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Solving Mathematical Problems among College Students: Process or Strategy?

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Abstract
Solving mathematical problems is quite unique as solution can be done with multiple ways of strategies and follow the process. The purpose of this paper is to review the definitions of solving mathematical problems among college students. This review article based on the terms of mathematical process, mathematical models or mathematical strategies of solving mathematical problems among researchers and scholars. There are two types of problem solving definitions among researchers and scholars which called process and strategy. In the problem solving processes there are also solving strategies but there are a lot of mathematical strategies that can be used to solve mathematical problems. Various terms are used by mathematicians despite the fact that the process performed to solve the problem is the same. This review paper will contribute to the literature on mathematical content of problem solving models. Solving strategies in this article only limited in mathematical problems. This study makes a new contribution to mathematical problem solvers to identify what are they used in making solutions whether it is a process or strategy.
Keywords: Mathematics, Problem Solving Strategy, Problem Solving Process, Solution Strategy, Review Paper

Introduction

Problem solving has generally been accepted as a means for advancing thinking skills and very important learning process in the mathematics curriculum (NCTM, 2010). Based on the Malaysian Education Development Plan, problem solving is an element contained in 21st century skills, which is one of the focus of mathematics learning (Kementerian Pendidikan

Malaysia, 2013). Therefore, problem solving skills among students must be taught from young as problem solving is closely related to word problems.

Problem solving is very important for intellectual development, gaining the ability and point of view of mathematics teaching (Karasel, Ayda, & Tezer, 2010). Problem solving is a specific and interesting field of mathematics as it is a topic that may be difficult for beginners as well as skilled as for those in the process of building a growing understanding of mathematics (Goldin, 1997). Furthermore, problem solving can be understood as a process where data are used in a new and unknown situation (Nik Pa, 2008).

Mathematical problem solving is difficult for most students, probably because solving it requires the problem solver to think abstractly about the situation, and then having to model the situation (object-based representation) using mathematical concepts (Corter & Zahner, 2007; Edens & Potter, 2008; Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016; Pelczer, Singer, & Voica, 2014; Perdomo-Díaz, 2017). Usually the problem solver will use the representation generates from the problem scheme. Problem schema is an important part of the knowledge we use to solve problems. This scheme is formed by the mental representation of a problem solver. When we think about a problem normally, the problem we represent has a goal and can also include representation from the start, the operator and the initial restriction, which is a medium of thought (Goldin, 1998; Hayes, 1981).

In the beginning of problem solving, students deal with problems where they need only to have one insight in order to find solution (Pehkonen, 2011). Some mathematical scholars named it as mathematical problem solving processes (Mayer, 1989; Polya, 1945; Schoenfeld, 1985). The process in solving problems fixed and used by practitioner in teaching. Recently, literature showed that one of mathematical problem solving process mean a lot to solution strategies (Charles, Lester, & O'Daffer, 2005; Posamentier & Krulik, 1998; Schoenfeld, 2013). Unfortunately, students did not identify any process or strategies in solving problems as long as they understand the concept and do the solution correctly in their way. So, in this article we try to review articles based on problem solving process and the strategies used by students and problem solver.

Mathematical Problem Solving Process

Mathematical problem solving involves several processes that have been used for centuries. Table 1 shows some of mathematical problem solving processes or models that have been introduced by mathematicians and practiced to this day. Based on the table, heuristic problem solving models commonly used in curriculum mathematics ranging from primary to secondary school show a detailed problem solving process.

The Polya model introduced in 1945 is a common problem-solving activity consisting of four steps namely; (1) Understand, (2) Plan, (3) Do, and (4) Check. The solution steps built form the basis of the problem-solving process not only in the field of mathematics but across other curriculum areas. Psychologists argue that effective problem-solving strategies are specific to the field, i.e. problem-solving strategies in mathematics are unique to mathematics, strategies in art are unique to art and so on (Woolfolk, 1998).

Table 1

Analysis of Mathematical Problem Solving Process

Polya (1945)	Schoenfeld (1985)	Mayer (1989)	Lester (1994)	Krulik & Rudnick (1995)
1.Understand problem 2.Plan strategy 3.Solve problem 4.Recheck	1.Understand problem 2.Choose strategy 3.Solve problem 4.Check solution	1.Translate problem 2.Problem integration 3.Plan solution 4.Implement solution	1.Identify problem 2. Understand problem 3.Analyze goal 4.Plan strategy 5.Solve 12 strategies 6.Evaluate the answer	1.Read and think 2.Explore and plan 3.Choose strategy 4.Find and answer 5.Reflect and change

Polya (1945) states that modern heuristics are trying to understand the problem-solving process, especially the mental operations commonly performed in mathematical problem-solving processes (Schoenfeld, 1985). However, Schoenfeld (1985) still retains the four steps of the problem-solving process by changing the second step, which is to choose an approach. The choice of approach requires the problem solver to choose a strategy or method to use to continue the problem-solving process. Schoenfeld states that choosing the right solution helps in getting a successful solution.

Meanwhile, the cognitive model of the process by Mayer (1989) for solving mathematical problems involves four steps; namely (1) the problem representation translated from the translation of the problem. This refers to adequate mental and physical development to represent problem information, (2) problem integration involves the selection of solution methods, (3) solution planning, (4) implementation solution that also involves iterative processes carried out throughout the overall solution process. The effectiveness of this model depends on the instructions given by the teacher to students especially for weak students. Therefore, this model is not suitable for use at all levels of students.

Lester (1994) argues that a problem can be used as a way to learn the content of mathematics. He also proposed a mathematical problem-solving process involving six steps; namely (1) identifying problems, (2) understanding problems, (3) analysing goals, (4) planning strategies, (5) implementing 12 strategies, and (6) re-evaluating final answers. The fifth step explains that there are various strategies that can be used to solve a problem. Lester states that the process of problem solving involves the use of correct and relevant strategies only to achieve a solution. Among the problem solving strategies discussed can be referred to in Table 2. However, he still stressed that the problem-solving process is different from the strategy to solve the problem because the strategy is a step by step process.

Later, Krulik and Rudnick (1995) introduced five levels of problem solving known as heuristics. First step; involves students having to identify facts, identify questions, describe situations, explain settings, and determine next actions. The second step; need to organize information, find out if there is appropriate or necessary information, determine if there is unnecessary information, describe problem models, as well as create charts, tables, or images. The third step involves the selection of strategies; such as finding or making patterns, working backwards, trying and doing, simulating or experimenting, simplifying or expanding, making sequential lists, logic, as well as dividing or categorizing the problem into simple things.

The fourth step; find and answer questions involving prediction, use numeracy, use algebraic abilities, use geometric abilities, and use calculators if necessary and the fifth step is to check answers, determine alternative solutions, apply answers to other situations, state responses (generalization or conceptualization), discuss answers as well as creating various problems from the root of the problem. This proposed problem-solving process gives the problem solver the freedom to use each step without having to follow it in order.

Table 1 summarizes the analysis made on the mathematical problem solving process. From the analysis, the steps used as a mathematical problem-solving process are the same, but different in name. The difference in the name of the step used may be due to the culture, place and environment of the relevant mathematician.

In Malaysia, the main model used in the mathematics curriculum is the Polya's Model which emphasizes four phases of problem solving including understanding of problems, planning strategies about problems, implementing planned strategies and recalling (Bahagian Pembangunan Kurikulum, 2012, 2013). Each phase of problem solving has several questions or suggestions asked to help students understand the problem and get a solution to the problem.

Mathematical Problem Solving Strategy

Other mathematicians are of the opinion that the problem-solving process is a strategy for solving problems. The term strategy or activity to solve this problem has various ways and methods. Table 2 gives a clear picture of some mathematical problem solving strategies introduced by researchers in the field of mathematics.

Based on the analysis of mathematical problem-solving process tables and mathematical problem-solving strategies, Lester (1994) introduced six problem-solving processes. He also introduced problem-solving strategies consisting of eleven types of strategies. All of the proposed strategies can be used during the mathematical problem-solving process. The solution strategy introduced by Lester also has the characteristics of a solution process but he does not equate a solution strategy as a problem-solving process.

Table 2
Analysis of Mathematical Problem Solving Strategies

Lester (1994)	Posamentier & Krulik (1998)	Charles, Lester, & O'Daffer (2005)	Schoenfeld (2013)
1.Guess, Check, Recheck 2.Draw picture 3. Action problem 4.Use object 5.Choose operation 6.Create table 7.Find patterns 8.List of outcome 9.Write equation 10.Logical reasoning 11.Work backward	1.Work backward 2.Find pattern 3.Adopt different views 4.Solve analog problem 5.Look at extreme cases 6.Draw picture 7.Trial and error 8.List of outcome 9.Arrange data 10.Logical reasoning	1.Guess, Check, Recheck 2.Draw picture 3.Action problem 4.Use object 5. Choose operation 6.Create table 7.Find pattern 8.List of outcome 9.Write equation 10.Logical reasoning 11.Work backward	1.Understand problem 2.Choose an approach i) Guess, Check, Recheck ii) Draw picture iii) Action problem iv) Use object v) Choose operation vi) Create table vii) Find pattern viii) List of outcome ix) Write equation x) Logical reasoning xi) Work backward 3.Solve problem 4.Check solution

Posamentier and Krulik (1998) also introduced specifically problem-solving strategies for students to practice using them. There are ten problem solving strategies that can be used in math class. They insist that there is no unique way to solve a problem. As a rule, students are encouraged to consider alternative solutions to problems. This means that solutions made in the classroom should be compared to standard solutions (examples of solutions found in textbooks or given by teachers). Many problems require more than one strategy as a solution. The nature of a problem is to determine the best strategy to use to solve the problem.

Past studies have shown a variety of solution strategies used in direct and indirect mathematics classes a decade ago (available in the teaching and learning of mathematics, textbooks and reference books) (Charles et al., 2005; Lester, 2007; Posamentier & Krulik, 1998; Schoenfeld, 2013). Thus, the use of strategies to solve problems is no stranger among students when solving problems. Charles et al. (2005) stated that in the 20th century the use of mathematical problem solving strategies was widespread and almost identical in mathematics classes with different strategy names depending on the situation and country. Therefore, to facilitate the use of strategies in mathematical problem solving classes, this researcher suggested that an evaluation of the use of solution strategies be made. Each type of solution strategy used by students is reviewed based on rubric scores to facilitate assessment (Charles et al., 2005).

Schoenfeld (1985, 2013) states that problem solving strategy is a tool to sort out something, see what makes a mathematical object with its relationship signs. Mathematics is about systematic exploration and research on mathematical objects, he added. In doing math, we explore, we make systematic, we make predictions and we use problem-solving techniques in the service of making and validating those predictions.

Thus, there is no unique way to solve a problem, moreover most problems may require more than one solution strategy (Posamentier & Krulik, 1998). Krawec, Huang, Montague, Kressler,

and De Alba (2013) argue that students know about solution strategies but do not know or do not realize which strategies are effective to use when performing mathematical tasks. The skill of choosing the right strategy depends on how much experience we have solving problems before.

Discussion

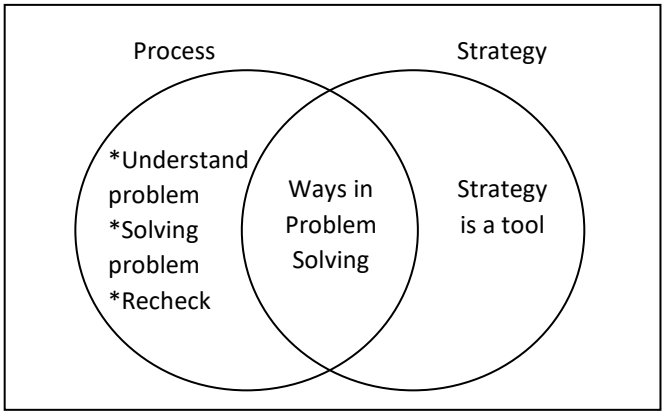

Figure 1: Diagram of the Review Finding

Based on Figure 1 above, the problem-solving process has similarities to problem-solving strategies, namely problem-solving methods (Charles et al., 2005; Lester, 2007; Posamentier & Krulik, 1998; Schoenfeld, 1985). Overall, the problem-solving strategy is more comprehensive because it covers a variety of things including how to solve problems of various types as in table 2, but problem solving strategies are still considered a helpful tool in the problem-solving process (Schoenfeld, 2013).

Based on previous surveys in mathematical problem solving, it can be concluded that the problem-solving process requires a set of systematic activities where it has logical planning and produces alternatives including strategies to be used and selection of appropriate methods to implement it. Each problem-solving process requires a method or strategy to solve the problem, perform calculations to achieve the goal and review the implementation that has been done to get the correct answer. The problem solver needs to choose the appropriate process and implement each plan as best as possible.

Based on previous studies in mathematics learning, especially on the difficulty of students solving problems, among the aspects that are given attention is the process used by students while solving problems. Although, instruction after instruction is given, students still do not successfully master the problem-solving skills. There are various problem-solving strategies suggested by mathematicians as a teaching aid or process that students need to use. However, these problem solvers or students are given the freedom to choose the way, technique, strategy, heuristic, step or whatever name to help them solve this problem. Whatever method is chosen depends on the willingness and existing knowledge to answer the problematic question.

Conclusion

Various terms are used by mathematicians despite the fact that the process performed to solve the problem is the same. Previous studies also show the importance of strategies used in problem solving. However, the lack of in-depth studies on the use of strategies in problem solving at the college level dominates past studies. The need for empirical research can

contribute to the increase in research in this field as well as improve the problem-solving process practiced in mathematics classes in Malaysia. Studies also can be suggested to the literature on mathematical content of problem solving models.

Contribution

Based on the review of this study explains in depth about the strategies and processes in solving mathematical problems. The two are interrelated in different terms based on the culture of the students. Those past researches that have been discussed provide complete evidence for further study in more depth.

Problem solving process and solution strategies can be as a tool for representing the problem mathematically or as a tool for calculating the actual solution to the problem.
The study is significant because it gives new knowledge to those not taking mathematics in college and also those who have no mathematics background.

Studying how students solve mathematics problems is a complex endeavour. This is because students can use any several solution methods or strategies for many problems. Also an individual student may switch approaches across similar problems or even during the solution of a single problem. These issues arise in studying how people solve other types of mathematics problem. Results from this research could prompt educators which terms they need to use in their daily mathematics problem solving lessons.

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