

# A Multi-Objective Model to Single-Allocation Ordered Hub Location Problems by Genetic Algorithm

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## Abstract

Most of supply chain and Hub location problems involve several conflicting objectives hence requiring a multi-objective formulation. Normally, Multi-objective approaches lead to the maximization of a weighted sum of score functions. Since normalizing these functions and quantifying the weights is not a straightforward process, such approaches are poor in practice. In this research, this difficulty is overcome by using a modified genetic algorithm for evaluation of solutions. Several qualitative and quantitative objectives are considered referring to layout model that also allows practical constraints take into account. Due to these constraints, many of strings in the population resulting from this model may be in infeasible regions; the common approach to solve such problems is to omit infeasible solutions. However, as these solutions may have useful criteria that can improve the average fitness of the population, they can be used to achieve better solutions. The proposed model uses a graded penalty term to penalize infeasible solutions to pressure the search towards feasible regions and subsequently uses their useful criteria.

**Keywords:** Supply chain management; Hub Location; Multiple objective programming; Genetic algorithms

## 1. Introduction

The Hub location can have a large impact upon the effectiveness and efficiency of supply chain. So the Hub location has generally been recognized as an important issue in modern manufacturing systems. The Hub location problem is a long term, costly proposition, and any

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modifications or rearrangements of the existing layout represent a large expense and cannot be easily accomplished. Hence, an efficient Hub location method can reduce these costs, and thus increase productivity (Sule, 1994).

Two basic approaches have most commonly been used to generate desirable Hub location: a qualitative one and a quantitative one. The qualitative ones provide a layout based on the closeness rating between the Hubs. The quantitative ones involve the minimization of the total material handling cost between the Hubs and the costumers. (Sha and Chien-Wen, 2001) However, the actual problem involves several conflicting objectives hence requiring a multi-objective formulation. Multi-objective approaches, recently proposed, in most cases lead to the maximization of a weighted sum of score functions. The poor practicability of such an approach is due to the difficulty of normalizing these functions and of quantifying the weights. (Aiello et al. 2006) Unlike classical approaches that tend to maximize the efficiency of the supply chain measured by a unique function; this paper presents a new approach for combining the quantitative and qualitative objectives to solve the Hub location problem.

When the number of Hubs is less than 15, mathematical approaches like Quadratic assignment are suitable to reach an optimal solution. However, when the number of Hubs is more than 15, it is impossible to solve problem with such approaches. As the number of departments increasing, the computational time is exponentially increased rapidly. (Sule, 1994) As a result suboptimal solutions need to be considered for large Hub location problems since optimal algorithms are computationally infeasible. There are lot of heuristic approaches has been developed to get the near-optimal solution, such as simulation annealing, tabu searching, and genetic algorithms. Genetic algorithms are one of the most well known classes of Evolutionary algorithms. The potential of such algorithms to yield good solutions even for hard optimization tasks has been demonstrated by various applications. (Bodenhofer, 2004)

The search space resulting from this highly constrained problem may include substantially large infeasible regions. Hence, many of strings in the population resulting from this search space may be in infusible reigns; the common approach to solve such problems is to omit infusible solutions. However, since these solutions may have useful criteria that can improve the average fitness of the population, they can be used to achieve better solutions. The proposed model uses a graded penalty term to penalize infeasible solutions, to pressure the search towards feasible regions and subsequently uses their useful criteria.

In previous studies, there are various approaches to generate solutions for the Hub location problem. The literature of hub location covers a large variety of models where the main goal is to minimize some globalizing function of the operation costs. A review of some classical approaches can be found in Puerto et al (2011).

Campbell (1994) classifies hub location problems according to their optimization criteria: (i) minimization of the total transportation cost,  $p$ -hub median problem (which is the original model proposed in O'Kelly, 1987); (ii) minimization of the total transportation cost and the fixed cost of establishing hubs, incapacitated/capacitated hub location problem, (iii) minimization of

the maximum transportation cost,  $p$ -hub center problem, and (iv) minimization of the number of hubs while serving each pair within a predetermined bound, hub covering problem.

This paper is organized as follows: Section 2 presents the problem formulation. While the proposed solving approach based on genetic algorithm is described in Section 3. The implementation and experimental results of the proposed approach are summarized in Section 4. Finally, the concluding remarks are given in Section 5.

## 2. Problem formulation

In this problem,  $n$  Hubs with the area of  $S_i$  ( $i=1,\dots,n$ ) must be located in  $m$  suggested locations with the area of  $b_j$  ( $j=1,\dots,m$  &  $\sum_{i=1}^n S_i \leq \sum_{j=1}^m b_j$ ) and the demand of  $o$  retailer must assign to them. The objective functions are minimization of transportation costs and minimization backlogs.

Due to this description the proposed model is as follows:

$n$  : Number of Hub

$m$  : Number of suggested locations

$o$  : Number of retailer

$S_i$  : Area of Hub  $i$

$ls_j$  : Area of suggested location  $j$

$d_{kj}$  : Distance between the retailer  $k$  and suggested location  $j$  using a pre-specified metric

$f_{ik}$  : Material flow between Hub  $i$  and retailer  $k$

$c_{jk}$  : The cost to move one unit load one distance from suggested location  $j$  to retailer  $k$

$Dr_k$  : Demand of retailer  $k$

$MD_i$  : Maximum capacity of Hub  $i$

- All of the areas of departments and suggested locations, distances, material flows, costs and perimeter lengths are equal or greater than zero.

$$x_{ij} = \begin{cases} 1 & \text{if Hob } i \text{ located in location } j \\ 0 & \text{Otherwise} \end{cases}$$

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}$$

$$b_{ki} = \begin{cases} 1 & \text{if Demand of retailer } k \text{ assigned to Hub } i \\ 0 & \text{Otherwise} \end{cases}$$

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1o} \\ \vdots & & \vdots \\ b_{o1} & \cdots & bk_{on} \end{bmatrix}$$

## Objective functions

Minimization of transportation costs:

$$\min : Z(X) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^o (x_{ij} \times b_{ki}) \times (f_{ik} \times c_{jk}) \times d_{kj} \quad (1)$$

Due to equation (1),  $(x_{ij} \times b_{ki}) = 1$  only if: Hub  $i$  located in suggested location  $j$  ( $x_{ij} = 1$ ) and demand of retailer  $k$  assigned to Hub  $i$  ( $b_{ki} = 1$ ). The material handling cost is material flow between hub  $i$  and retailer  $k$  ( $f_{ik}$ ) multiplied by cost to move one unit load one distance from suggested location  $j$  to and retailer  $k$  ( $c_{jk}$ ) multiplied by distance between the center of suggested location  $j$  and retailer  $k$  ( $d_{kj}$ ).

Minimization backlogs:

$$\min : Z(X) = \sum_{k=1}^o Dr_k - \sum_{i=1}^n \sum_{k=1}^o f_{ik} \quad (2)$$

Due to equation (2), backlog is total sum of retailers' Demand ( $\sum_{k=1}^o Dr_k$ ) minus total material

flow between Hubs and retailers.

### Feasible solution:

Two matrixes:  $X = \begin{bmatrix} x_{11} & \dots & x_{1m} \\ \vdots & & \vdots \\ x_{n1} & \dots & x_{nm} \end{bmatrix}$  and  $B = \begin{bmatrix} b_{11} & \dots & b_{1o} \\ \vdots & & \vdots \\ b_{o1} & \dots & b_{on} \end{bmatrix}$  where  $x_{ij}$  &  $b_{ij} \in \{0,1\}$  such that:

$$\sum_{j=1}^m x_{ij} = 1 \quad \forall i = 1, 2, \dots, n \quad (3)$$

$$\sum_{i=1}^n b_{ki} \times f_{ik} \leq Dr_k \quad \forall i = 1, 2, \dots, n \quad (4)$$

$$\sum_{k=1}^o b_{ki} \times f_{ik} \leq MD_i \quad \forall k = 1, 2, \dots, o \quad (5)$$

$$x_{ij} s_i \leq b_j \quad \forall i = 1, 2, \dots, n \ \& \ \forall j = 1, 2, \dots, m \quad (6)$$

**Optimal solution:** A feasible matrix that gives the best value for objective functions.

## 3. Solving Approach

### 3.1. Adjusting the Model for Genetic Algorithm

The problem needs to be encoded in such a way that the genetic algorithm can be applied to it. The matrixes are used in the encoding step. Each string  $X = \begin{bmatrix} x_{11} & \dots & x_{1m} \\ \vdots & & \vdots \\ x_{n1} & \dots & x_{nm} \end{bmatrix}$  in the population represents a possible solution. If the position  $ij^{th}$  has the value of 1, i.e.  $x_{ij} = 1$  then

the hub  $i$  located in suggested location  $j$  otherwise it is not.

Similarly, each string  $B = \begin{bmatrix} b_{11} & \dots & b_{1o} \\ \vdots & & \vdots \\ b_{o1} & \dots & b_{on} \end{bmatrix}$  in the population represents a possible solution. If

the position  $ij^{th}$  has the value of 1, i.e.  $x_{ij} = 1$  then Demand of retailer  $k$  assigned to Hub  $i$ .

### 3.2. ELECTRE Method

ELECTRE is a multi-criteria decision-making procedure that can be applied when a set of alternatives must be ranked according to a set of criteria reflecting the decision maker's preferences. Relationships between alternatives and criteria are described using attributes referred to the aspects of alternatives that are relevant according to the established criteria. In multi-criteria decision problems, although logical and mathematical conditions required to determine an optimum do not exist, a solution representing a good compromise according to the conflicting criteria established can be individuated. ELECTRE method is based upon pseudo-criteria. A pseudo-criterion allows, by using proper thresholds, to take into account the uncertainty and ambiguity that can affect the evaluation of the performance, so that, if the difference in the performance of two alternatives is minimal, according to a certain criterion, such alternatives can be considered indifferent according to that criterion. Another peculiarity which differentiates ELECTRE from other methodologies is that it is not compensative, which means that a very bad score in one objective function is not compensated by good scores in other objectives. In other words, the decision maker will not choose an alternative if it is very bad compared to another one, even on a single criterion. This occurs if the difference between the values of an attribute of two alternatives is greater than a fixed veto threshold.

ELECTRE is based upon outranking relations: an alternative  $a$  outranks another alternative  $b$  if sufficient reasons exist to assert that  $a$  is as good as  $b$  and good reasons to reject such assertion do not exist. Outranking is therefore based upon concordance/discordance principle, which consists in the verification of the existence of a concordance of criteria in favour of the assertion that an alternative is as good as another one, and upon the verification that strong discordance among the score values that may reject the previous assertion does not exist.

For each criterion, the following thresholds are introduced:

$q_j$  : Indifference threshold

$p_j$  : Preference threshold

$v_j$  : veto threshold

Where:  $q_j \leq p_j \leq v_j$

### 3.3. Suggested Algorithm

The suggested algorithm is as follows:

**Step 1:** Randomly create initial population ( $P_0$ ) of  $\mu$  individuals

**Step 2:** At each generation  $j$ , randomly couple all solutions to form  $\frac{\mu}{2}$  couples. Generate two children from each couple by crossover operator to obtain offspring population of  $\mu$  individuals

**Step 3:** combine these two populations together and form a unique population  $P_j$  of size  $2 \times \mu$

**Step 4:** keep the number of clones below a fixed value by means of recursive mutations

**Step 5:** Select the best  $\mu$  individuals from the population  $P_j$  to constitute the new parent population  $P_{j+1}$  by ELECTRE method.

**Step 6:** apply mutation operator to the individuals with a fixed probability

## 5. Conclusion

In this paper, a new approach to un-equal Hub location with both quantitative and qualitative objectives has been proposed. To achieve better solutions, infeasible regions were also taking in to account and subsequently a graded penalty term was used to penalize infeasible solutions. The proposed approach seems to be an effective and efficient tool to support supply chain managers in determining the most optimal location for supply chain hubs through a multi-objective approach.

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