory, whereas squared returns power transformations of absolute returns appear to present long memory, do not find evidence of long memory for the S&P index, using daily data for a sample for the period July 1962 to December 1994. Also examining the behavior of the S&P 500 with a large sample of 1,700 observations, Caporale and Gil-Alana (2004) find little evidence of fractional integration. Nevertheless, using squared returns, i. e., volatility, Barkoulas, Baun and Travlos (2000) find evidence of long memory, which confirms the conclusions by Ding *et al* (1993), assertion that both returns and volatility from financial markets are adequately portrayed by long memory processes. Nonetheless, feeding the controversy, Sadique and Silvapulle (2001) present mixed results in their results examining a sample of six countries: Japan, Korea, Malaysia, Singapore, Australia, New Zeland and United States. Their results suggest that returns from the markets from Korea, Malaysia, Singapore and New Zeland, essentially emerging markets, show long-run dependency in returns. Analogous results are presented by Henry (2002) about long run dependency about the returns from nine markets. Henry found evidence of long memory in four markets, two of them developed, Germany and Japan and in the emerging markets from South Korea and Taiwan; he did not find long memory in the markets from the United States, United Kingdom, Singapore, Hong Kong and Australia.

In the case of the emerging markets, consistent with their lower level of efficiency, in general, the presence of long memory is confirmed in most markets analyzed Assaf (2004, 2006), Assaf and Cavalcante (2005), Bellalah *et a*l (2005), Kilic (2004), and Wright (2002) apply a FIGARCH model to determine long-run dependency in the volatility of five emerging markets (Egypt, Brazil, Kuwait, Tunisia, Turkey) and United States. In all cases the FIGARCH estimations yield a long memory parameter very significative, confirming the presence of long memory in the volatility of these markets. Jayasuriya (2009) finds long memory in the volatility in a wide sample of 23 emerging and frontier markets from various regions. Applying an EGARCH fractional integration model his evidence reveals long memory in the returns for a wide sample covering the period January 2000 to October 2007. However no evidence of long memory is found for analyses carried out for two sub periods; this is true particularly for the most recent period for most markets analyzed, signaling a trend towards greater efficiency induced by their own development as well as from international stock market competition.

Analysis from emerging stock markets at individual levels yield similar results, with some notorious exceptions. Thupayagale (2010) finds evidence of long memory for the returns of 11 African capital markets; evidence about long memory concerning volatility is mixed. DiSario *et al* (2008) and Kasman and Torun (2007) show evidence about the existence of long memory in the returns and volatility in the Istanbul stock market. Nevertheless, applying parametric FIGARCH models and non parametric methods Kilic (2004) finds opposite evidence to what is generally reported for emerging markets, including the case of Turkey. His study reveals that daily returns are not characterized by long memory; however his study reveals that, similar to the case of developed markets, emerging markets present a dynamic long memory in the conditional variance, which can be adequately modeled by a FIGARCH model. Kurkmaz, Cevic and Özatac (2009) confirm these results. Using structural rupture tests for the variance and the model ARFIMA-FIGARCH they do not find evidence of long memory in the returns of the Istanbul market; but they did find evidence of long memory in the volatility of returns.

In relation to the emerging Asian capital markets, Cajueiro and Tabak (2004) show that the markets from Hong Kong, Singapore and China present long-run dependency in the returns from their stock markets, which has been confirmed for the case of China. Analyzing the stock market index for the Shenzhen market, Lu, Ito and Voges (2008) find significant evidence pointing out to the presence of long memory and lack of efficiency in this market. Applying fractionally integrated models Cheong (2007; 2008) presents evidence of long memory in the absolute returns, squared returns, and the volatility from the stock market from Malaysia, Also investigating the Kuala Lumpur market for the period 1992 a 2002, Cajueiro and Tabak (2004) find long memory in the volatility of returns; they report and Hurst index of 0.628. Also for this market, Cheong *et a*l (2007) prove with GARCH modeling the presence of asymmetry and long memory in the volatility of returns using daily returns for the period 1991-2005, subdividing also the series into four sub periods. Tan, Cheong and Yeap (2010) also report long memory for the Kuala Lumpur stock exchange. Applying the model by Geweke and Porter-Hudak (1983) the authors find that during the 1985-2009 period during which took place several upward and downward periods, the persistence of long memory was longer during the periods previous to the 1997 crisis.

In the case of India, Kumar (2004) proves the existence of long memory due to the presence of conditional heterokedasticity in the series. Kumar applies ARFIMA.GARCH models obtaining robust results. Similarly, Banerjee and Sahadeb (2006) find evidence of long memory in India analyzing return series SENSEX index. In his study the fractionally integrated GARCH model is the most appropriate to represent volatility.

Confirming these results, Barkoulas, Baum and Travlos (2000) analyze the long run memory in the Athens stock market using spectral regression analysis. The authors present significant statistical evidence about the existence of long memory in the Greek stock market. However, Vougas (2004) finds weak evidence concerning the presence of long memory in the Athens markets, applying an ARFIMA-GARCH model, estimated via maximum conditional likehood.

In the case of the Latin American emerging stock markets, research about long memory in these markets is limited. Cavalcante and Assaf (2002) examine the Brazilian stock market and conclude emphatically that volatility in these markets is characterized by the presence of long memory, while they find weak evidence about the existence of long memory in the returns series of this market. Cajueiro and Tabak (2005) assert that the presence of long memory in the time series from financial assets is a stylized fact. Examining a sample of individual shares listed at the Brazilian stock market they find that specific variables from the firms explain, at least partially, long memory in this market. Finally, pioneer studies account for the presence of long memory in the Mexican stock market. Islas Camargo and Venegas Martinez (2003) applying a model of stochastic volatility find evidence of long memory in the volatility of returns from the stock market index: additionally, they show the this behavior may have negative impacts on hedging with European options. Venegas Martínez and Islas Camargo (2005) present evidence of long memory in the markets from Argentina, Brazil, Chile, Mexico, and United States. Finally, López Herrera, Venegas Martínez and Sánchez Daza (2009) examine the existence of long memory in the volatility of returns from the Mexican stock market. Their evidence based on several non parametric models and parametric models with fractionally integrated models suggest the presence of long-run time dependency both in returns and volatility in this market.

Recent research has also dealt with the benefits of determining the existence of long memory for risk analysis. Giot and Lautent (2001) model VaR for the daily returns of a sample that includes stock market indexes from five developed countries: CAC40 (France), DAX (Germany), NASDAQ (United States), Nikkei (Japan) and SMI (Switzerland). They also estimate the expected shortfall and the multiple average to masseur VaR. They APARCH model produces considerable improvements in VaR prediction for one day investment horizons for both the long and short positions. In a similar study So and Yu (2006) also examine the performance of several GARCH models, including two with fractional integration. Their considers return series from the NASDAQ index from United States and FTSE from the United Kingdom and prove that VaR estimations obtained with stationary and fractional integration are superior to those obtained with the Riskmetrics model at 99.0 percent confidence levels. VaR analysis carried out by Kang and Yoon (2008) applying Riskmetrics reveal the importance of taking into account asymmetry and fat tails in the distribution of returns of corporate shares of thee important firms listed in the South Korean stock markets.

Analyzing the importance of skewness and kurtosis for determining VaR with greater precision Brooks and Pesard (2003) compare VaR estimates for the case of five Asian markets and the S&P 500 index. Models applied are Riskmetrics, semi-variance, GARCH, TGARCH, EGARCH and multivariate extensions of the considered GARCH models. Their results suggest that incorporating asymmetry generate better volatility predictions which in turn improves VaR estimations. Tu, Wong and Chang (2008) scrutinize the performance of VaR models that take into account skewness in the process of innovations. They apply the model APARCH based on the skewed t distribution; the study includes the markets from Hong Kong, Singapore, Australia, Korea Malaysia, Thailand, Philippines, Indonesia, China and Japan, albeit performance of this model is not satisfactory in all cases. A similar study by McMillan and Speigh (2007) examine daily return series for eight markets from the Asia-Pacific area, and in addition from the U.S. and U.K. markets to have a comparative frame of reference. Applying very restrictive levels of confidence, the authors find that the models that take into account long memory mitigate common under estimations from models that do not consider skewness and kurtosis in the distribution of financial series.

Summing up, the presence of long memory in the returns and volatility of final assets has important implications both for the valuation of assets, as well as for risk analysis. Several methodologies have been applied to determine the existence of long memory in returns, among them models using autoregressive fractional integration. The impact of long memory of VaR analysis also led to mixed results, particularly in the case of developed markets. In the case of emerging markets research has also led to mixed research, albeit it is important to acknowledge that research dealing with these markets is still limited.

Research Methodololgy

ARFIMA Models

Granger (1980), Granger and Joyeux (1980) and Hosking (1981) advance the concept of fractional integration to model financial time series characterized by long memory processes. These models, denominated ARFIMA (*autoregressive fractionally integrated moving average*) differ from the common stationary ARMA and ARIMA models in the lag function of the residuals; in the ARFIMA models this function is represented by $(1 - L)^d$ where d is different from cero, as is in the ARMA stationary processes or else from 1, like in the case of integrated ARMA models, i.e. ARIMA or unit root processes. A process ARFIMA (p,d,q) is generated by:

$$\phi(L)(1-L)^d = \psi(L)\varepsilon_t, \tag{1}$$

where d is not an integer and

$$(1-L)^d = \sum_{j=0}^{\infty} b_j L^j \tag{2}$$

where $b_0 = 1$ and the nth j autoregressive coefficient b_j, is given by:

$$b_j = \frac{-d\Gamma(j-d)}{\Gamma(1-d)\Gamma(j+1)} = \frac{j-d-1}{j} b_{j-1}, \qquad j \geq 1. \tag{3}$$

ARCH Models

It is a well known fact that share returns, as well as those from other financial series are characterized by time varying volatility; furthermore, large price positive changes are followed by large negative changes; similarly, small price changes are followed by small price changes; therefore, changes tend to cluster which derives in time dependency of returns. It has been also observed that the distribution of daily financial returns tend to show fat tails which is absent in the normal distribution. For that reason ARCH (autoregressive conditional heterokedasticity) models have been used extensively to analyze financial time series. The original ARCH model was put forth by Engel (1982) and soon after Bollerslev (1986), advanced a generalized version, commonly known as GARCH model. In its original version the GARCH (*p,q*) model can be expressed as follows:

$$\varepsilon_t = z_t \sigma_t$$
$$z_t = i.i.d.N.(0,1) \qquad (4)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

In this GARCH (*p,q*) model the conditional variance is explained as a lineal function from the square form past errors and from the conditional past variances. To make sure that all conditional variances are positive for all *t*, it is required that $w > 0$, $a_i \geq 0$ for $i = 0,1,2,\ldots,q$ and $b_j \geq 0$ for $j = 0,1,2,\ldots,p$. Additionally, to ensure that the model is stationary of second order it is required that $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$, since if the sum is greater or equal to 1 the process is said to be characterized by strong persistence. The case in which $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j \approx 1$ derives in a process known as integrated GARCH, or simply IGARCH.

Although the models ARCH and GARCH can capture adequately the changing behavior from volatility and clusterings from returns, as well as the fat tails from their distributions, they cannot capture the consistent asymmetric trends from negative returns, which comparatively are greater than positive returns, even though the magnitude of the shocks that lead to them might be equal for both of them. This is known as the leverage effect and to capture it several asymmetric models from the GARCH family have been developed. Glisten, Jagannathan and Runkle (1993) have advanced one of the most important models, commonly known as GJR model, which can be expressed as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-i}^2 + \gamma_i S_{t-i}^- \varepsilon_{t-i}^2\right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \qquad (5)$$

Here S_{t-1}^- is a *dummy* variable with a value of 1 when the shock is negative and of 0 otherwise.

Asymmetric Power ARCH (APARCH) is a more general model and was original presented by Ding, Granger and Engle (1993); the model combines a changing exponent with the coefficient of asymmetry which is required to capture the leverage effect. The APARCH model can be represented by:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i \left(\left|\varepsilon_{t-i}\right| - \gamma_i \varepsilon_{t-1}\right)^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta . \qquad (6)$$

An additional advantage to its flexibility is that several particular models can be nested as particular cases into the APARCH model.

VaR and Backtesting

In terms of risk analysis and management, potential losses are also associated with the long memory from the volatility of returns of financial assets traded in a market. Thus, prediction of potential losses identifying the long memory behavior of returns and volatility from a financial asset should lead to more precise and thrust worthy estimates than those obtained with traditional VaR analysis. To test the benefits of AFIRMA models on VaR analysis this paper applies backtesting using the Kupiec model (1995). VaR estimates for the case of returns from the Mexican stock market index, for a one day ahead time horizon, are obtained by an internal (in-sample) application from G@ARH 4.2 (Laurent y Peters, 2006) in the Ox V5.0 matrix program developed by Doornik (2001; 2007). Obtained estimates are tested with the backtesting methodology.

Backtesting can be summarized as follows. Assuming that $n = \sum_{t=1}^{T} I_{t+1}$ represents the number of days within a period T, where losses on the investment exceeded the estimated VaR value, while I_{t+1} is a series of failures from VaR that can be expressed in the following way for the long and short positions:

$$\text{Long: } I_{t+1} = \begin{cases} 1, \ \text{si } R_{t+1} < \text{VaR}_{t+1}|t \\ 0, \ \text{si } R_{t+1} \geq \text{VaR}_{t+1}|t \end{cases}$$

$$\text{Short: } I_{t+1} = \begin{cases} 1, \ \text{si } R_{t+1} > \text{VaR}_{t+1}|t \\ 0, \ \text{si } R_{t+1} \leq \text{VaR}_{t+1}|t \end{cases}$$

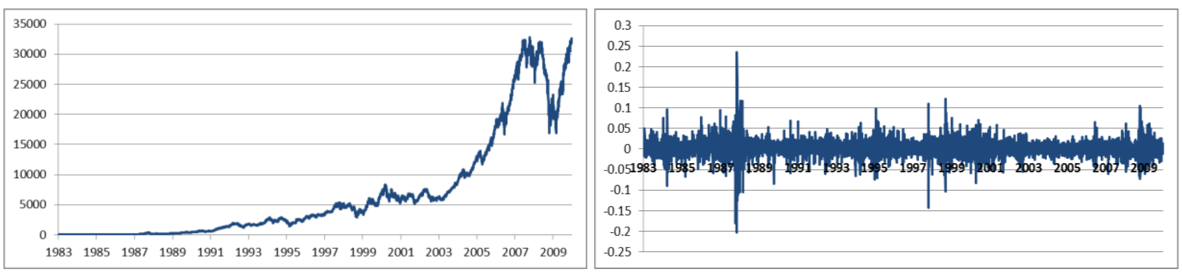
The coefficient of likehood proposed by Kupiec shows how many times a VaR is violated in a given spam of time; when the failure rate α is equal to the expected coefficent, that is, $\alpha = 1 - p$, where p is the confidence level used to estimate VaR. If T represents the total number trials, then the number of failures n follows a binomial distribution with probability α. Thus Kupiec's likehood statistic can be defined as follows:

$$LR = 2ln\left[\left(\frac{n}{T}\right)^{n}\left(1-\frac{n}{T}\right)^{T-n}\right] - 2ln[(\alpha)^{n}(1-\alpha)^{T-n}],$$

Which follows a Chi square distribution with one degree of freedom and the null hypothesis $H_0: \frac{n}{T} = \alpha$. In other words, the null hypothesis implies that the VaR model is highly significant to estimate expected losses in a given time horizon and a given confidence level; the alternative hypothesis rejects the VaR model when it generates a number of failures too large or else too small.

Empirical Evidence

Figure 1 depicts the behavior of the Mexican Stock Market for the period under study. Panel 1 shows its explosive growth from an isolated rather stagnant local market into, thanks to financial liberalization, an amazingly increasing emerging market reaching price levels amounting to 32,120.47 points in 2009 from its original level of 0.67 points in 1983 (September 1979 = 1.0). Panel 2 shows the behavior of returns. High volatility is present (standard deviation of 8,882.21 points). Additionally, important clusters of volatility associated with local and international turbulences in the financial markets can be distinguished. The greater spread in returns surprisingly did not take during the peso crisis of 1994-1998, but place during the 1987 world financial crisis; the lowest return was of minus 20.24 percent on November 16 and the highest return amounted to 23.58 percent on November 11, certainly a mind-boggling spread of 47.16 percent.



Panel A: Stock Market Index Panel B: Stock Market Returns

Figure 1: Behavior of Prices and Returns of the Mexican Stock Market (Daily series, 1983 -2009)

Employing the Mexican Stock Market data, log returns are used to apply the models proponed in this work: $100*(\ln P_t - \ln P_{t-1}) = r_t$. The daily return series starts the first working day from 1983 and ends the last operating day of 2009; the total sample includes 6755 observations. Volatility of returns is estimated applying the ARCH model described previously; their parameters are estimated taking into consideration the error terms from three different distributions: normal, Student-t and Student-t asymmetric: in all cases the equation for the mean was estimated using an AR(2) fractionally integrated model.

Table 1 presents the estimates from the GARCH model. In general, all the estimated parameters show highly significant statistical values, including at a one percent level of significance. The numerical values are similar and statistically equal in the case of the three distributions. In all three cases the long memory parameter for the mean equation is highly significant. However the value of this parameter is smaller for the Gaussian estimates, but presents a larger standard error. Tables 2, 3 and 4 summarize results for the estimates from the other models from the ARCH family, also for the case of the three distributions under analysis.

Table 1. Model ARFI (2)-GARCH (1, 1)

		Coefficient	*Standard error*	*T*	*p- value*
Errors with gaussian distribution	*m*	0.210049	0.032327	6.498	< 0.01
	d_{Arfima}	0.058881	0.022066	2.668	< 0.01
	f_1	0.159626	0.024567	6.497	< 0.01
	f_2	-0.076916	0.015093	-5.096	< 0.01
	w	0.102092	0.025921	3.939	< 0.01
	a	0.133607	0.020685	6.459	< 0.01
	b	0.835582	0.023572	35.45	< 0.01
Errors With student_t distribution	*m*	0.195432	0.031165	6.271	< 0.01
	d_{Arfima}	0.079041	0.017234	4.586	< 0.01
	f_1	0.140169	0.020034	6.997	< 0.01
	f_2	-0.093573	0.013094	-7.146	< 0.01
	w	0.092756	0.018665	4.969	< 0.01
	a	0.136205	0.015997	8.514	< 0.01
	b	0.837757	0.019003	44.09	< 0.01
	g.l.	5.874431	0.41977	13.99	< 0.01
Errors with distribution Skewed Student-t Distribution	*m*	0.19779	0.03359	5.88800	< 0.01
	d_{Arfima}	0.07921	0.01670	4.74400	< 0.01
	f_1	0.14021	0.02031	6.90300	< 0.01
	f_2	-0.09346	0.01383	-6.75700	< 0.01
	w	0.09278	0.01538	6.03500	< 0.01
	a	0.13615	0.01335	10.20000	< 0.01
	b	0.83783	0.01495	56.04000	< 0.01
	$x_{(\text{asymmetry})}$	0.00331	0.01693	0.19550	0.84500

Table 2. Model ARFI (2)-IGARCH (1, 1)

		Coefficient	*Standard Error*	*T*	*p.value*
Errors with gaussian Distribution	*m*	0.209365	0.032733	6.396	< 0.01
	d_{Arfima}	0.057954	0.021252	2.727	< 0.01
	f_1	0.159542	0.023532	6.78	< 0.01
	f_2	-0.075585	0.014281	-5.293	< 0.01
	w	0.066583	0.01839	3.621	< 0.01
	a	0.15499	0.024227	6.397	< 0.01
	b	0.84501			
Errors	*m*	0.19503	0.03072	6.349	< 0.01

With student_t distribution	d_{Arfima}	0.077855	0.016745	4.649	< 0.01
	f_1	0.1408	0.019534	7.208	< 0.01
	f_2	-0.09323	0.012751	-7.311	< 0.01
	w	0.068614	0.014856	4.619	< 0.01
	a	0.15591	0.018696	8.339	< 0.01
	b	0.84409			
	g.l.	5.280416	0.35709	14.79	< 0.01
Errors with distribution Skewed Student-t distribution	m	0.19824	0.03370	5.882	< 0.01
	d_{Arfima}	0.07808	0.01671	4.673	< 0.01
	f_1	0.14084	0.02032	6.93	< 0.01
	f_2	-0.09310	0.01382	-6.736	< 0.01
	w	0.06867	0.01214	5.65900	< 0.01
	a	0.15583	0.01440	10.82	< 0.01
	b	0.84417			
	$x_{(asymmetry)}$	0.00412	0.01737	0.2375	0.8123

Table 3. Model ARFI (2)-GJR (1, 1)

		Coefficient	*Standard Error*	*T*	*p-value*
Errors with gaussian distribution	m	0.138539	0.037068	3.737	< 0.01
	d_{Arfima}	0.085686	0.025267	3.391	< 0.01
	f_1	0.137237	0.02635	5.208	< 0.01
	f_2	-0.082367	0.015032	-5.479	< 0.01
	w	0.105078	0.02342	4.487	< 0.01
	a	0.076207	0.013083	5.825	< 0.01
	b	0.837309	0.019944	41.98	< 0.01
	g	0.109769	0.024106	4.554	< 0.01
Errors With student_t distribution	m	0.150934	0.034591	4.363	< 0.01
	d_{Arfima}	0.091588	0.017994	5.09	< 0.01
	f_1	0.128465	0.020502	6.266	< 0.01
	f_2	-0.094231	0.013154	-7.164	< 0.01
	w	0.096399	0.01756	5.49	< 0.01
	a	0.086927	0.011644	7.465	< 0.01
	b	0.834155	0.017488	47.7	< 0.01
	g	0.104839	0.018761	5.588	< 0.01
	g.l.	6.132263	0.45675	13.43	< 0.01
Errors with distribution	m	0.152703	0.036977	4.13	< 0.01
	d_{Arfima}	0.091547	0.016926	5.409	< 0.01
	f_1	0.128559	0.020442	6.289	< 0.01

Skewed Student-t distribution	f_2	-0.09415	0.013869	-6.789	< 0.01
	w	0.096365	0.014936	6.452	< 0.01
	a	0.086865	0.011461	7.579	< 0.01
	b	0.834215	0.01442	57.85	< 0.01
	g	0.104789	0.017521	5.981	< 0.01
	$x_{(asymmetry)}$	0.0024	0.016821	0.1427	< 0.01

Table 4. Model ARFI (2)-APARCH (1, 1)

		Coefficient	*Standard Error*	*T*	*p-value*
Errors with gaussian distribution	*m*	0.139045	0.036485	3.811	< 0.01
	d_{Arfima}	0.085162	0.0252	3.379	< 0.01
	f_1	0.137845	0.026315	5.238	< 0.01
	f_2	-0.082235	0.015024	-5.474	< 0.01
	w	0.107979	0.030356	3.557	< 0.01
	a	0.123564	0.017363	7.117	< 0.01
	b	0.835758	0.022072	37.86	< 0.01
	g	0.215185	0.03944	5.456	< 0.01
	d	2.065445	0.24827	8.319	< 0.01
Errors With student_t distribution	*m*	0.150534	0.0347	4.338	< 0.01
	d_{Arfima}	0.091746	0.018056	5.081	< 0.01
	f_1	0.128322	0.020542	6.247	< 0.01
	f_2	-0.094261	0.013155	-7.166	< 0.01
	w	0.094195	0.019304	4.88	< 0.01
	a	0.13529	0.014601	9.266	< 0.01
	b	0.835505	0.018452	45.28	< 0.01
	g	0.19799	0.032276	6.134	< 0.01
	d	1.949414	0.1791	10.88	< 0.01
	g.l.	6.134289	0.45782	13.4	< 0.01
Errors with distribution Skewed Student-t Distribution	*m*	0.15230	0.03706	4.11000	< 0.01
	d_{Arfima}	0.09171	0.01695	5.41100	< 0.01
	f_1	0.12841	0.02045	6.27800	< 0.01
	f_2	-0.09418	0.01387	-6.79100	< 0.01
	w	0.09415	0.01673	5.62800	< 0.01
	a	0.13521	0.01342	10.08000	< 0.01
	b	0.83557	0.01518	55.04000	< 0.01
	g	0.19804	0.03266	6.06300	< 0.01
	d	1.94910	0.19131	10.19000	< 0.01
	$x_{(asymmetry)}$	0.00239	0.01682	0.14230	0.8868

As previously stated the statistical robustness of each VaR model to adequately estimate market risk is determined in terms of the failure rate and the p-values from the Kupiec test. The failure rate is defined as the percent of empirical returns that exceeds the estimated VaR for any investment position. In this respect, a failure rate larger than the α% level leads to sub estimate risk from the return series; similarly, a failure rate smaller than α% level overestimates risk measures from the applied VaR model. Furthermore a p-value smaller or equal to 0.05 is enough evidence to reject the null hypothesis about the statistical robustness of the VaR models to measure risk exposure.

Table 5. Frequency of exceptions that returns exceed VaR levels

α(%)	5%		2.5%		1%		0.5%	
Positions	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**
Panel A								
GARCH_n	4.77(4)	4.38(10)	2.86(8)	2.44(2)	1.70(8)	1.26(5)	1.14(8)	0.84(6)
GARCH_t	5.29(5)	4.94(1)	2.66(6)	2.30(5)	1.15(4)	0.86(2)	0.53(2)	0.37(2)
GARCH_st	5.30(6)	4.92(2)	2.66(6)	2.26(6)	1.17(5)	0.86(2)	0.55(3)	0.37(2)
Panel B								
IGARCH_n	4.50(7)	4.09(11)	2.69(7)	2.12(7)	1.51(6)	1.11(1)	1.01(6)	0.67(3)
IGARCH_t	4.99(2)	4.59(8)	2.52(1)	2.03(8)	0.93(2)	0.67(6)	0.38(4)	0.25(4)
IGARCH_st	4.99(2)	4.57(9)	2.52(1)	1.97(9)	0.95(1)	0.67(6)	0.38(4)	0.25(4)
Panel C								
GRJ_n	4.40(8)	4.77(6)	2.69(7)	2.64(3)	1.54(7)	1.39(8)	0.99(5)	0.86(7)
GRJ_t	4.93(3)	5.23(6)	2.56(2)	2.47(1)	1.08(3)	0.83(3)	0.49(1)	0.40(1)
GRJ_st	4.99(2)	5.17(3)	2.59(4)	2.44(2)	1.08(3)	0.81(4)	0.49(1)	0.40(1)
Panel D								
APARCH_n	4.40(8)	4.75(7)	2.69(7)	2.67(4)	1.54(7)	1.38(7)	1.02(7)	0.83(5)
APARCH_t	4.93(3)	5.21(5)	2.58(3)	2.44(2)	1.08(3)	0.83(3)	0.49(1)	0.40(1)
APARCH_st	5.00(1)	5.18(4)	2.61(5)	2.44(2)	1.08(3)	0.83(3)	0.49(1)	0.40(1)

Numbers in parenthesis indicate the best model for the long/short position and the ? (%)

Table 6. Results from the *p*-values for the Kupiec Test

VaR	95%		97.5%		99%		99.5%	
Positions	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**	**Long**	**Short**
Panel A								
GARCH_n	0.3757	0.0174	0.0659	0.7618	0	0.0402	0	0.0003
GARCH_s	0.2868	0.8339	0.3909	0.2730	0.2123	0.2314	0.7041	0.1125
GARCH_st	0.2627	0.7476	0.3909	0.2088	0.1727	0.2314	0.5838	0.1125
Panel B								
IGARCH_n	0.0555	0.0004	0.3123	0.0384	0.0001	0.3707	0.0000	0.0655

IGARCH_t	0.9666	0.1164	0.9302	0.0103	0.5736	0.0033	0.1621	0.0014
IGARCH_st	0.9666	0.1037	0.9302	0.0037	0.6614	0.0033	0.1621	0.0014
Panel C								
GRJ_g	0.0203	0.3757	0.3123	0.4808	0	0.0023	0	0.0002
GRJ_t	0.7904	0.3979	0.7488	0.8836	0.5106	0.1456	0.8932	0.2258
GRJ_st	0.9666	0.5321	0.6351	0.7618	0.5106	0.1128	0.8932	0.2258
Panel D								
APARCH_n	0.0203	0.3459	0.3123	0.3909	0	0.0032	0	0.0005
APARCH_t	0.7904	0.4293	0.6910	0.7618	0.5106	0.1456	0.8932	0.2258
APARCH_st	0.9889	0.4965	0.5813	0.7618	0.5106	0.1456	0.8932	0.2258

Examining results reported in Tables 5 and 6 it can be observed that the symmetrical models based on the normal conditional distribution show a low statistical potential to estimate VaR for both the long and short positions. In this case sub estimates of risk exposure are highly noticeable at the 99% y 99.5% confidence levels. The GARCH, IGARCH and APARCH models overestimate VaR at confidence levels of 95 percent for both positions. This fact is frequently expected as a result from the excess in kurtosis and the different levels of asymmetry that present returns from financial series. Estimates from the symmetrical models based on the normal conditional distribution are not rejected at a (lower) confidence level of 97.5%, since they show a high rate of successes to measure risks for both positions, except for the case of the model IGARCH for the short position. Thus, estimates from the models based on the assumption of normality can cause investors to experience very large losses due to sub estimations of risk.

The asymmetrical models based on the conditional *student-t* distribution proportionate better estimates about risk for both positions for all confidence levels, except for the IGARCH model that overestimates risk for the short position at confidence levels of 97.5%, 99% y 99.5%. Generally, the p-values support the contention that these alternative models correctly capture the heavy or fat tails behavior from the distribution of returns, caused by atypical movements, particularly for the right tail where for the coefficient of successes a rate of 100 percent is obtained for the ARCH models, which is shown in Table 6. Nevertheless, it is worth pointing out that financial theory has empirically demonstrated inefficiency of these models to correctly model volatility clusters of financial returns (Bollerslev, 1986; Baillie and DeGenaro, 1990; and De Jong, Kemma y Kloek, 1992)

Analyzing results from the failure rate and the *p*-values. It can be observed that the models based on GARCH, GRJ, APARCH volatilities for both the student-t and the skewed student-t distributions produce not only better, but also similar VaR estimates. This fact is confirmed statistically by the closeness between the failure rates and the p-values from the Kupiec test, as it can be observed in Panels A and B. However, this high performance is not fully satisfactory for all the models. For instance the GJR model with normal innovations underestimates risk quantifications for negative and positive returns for confidence levels of 99.0 and 99.9 percent, while it overestimates risk for negative returns at confidence levels of 95 percent. Similarly the

IGARCH model applied with a skewed student-t distribution overestimates risk for the short position for confidence levels of 97.5%, 99% and 99.5%.

Finally, results in parenthesis from Table 5 confirm the potential of the asymmetrical models to quantify VaR satisfactorily, particularly for the skewed student-t distribution, for all confidence levels and financial positions; these models are always on top of a ranking of all models tested, as shown by the proximity between the expected and real failure rates and the high p-values from the Kupiec tests.

Conclusions

This work employed ARCH models from the ARCH family, based on the normal, Student-t and skewed Student-t distributions in order to test the behavior of volatility in the presence of long memory effects on the returns, as well as for analyzing Value at Risk applying those models for both the long and short positions. For the case of the Mexican stock market returns. Although empirical results vary for each position and confidence levels, the differences among the applied models is not significant among several cases.

It is worth noting the significance of long memory of returns from the return series from the Mexican shares market, that is time dependency of returns, signals the presence of significant autocorrelations among returns even though observations might be distant over time. This implies that it is possible to predict future prices and extraordinary gains could be obtained trading in this market, contrary to what the efficient markets theory points out. Similarly in terms of risk analysis and administration potential losses are also linked to the long and short memory of a market. Thus, predicting potential losses integrating into the analyses the long and short memory for the returns and volatility of financial assets must produce better, more conservative and reliable results than those obtained applying traditional VaR methodologies.

Nevertheless, the empirical evidence shows that the asymmetrical models show a great potential to quantify in a more precise way real risk from positive and negative returns for any confidence level, particularly when the skewed student-t distribution is used, for the case of emerging markets such as the Mexican bourse where returns are characterized by high time dependent volatility, excesses in kurtosis, and different levels of asymmetry. This study is therefore relevant for institutional investors participating at emerging markets because lots of empirical studies have demonstrated that these economic agents are exposed to risks derived not only from stock markets cracks, but also from economic booms.

In short, results from this study provide strong evidence signaling that that the asymmetrical GARCH models are more reliable to estimate VaR than that the GARCH symmetrical models, as well as for estimating minimum capital requirements for financial institutions, for any confidence level and for both the long and short positions. This is particularly the case of the GJR and APARCH models implemented with the skewed student-t distribution. Findings reported in this paper clearly show the high potential of the GARCH asymmetrical models to capture more efficiently different levels of asymmetry and excesses in kurtosis from returns from the Mexican stock market, generated by downfalls and booms from the economy.

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